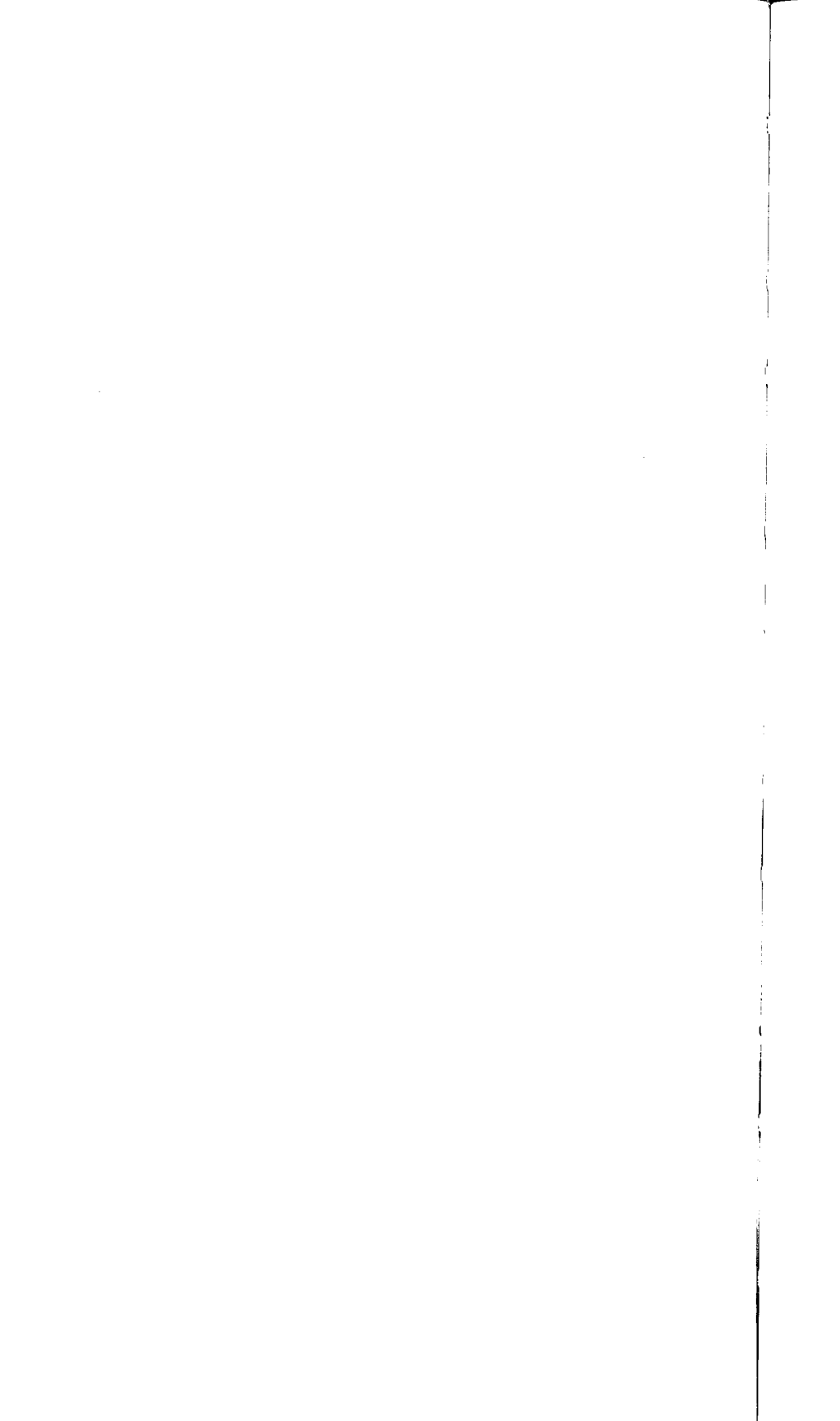


NECESSITY
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NECESSITY
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CONTINGENCY
THE MASTER ARGUMENT

Jules Vuillemin

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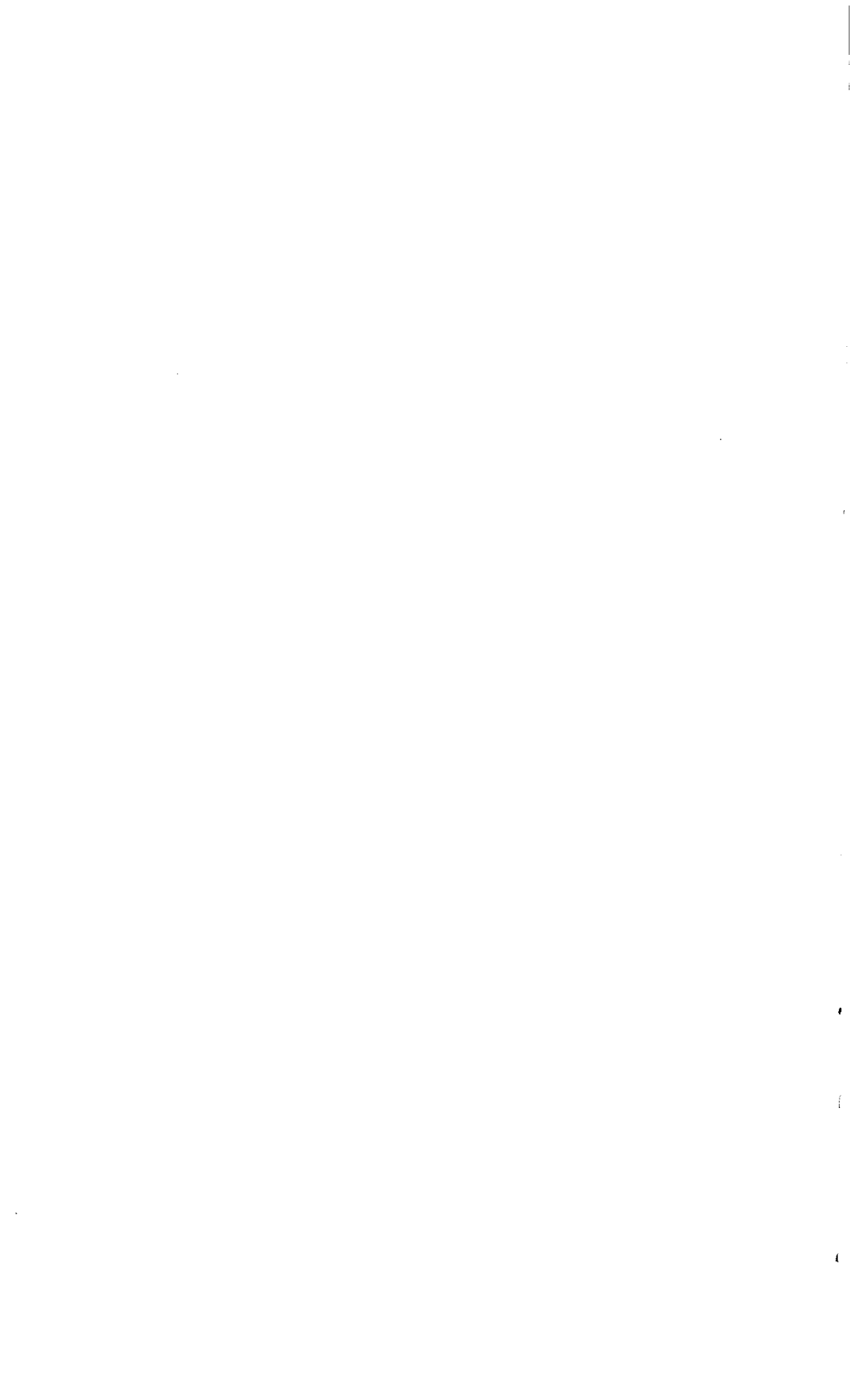
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Introduction

The Master Argument, recorded by Epictetus, indicates that Diodorus had deduced a contradiction from the conjoint assertion of three propositions. Epictetus adds that three solutions of the aporia had been obtained by denying one or another of the propositions advanced while maintaining the other two.

The argument, which has to do with necessity and contingency and therefore with freedom, has attracted the attention of logicians above all. In any case there have been many attempts at reconstructing it in logical terms, without excessive worry about historical plausibility and with the foregone conclusion that it was sophistic since it directly imperiled our common sense notion of freedom.

On both of these counts I have taken exception to recent tradition. The success of the argument with the Ancients, and with Ancients who were no mean logicians, seemed reason for presuming that the Master Argument is not sophistic and that the contradiction it produces is a real one. On the other side, I looked for a classical text containing the propositions stated by Epictetus and which could have furnished Diodorus with the material for his argument. I believe to have found such a text in Aristotle's *De Caelo*.

In order to demonstrate the contradiction in the propositions thus restored, I had in my turn to translate them into logical terms. It is unlikely that Diodorus proceeded in such a way. Although the translation I have proposed tries to remain faithful to its models as they have been handed down to us, it inevitably gives them a precision they did not have in themselves. This indulgence in precision amounts to historical inexactitude but seemed necessary nevertheless as it had to do with restoring a reduction to the absurd.

There is one distinctive feature of the translation that must be noticed. The propositions figuring in the Master Argument are in-

terpreted in terms of temporal modal logic where both the modalities and the statements they govern have chronological indices. This means that the force of the argument comes not from purely logical or modal considerations, but from our experience of time.

To bring to its complete end the research undertaken here, I would have had to assign to the Master Argument and to each of the solutions it is susceptible of, an explicit axiomatic system formalized according to a set of rules. For want of competence, of stamina, of time, I have been content to formulate only what was needed to elucidate the argumentation.

Justification for the first four chapters is to be found in the Epictetus passage. The remaining chapters extend the debate about the Master Argument to Greek philosophy at large. In this way it is seen that principles are challenged—even logical ones at that—which are not mentioned in the Epictetus passage but which must have played their role in the argument. The reader will judge whether that extension is legitimate or not. As one well imagines, the debate on the issue was continued by the philosophers of the Middle Ages and the Moderns. This was quite generally done in ignorance of the Master Argument itself; but I have had no hesitation in appealing to them where they might be apt to help explain or specify the position of the Ancients.

Acknowledgements

This book builds on earlier studies (Vuillemin, 1979, 1983) and especially on *Nécessité ou contingence, l'aporie de Diodore et les systèmes philosophiques* (Les Éditions de Minuit–La Fondation Singer Polignac, Paris, 1984). I considerably shortened this last work and eliminated the systematic considerations not directly pertinent to the Master Argument. I corrected it on the occasion of an objection raised by M.H. Angstl (his letter and my reply to it were published in the *Allgemeine Zeitschrift für Philosophie*, XI 3, 1986, pp. 79–87). Finally I added the present, unpublished Epilogue. M. Thomas F. Morran has translated into English this new French version.

My thanks go to Professor J. Moravcsik for his criticism, Bill Graham, who revised the text, Mrs. L. von Kampen, who transformed a manuscript into a book, and to Patrick Suppes, my good friend.

Part I

The Master Argument

The Master Argument. On the Shortcomings of Some Past Interpretations. Conditions to be Fulfilled by any Acceptable Interpretation.

1.1 The text of Epictetus.

Epictetus gives the following account of the argument.

Here, it seems to me, are the points upon which the Master Argument was posed: there is, for these three propositions, a conflict between any two of them taken together and the third: 'Every true proposition about the past is necessary. The impossible does not logically follow from the possible. What neither is presently true nor will be so is possible'. Having noticed this conflict, Diodorus used the plausibility of the first two to prove the following: 'Nothing is possible which is not presently true and is not to be so in the future'. Another, for the two propositions to keep, will maintain these two: 'There is a possible which neither is presently true nor will be so; the impossible does not logically follow from the possible'; but then it is not exact to say that every true proposition about the past is necessary; that is what the school of Cleanthes seems to maintain (SVF I 489) with whom Antipater is generally in agreement (SVF III Ant. 30). Others (namely Chrysippus, SVF II 283) admit the two other propositions: There is a possible which neither is presently true nor will be so; every true proposition about the past is necessary'; but then the impossible follows logically from the possible. But there is no way to maintain the three propositions

at once, since in every case there is a conflict between one and the other two.¹

To this account Epictetus adds an ironic commentary on the virtues of erudition. To one who boasted of having read Antipater's treatise on the Master Argument he exclaims: "What more do you have, you, for having read it? What opinion have you formed on the question? You might just as well speak to us of Helen, of Priam and of that isle of Calypso that hasn't existed in the least and never will".²

The irony of Epictetus is aimed neither at the doctrine nor at the moral consequences that could be drawn from the argument, but only at the vanity of logical quibbles. We cannot speculate then on any opposition of principle that Epictetus might have had to the supposedly morally disastrous consequences of the argument.

Such is the only explicit text on the Master Argument.

Most interpretations and reconstructions of the argument that have been given can be put into one of three categories. They all agree, implicitly at least, on denouncing its disastrous moral consequences and on flushing out some ambiguity or other in the premises, which they in turn hold responsible for these consequences. In order to save human freedom called into question by the Master Argument it has been postulated that Diodorus was guilty of making one of the following confusions. He took the word 'follow' in two different senses in the first two premises. He played, in the first premise, either on two possible senses of the way in which a proposition can be concerned with the past or on two senses of the word 'necessary'. Or finally, he played on a more general ambiguity hidden in the usage of indeterminate grammatical tenses.

1.2 Zeller's interpretation. Confusion of the logical and the chronological.

Zeller³ has given the following syllogistic interpretation of the argument. "If something was possible which neither is nor will be, an impossible would result from a possible. But an impossible cannot result from a possible. Thus nothing is possible which neither is nor will be". The minor premise illustrates Diodorus' second proposition. The conditional major premise has Diodorus' third proposition as antecedent

¹Epictetus, 1916, II, 19 1-4; Döring, 1972, p. 131; the parentheses refer to the von Arnim collection, 1905; I follow the Bréhier translation, revised by Goldschmidt in Bréhier, 1962, pp. 932-933. Bréhier translates ἀκολουθεῖν correctly by 'logically follows'.

²Ibid., p. 933.

³Zeller, 1910, p. 254.

and the negation of his second as consequent. The correlation of the two must evidently express the argument's first proposition. "The hypothetical major premise was in need of some basis; and it found it by way of the proposition saying that every past fact is necessary. Indeed, when of two mutually exclusive cases one has turned out to be realized, the possibility of the other finds itself cancelled out, since that which has happened can no longer be changed... This second case, then, is now impossible; had it been possible beforehand, then, in the opinion of Diodorus, an impossible would have resulted from a possible".

This reconstruction supposes a *quaternio terminorum* in the argument of Diodorus: one based on the ambiguity of the words ἀκολουθεῖν and ἀδύνατον. In the major premise, the first of these words does indeed have the temporal sense of 'follow in succession', whereas, in the minor premise, on the contrary, it has the logical sense of 'follow logically'. In the major premise, from the realization of a possible can and must result the *de facto* impossibility of another possible, since the event realized excludes the contrary of what it produces. But the minor premise signifies that nothing is possible of which the realization would result in something impossible *in itself*.

According to P.M. Schuhl, it is possible to restore a purely logical meaning to the conditional major premise, thereby validating the argument in rendering its elements homogeneous. It is not that the impossibility of an event simply succeeds its possibility: it is rather a strict consequence of it, given the realization of its contrary.⁴ To illustrate with an example from Kurt von Fritz,⁵ the conditional major premise would mean "If the Carthaginians win today at Cannes, it follows that the Romans don't win today at Cannes, which, beforehand, had seemed possible". But as von Fritz points out, Zeller would have answered that the impossibility referred to by Schuhl is not an impossibility *in itself*, in the logical sense, but on the contrary a *de facto* impossibility, which is not actually far removed from the impossibility due to temporal succession, as understood by Zeller.

Mr. G.H. von Wright has given an elegant and logically impeccable, formally modified version of the intuition contained in this type of interpretation.⁶ He begins by supposing p to be possible. As a consequence of that supposition, for some future moment, t , it is possible that p at t . But, in virtue of Diodorus' third premise, it is true that not- p at t . As a consequence of his first premise, for every moment, t' ,

⁴Schuhl, 1960, pp. 74-75.

⁵Von Fritz, review of Schuhl, 1962, pp. 138-152.

⁶von Wright, 1979a. Compare, however, von Wright's interpretation of the second premise with the interpretation that will be given in 2.3 and 2.4.

later than t , it will be impossible that p at t' . Thus the supposition of the non-realization of the possible leads to an impossibility, if one interprets the second Diodorean premise as follows: if the proposition p at t is prospectively possible, that same proposition cannot become impossible at some later moment of time.

In interpreting the second Diodorean premise as a prohibition of the proposition p 's changing modality over time, Mr. von Wright brings to light the metaphysical, as opposed to the logical, sense that this first type of interpretation is obliged to suppose. The difficulty is that the retrospective impossibility of p , which, according to the first premise, follows upon the non-realization of p at t begins at a moment, t' , later than t , no matter what hypothesis might be made as to the continuity of time. The Master Argument would be incontestable only if t' coincided with t , and the second premise is then supposed to be meant to assure the validity of the synthetic judgment by which the conservation of the modality of p from t to t' is guaranteed.

In all these cases, the commentators reject the validity of the Master Argument in accusing Diodorus of confusing a *de facto* necessity with a necessity of principle; and in so doing they are immediately in conflict with all of the ancient commentators. The translation of Epictetus accepted here, and against these modern commentators, disqualifies these interpretations from the start, in taking ἀκολουθεῖν as 'follow logically'. In this way the tradition of the Ancients is respected, though without, of course, an *a priori* refusal to examine whether the notions of possible and of impossible here have a univocal meaning or not.⁷

As Mr. Boudot points out, "the term rendered by 'follow [logically]' (ἀκολουθεῖν) is that which the Megaro-Stoics use for designating what we call implication. Certainly the definition varies from author to author, but no one identifies implication and temporal succession.⁸ For Diodorus, 'q' follows from 'p' if at no time do we have both 'p' true and 'q' false. Moreover, and this is the most important point, the principle 'the impossible does not follow from the possible' is false in the Diodorean system if interpreted in the sense of temporal succession. A statement of the type 'it is possible that p ' can be true at present and false in the future".⁹ Though a temporal sense of the term "follow" is excluded here, it is however not decided whether the logical connection between the antecedent and the consequent and the consequent

⁷For example, Kneale, 1962, p. 121.

⁸On this point, cf. *ibid.*, pp. 128-138, especially, p. 132.

⁹Boudot, 1973, p. 445. As the author points out, Rescher and Urquhart, 1971, p. 192, agree with Zeller in taking 'follow' in the sense of 'succeed'.

themselves express temporal modalities connected by a simple material implication.

1.3 Ambiguity in the first premise: Necessity and irrevocability. Signification of the first premise.

The first premise of the Master Argument has seemed ambiguous, and even doubly so.

It says, literally, that all that is past and true (πᾶν παρεληλυθὸς ἀληθές) is necessary. It seems, then, open to two different interpretations accordingly as it signifies that every statement grammatically in the past is necessary if true, or that every statement about a past event is necessary if true. Since we can give a grammatically past form to every statement about a future event, in going, for instance, from the form 'it will rain tomorrow' to the form 'it was true yesterday that it would rain in two days', the immediate result of taking the first premise in the grammatical sense would be universal necessity.

Is it believable that such a gross confusion could have escaped Aristotle, Epicurus and Chrysippus or that they would have preferred incurring the risk of logically onerous solutions to denouncing this simple sophism? The translation retained here: 'about the past', eliminates the incriminated ambiguity from the outset.

There is a second type of ambiguity resulting from the use of the word 'necessary'. It might be said that a past event is simply irrevocable. When a thrown dice has come up six, the throw doesn't cease to have been aleatory. The proposition saying that the dice turned up six could in no way be necessary, although it is about an irrevocable event.

Such an objection, which is entirely justified, obliges us to reconsider the sense of the Master Argument's first premise and to ask ourselves what could be the meaning of the necessity accorded to true propositions about the past.

The modern logicians who have studied the relations between the modalities and time have universally accepted the so-called axiom of necessity, by virtue of which that which is necessary is *a fortiori* existent (*Ab oportere ad esse valet consequentia*) and that which is existent is *a fortiori* possible (*Ab esse ad posse valet consequentia*). This axiom seems logically well-founded when the modalities are without relation to time. If, in mathematics, a conclusion is necessary, it is *a fortiori* possible. The logical sense of the modalities, as such, excludes time. Naturally, if a logical necessity had a temporal statement as its object,

it would necessarily entail, *ipso facto*, the logical possibility having that same temporal statement as its object. Supposing it necessary that a certain thing should have happened, it is *a fortiori* possible that that thing should have happened. But when the modalities are applied to temporal events, they are generally understood, and rightly so, in a different sense. They are taken in a real, rather than in a logical sense. Irrevocability, which is a factual kind of necessity, applies to any event whatsoever, even a contingent one, once it has come to pass. It follows then that the real modality itself must be assigned a temporal index distinct from the one affecting the event to which the modality applies. At present it is irrevocable, or necessary in the factual sense, that the battle of Salamis took place. Factual possibility, the modal counterpart of this factual necessity, will likewise be assigned a temporal index of its own. But it is notable that there is no way of getting from the past conceived of as a factual necessity to the corresponding factual possibility, where that factuality is taken to be that of a future or, at most, a present event, to the exclusion of any event having taken place. We shall see that for Aristotle this privileged temporal direction of the possible constitutes the entire content of the Master Argument's first premise.

The Master Argument then could not be accused of having an ambiguous first premise, unless, in order to be demonstrative, it confused either the grammatical form of the past with the factual past or temporal necessity with irrevocability. Neither the one nor the other of these confusions is required. Each axiom of Epictetus will, in the same way, be given an interpretation in terms of temporal modalities. As to the logical demonstration that these axioms are incompatible, it will be shown to simply obey the laws of first order extensional logic without having to resort to arguments borrowed from modal logic. Therefore there will be no risk of subreption between the logical and the factual meanings of modalities.

1.4 Prior's interpretation: It supposes two supplementary premises, one of which is explicitly rejected by Aristotle; it supposes the first premise ambiguous.

A. N. Prior has proposed a formal reconstruction of the Master Argument giving a purely logical sense to the word 'follow', though without deeming it useful to distinguish between logical necessity and the irrevocability of the past.

He shows that Diodorus' first two premises, A and B, joined to two others, D and E, allow the derivation of the denial of the third premise, C. What he proposes is roughly the following.¹⁰

- A. If it has been true that something is the case then it is not possible that it has never been true that it is the case.
- B. If the consequence q necessarily follows from the premise p , then if q is not possible, p is not possible.
- D. Of whatever is the case it has never been true that it will never be the case (i. e. it has never-been never-going-to-be the case).
- E. Of whatever neither is nor ever will be the case, it has been true (at some moment) that it will never be the case.

Consider the propositions B and D. Let the premise p in B be the proposition 'something is the case' and the consequence q 'this something has never been never going to be the case'. We are justified in saying that if it is necessary that whatever is the case has never been never going to be the case, then if it is not possible that it has never been never going to be the case, then it is not possible that it is the case. But since D gives us the antecedent of this proposition as true,

¹⁰Prior, 1967, pp. 32-33 [Pp: it has been the case that p (it has been true that p); Fp: it will be the case that p (it will be true that p):

- A. $Pp \supset \sim M \sim Pp$
- B. $L(p \supset q) \supset (\sim Mq \supset \sim Mp)$
- \sim C. $(\sim p \cdot \sim Fp) \supset \sim Mp$
- D. $p \supset \sim P \sim Fp$
- E. $(\sim p \cdot \sim Fp) \supset P \sim Fp$

Prior (1967, p. 34) starts from the system T of R. Feys, who takes L as undefined, defines M as $\sim L \sim$, and adds to propositional calculus only the one new rule $\vdash \alpha \rightarrow \vdash L\alpha$ and the two axioms: $Lp \supset p$ and $L(p \supset q) \supset (Lp \supset Lq)$. First let us demonstrate B:

- 1. $(p \supset q) \supset (\sim q \supset \sim p)$ (Contraposition)
- 2. $L[(p \supset q) \supset (\sim q \supset \sim p)]$ (1, rule of necessitation)
- 3. $L(p \supset q) \supset L(\sim q \supset \sim p)$ (2, second added axiom, *Mod. Pon.*)
- 4. $L(p \supset q) \supset (L \sim q \supset L \sim p)$ (3, second added axiom, *Mod. Pon.*)
- 5. $[L(p \supset q) \supset (\sim Mq \supset \sim Mp)] \equiv B$ (4, definition of M)

Let us demonstrate that (A. B. D. E.) $\supset \sim C$

- 1. $L(p \supset \sim P \sim Fp)$ (D, rule of necessitation)
- 2. $L(p \supset \sim P \sim Fp) \supset (\sim M \sim P \sim Fp \supset \sim Mp)$ (B with substitutions $p/p, \sim P \sim Fp/q$)
- 3. $\sim M \sim P \sim Fp \supset \sim Mp$ (1,2, *Mod. Pon.*)
- 4. $\sim p \cdot \sim Fp \supset P \sim Fp$ (E)
- 5. $P \sim Fp \supset \sim M \sim P \sim Fp$ (A)
- 6. $\sim p \cdot \sim Fp \supset \sim M \sim P \sim Fp$ (4,5 *Mod. Pon.*)
- 7. $(\sim p \cdot \sim Fp \supset \sim Mp) \equiv \sim C$ (6,3, *Syll., Mod. Pon.*)

we can detach its consequent to get 'if it is not possible that something has never been never going to be the case, then it is not possible that it be the case'.

On the other hand, given E and substituting 'something will never be the case' for 'something is the case' in A, the rule of syllogism will allow us to write:

Of whatever neither is nor ever will be the case it is not possible that it has never been never going to be the case. But we have demonstrated (B and D) that if it is not possible that something has never been never going to be the case then it is not possible for that something to be the case. Hence, whatever neither is nor ever will be the case is impossible. This is the denial of the Master Argument's premise C according to which there is a possible that will never be realized.

Once the ambiguous interpretation of premise A is admitted, the whole question comes down to one of justifying the introduction of D and E.

In favor of D one might invoke¹¹ Chapter IX of *De Interpretatione*,¹² or again Cicero's *De Fato*.¹³ But in one case it is the Megarians, in the other the Stoics, to which this principle of retrogradation is attributed. Aristotle refuses it implicitly.

As for the premise E, it means that "... if a statement is false and will always remain so in the future, then there has been a past moment at which it was true that that statement would always subsequently be false".¹⁴ What makes this premise seem plausible is that "... if *p* is now and always will be false then it has already been true in the past, at least at the moment *just* past, that *p* will never be true anymore—it hasn't always been true, because at least in the moment *just* past it wasn't true, that *p* would be true again".¹⁵ This thesis is valid only if time is discrete and non-dense,¹⁶ in other words, only if, as Diodorus holds,¹⁷ every instant, and in particular the present one,

¹¹Becker, 1961, pp. 250–253.

¹²See below, 6.4, pp. 140–141.

¹³*Potest factum quicquam igitur esse, quod non verum fuerit futurum esse?* (Cicero, *De Fato*, XII (27)).

¹⁴Boudot, 1973, p. 447.

¹⁵Prior, 1967, p. 49; but cf. also p. 8.

¹⁶Boudot, 1973, pp. 447–448 (for an intuitive résumé); Prior, 1967, pp. 49–50.

¹⁷Döring, 1972, frag. 116–120, p. 129 *sq.* According to Fraenkel (1960, pp. 204–211) the arguments of Zeno treat of space represented as continuous just as well as discontinuous, whereas Diodorus envisaged solely the case of its discontinuity. In the third proof of Festus, the ἀμερῆ, as logical conceptions, are introduced only to render motion impossible. According to Sedley, 1977 (pp. 88–89), basing himself on Chalcidius, Diodorus' influence on some of the Stoics was so great that they would have incorporated the notion of indivisible units of matter into their doctrine. For

has an immediate predecessor and successor. But suppose that time is continuous or, what suffices for the argument, dense. Then between the present moment, t_0 , time from which p is supposed by virtue of the antecedent no longer to be the case, and the past moment, t_1 , chosen arbitrarily near so as to verify the consequent ' p has been never going to be the case', there will inevitably be an infinity of instants and so at least one instant, t_2 , different from t_1 and t_0 . But thesis E, which affords no condition on the status at time t_2 of the future case in question, inevitably leaves the truth-value of the statement about the future case indeterminate at t_2 . Taking the value of this statement to be true we falsify the thesis. Thus E is valid only if it is excluded *a priori* that that statement chosen as false at t_1 and confirmed as false at t_0 be able to change its truth-value between t_1 and t_0 . This exclusion is satisfactory only if there are no intermediate instants between t_1 and t_0 , which implies that t_1 is the immediate predecessor of t_0 and that time is thus discrete.

With the conclusion established, it can be shown that Diodorus' propositions A and B are, in turn, demonstrable. Mr. Boudot concludes; "If we admit four principles, three of which were in common use in antiquity -viz. A, B and D- the fourth -viz. E- expressing the discreteness of time, we conclude the Diodorean definition of the possible.¹⁸ On the other hand, if we begin with that definition, the premises used, linking the modalities with temporal determinations, are demonstrable. Looked at in this way, the Master Argument reveals such a perfection in its alliance of coherence and simplicity that it becomes easy to understand the reputation it established for its author and the respect it inspired among the Ancients".¹⁹ What is more, conceived of in this way, the Master Argument does not lead, not directly at least, to determinism: a statement true sometimes, but not always, is true without being necessary.

Two shortcomings, however, prevent us from accepting Prior's reconstruction. The first lies in the introduction of premises D and E, the second in the hypothesis of an ambiguity in the interpretation of A which would be imputed to both Diodorus and his opponents.

In the first place, premise E, of which an equivalent is supposed

Diodorus there is a relation between these unities and the definition of the possible: since no body will be divided into an infinite number of parts, why call such a division possible? Chrysippus holds that the division is indefinite, not reaching infinity. It is to him that our explicit sources trace back the affirmation of infinite divisibility (note 80, pp. 111-112).

¹⁸That is to say: the possible is whatever is or will be and only that.

¹⁹Boudot, 1973, p. 448 and p. 449.

to be attributable to Diodorus, is, like premise D, in direct opposition to the Aristotelian theory of the continuity of time in book V of the *Physics*. Aristotle distinguishes, according to an increasing degree of 'neighbourhood', the three notions of succession, contiguity and continuity. "The Aristotelian continuum is defined as a collection, in a certain sense well-linked (Ephexes and Echomenon), of parts potentially separated by limiting points".²⁰ The infinite divisibility of the continuum thus involves its density. So Diodorus' argument, on Prior's reconstruction, would fail to touch Aristotle in the least. Not only would it not touch him in fact, but the introduction of clause E of the discontinuity of time would be tantamount to a conscious admission of defeat. If we think of Diodorus as having distinguished himself from the "ancient" Megarians in saving the modal distinctions, it would have to be added that as soon as confronted with a dense time—and we can presume that on this point the Aristotelian representation had rallied the assent of the scientific community—those distinctions collapse. The Master Argument is thus demoted to the rank of a particular school's argument.

In the second place, if we accept Prior's reconstruction, we have the right to accuse Diodorus of having confused "a past-tense statement" with a "statement about the past"²¹ and of having, in this way, exploited an ambiguity of language. But this is to fly in the face of the tradition which has it that, contradicted on this point by Chrysippus, he conceived language to be free of any equivocation.²² Surely, one might argue, this premise does not in itself produce necessity, for Diodorus distinguishes the necessary and the possible. It does, however, produce it if we go from a tense-logic to a dated or 'pseudo-dated'

²⁰Granger, 1976, p. 306; for the philosophical analysis of this Aristotelian continuum cf. Vuillemin, 1962, pp. 185-198.

²¹Boudot, 1973, p. 470; Mrs. Kneale denounces the confusion, 1962, p. 121.

²²AulusGellius, *Noctes Atticae*, Bk. XII, c.1-3 in Döring, 1977, 111, p. 31 and p. 128.

event-logic.²³ Prior and Boudot²⁴ have shown that in constructing an Ockhamist model in which the suspect confusion is avoided, necessitarianism too can be avoided, even on a dated event-logic.

But should it turn out, as we hope to show, that there is no ambiguity in Diodorus' first premise, it would no longer be possible to avoid necessitarianism in a logic of dates by recourse of a rejection of that supposed ambiguity.²⁵ It would in fact suffice in this case to limit ourselves to authentically past events, where there is no room for formal confusion, to derive necessity. And if one wanted then to conclude to necessitarianism, Ockham's distinction would no longer block the move, for it would no longer be simply a grammatical mistake that suggested necessitarianism.

Let us, on the contrary, follow historical likelihood; take the universal favor with which the Master Argument was met in Antiquity as a presumption in favor of its solidity; dismiss the supposition that crude ambiguities would have crept into its premises. Let us give, along with Prior, a purely logical sense to 'follow'—though not necessarily the sense of a thesis of modal logic—; and let us suppose that the past mentioned in the first premise is that of events to show the incompatibility of the three premises without having either to postulate the discreteness of time, or to confuse the irrevocability of the past with logical necessity, or even to invoke a retrogradation of the true, of which Epictetus makes no mention and which the Stagirite expressly calls into question. Above all, in order to avoid introducing principles inappropriate to the interpretation, it is important to find a text contemporaneous with the Master Argument as formulated by Diodorus and apt, by way of comparison, to shed light on the objective sense of the latter.

²³Boudot, 1973, p. 451: "In tense-logic the conclusion from the truth to the necessity of 'Socrates will be sitting' does not follow. But in metric tense-logic, the conclusion from the truth to the necessity of 'Socrates will be sitting tomorrow' does follow, since that statement is equivalent to 'It was true yesterday that Socrates will be sitting the day after tomorrow', which is itself necessary because past". This variation of modal status with the formal expression of the tense of the statements in question assures the originality of Diodorus with respect to the 'ancient' Megarians (Blanché, 1965, pp. 133-149). It also limits the importance of that originality. Sedley (1977, pp. 74-120) has contested Diodorus' adhesion to the Megarian 'school' (pp. 74-78), making him rather a representative of the Dialectical 'school', which had a separate existence. But even accepting this thesis, there remains sufficient affinity between the two schools, from the point of view both of the theory of motion and of that of the modalities, for us to still consider Diodorus to be a 'Megarian' philosopher.

²⁴Prior, 1967, pp. 121 *sq.*

²⁵This constitutes the essentials of Boudot's elegant solution (1973).

There is such a text. It is that of an argument of Aristotle's concerning demonstrations by the absurd in modal logic for propositions involving time. It is a text that has not attracted as much attention as it deserves on the part of modern commentators.²⁶

²⁶With the exception of Cherniss (1962) and Hintikka (1973), though even here, none of Hintikka's references to this passage (p. 94, p. 152, p. 164, p. 183) actually analyzes Aristotle's manner of reasoning.

Reconstruction of the Master Argument.

2.1 An Aristotelian paradigm: *De Caelo*, I, 283^b6-17; its context.

/6/ "It is not true to say now /7/ that a thing exists last year, nor last year that it exists now. It is therefore impossible /8/ that that which does not exist at some moment should later be sempiternal. For it will have later as well /9/ the capacity of not existing, though not that of not existing while it exists (that which is /10/ in fact exists in actuality), but of not existing last year /11/ and in the past. Now suppose this capacity it has to be realized in actuality. /12/ It will then be true to say now that it does not exist last year. But /13/ that is impossible. For there is no potentiality of the past, but /14/ only of the present and the future. The same goes too if that which is formerly sempiternal /15/ passes later to non-existence; for it will have the capacity for what is not in actuality. /16/ Let us then actualize the possible: it will be true to say now that this exists last year /17/ and generally in the past."

In this passage Aristotle proposes a refutation of the Platonic thesis set forth in the *Timaeus*, according to which the demiurge created the world in imitating the eternal Ideas and in informing the receptacle after their pattern. That creation accomplished, the world is then supposed to exist sempiternally.¹

¹That, in any case, is the "literal" interpretation Aristotle gives of Plato (Cherniss, 1962, pp. 414-417). This text is fundamental for insuring the separation of the eternal, though sensible, substances which are not subject to generation and decay from the properly material substances which are subject to generation and decay. It is here that the separation of Heaven and Earth and astronomical theory find their *raison d'être*.

The passage here is composed of two parts. In the first, Aristotle shows that nothing that has been generated can last sempiternally, in the second, that nothing ungenerated can decay. He consequently concludes the equivalence of the terms 'generated' and 'perishable'.

This difficult passage brings four principles to bear. The last three of these, with the exception of changing a negation into an affirmation, will serve to constitute the Master Argument as well. Since the first of these last three principles has the form of a logical conjunction, it will be analyzed, for clarity's sake, as two independent principles. The complete list then will consist of 1) the principle of conservation of the modalities, 2) the principle of the possible realization of the possible or principle of the diachronic expansion of the modality together with the principle of conditional necessity, 3) the principle of the impossibility of realizing a contingent possible in the past, 4) the principle of the subsistence of a possible that is never realized. All of these principles, and these alone, are explicitly used in the *De Caelo* text. All but the first, and these alone, are to be explicitly mentioned in Epictetus' text on the Master Argument.

2.2 The principle of the conservation of modal status.

This is the principle proper to the *De Caelo* to which the Master Argument will have no recourse. The modalities, according to the Stagirite, are not simple operators governing statements or propositions. They are the characteristic properties of substances. There are thus two sorts of substances: a) necessary substances which are either eternal, that is to say, have no relation to time, or sempiternal, that is to say, are permanent in time, participating in topic matter alone and endowed with perpetual motion, b) contingent substances, now existing, now not existing, subject to full materiality, to generation and decay. It is an axiom of Aristotle's philosophy that a substance cannot change its modal status. Whether necessary or contingent, it is so by nature. It will retain its modal status then and will never be seen to change it.

The consequence of this principle is used in lines 7-9 of the text. Suppose that something that did not exist in the past should come, or should have come, to be. It thus belongs to the category of contingent things. By virtue of the principle of the conservation of modal status it will then retain later the capacity of not existing.

This principle and its consequence would merit an investigation on their own; but since they play no role in the Master Argument and

since its validity in no way depends on them, they will simply be taken here as given.²

The three following principles, on the other hand, broken down into four for the sake of clarity, are the very ones operative in the Master Argument.

²The principle of the conservation of modal status is a consequence of the Aristotelian distinction between three kinds of substance (*Metaphysics* Λ , 1069^a30–1069^b2): a) the sensible corruptible substances, b) the sensible incorruptible substances, c) the immaterial and immobiles substances. The three kinds are such that, if something belongs to one of them, it can never belong to another one. But belonging to b) or to c) means being a necessary being, and belonging to a) means being a contingent being. Themistius, in a subtle passage of his *Commentary*, lays down the principle and shows the absurdities that its denial would entail. To allow perishable substances to transgress their essence and accede to immortality and indestructibility would be to destroy the very limits that define the nature of things and to make contrary capacities persist indefinitely. “As the generable and the perishable are not so by chance and by fortune, it is seen that they are so by nature. Indeed, all that there is is either by nature or by chance—for we leave aside here that which art produces; but all natural things maintain (*custodiunt*) the capacities that are proper to them. If they don’t keep them and they change into other dispositions, their modification will be either the result of chance—in which case there would be but one and the same disposition for both natural things and for those resulting from chance and fortune—or the modification will come about, even for chance things, according to nature. That is why, since there is conservation of the modification even if the things themselves are to change, the modification in turn will take place either by nature or as a result of chance. This is why natural capacities must have limits. But if these capacities have limits, then that also, which has a birth and is subject to death, will be so by nature before that death. It can then be seen that what happens now to the nature and the matter thus subject to these two dispositions, namely, on the one hand the generation of things and their existence, on the other their privation and their inexistence, has a limit beyond which it changes no more. That is why it is necessary that that which is engendered should not be deprived of this power of being changed; that is why death too will come to it, in its time. Likewise, we do not surprise that which is always subject to death transgressing its proper nature. Otherwise, since it will have persisted for some time by virtue of its nature, it will equally persist in the disposition that renders it immortal and, keeping its nature, it will also retain the capacity in virtue of which it was changed. That is why there must then be several capacities at once for an infinite time, and since we have established that the action of this capacity persists, then what we have established will be false, which can no more be the case than the rest we have spoken of so often.” (Themistius, *De Caelo* A 12 [Arist. p. 283^b2-12] 1902, p. 86, lines 4-29).

2.3 The principle of the possible realization of the possible interpreted as a principle of pure modal logic.³

The reasoning, or rather the two parallel lines of reasoning that compose our text are among those *reductio ad absurdum* arguments applied to singular propositions⁴ and obtained by showing that a supposed possible cannot be realized.

In the text at hand Aristotle applies this manner of reasoning to singular propositions of existence. What there is actually question of is a determinate individual, the Heavens; but the reasoning has to do with an individual in general, which happens at one time not to exist (1.7-8). The reasoning is seen to be transcendental and universal, since the only 'predicate' under which such indeterminate individuals are subsumed is that of being and non-being which is not a genus. But that very universality, transcending the categories, is the hallmark of the syllogistic form of science and particularly of the modal syllogistic.

The directing principle of this mode of reasoning has generally been expressed in terms of pure modal logic. It is this conception that is to be set out first. But, on subsequently examining the Aristotelian text, this first conception will be shown to be inadequate and it will further be seen that the principle of the possible realization of the possible comes down to none other than the diachronic and synchronic movements of the modalities.

From the possible as such, nothing can be concluded, for a modal conclusion is legitimate only if it proceeds *a fortiori*, that is to say, from the necessary to the actual or from the actual to the possible.

Starting from the consideration of a possible then, the only thing we can do is to analyze the consequences that would result from its realization. If these consequences follow necessarily they would themselves have a necessary character, if the realization was necessary. By contraposition, if these consequences which result necessarily from the realization turn out to be impossible, that realization is impossible in turn. By a second contraposition it can further be shown that, if the consequences are supposed to follow necessarily from the realization of the possible, then if that realization is possible the consequences will be so as well.

³Attested to, for example, in the *De Caelo* (281^a15): Συμβαίνει δ' ἀδύνατον ἐξ ἀδυνάτου ("The impossible comes from the impossible").

⁴The principle applied to general statements and according to which one must be able to realize a possible is called "ecthesis by demodalization" by G. Granger (1976, p. 193, p. 214).

Consider the possibility of a condition p . Let us posit p as existent and examine the consequence q that follows necessarily from it. If q is contradictory, *i.e.* not realizable in any possible world, we have the right to conclude that p is also impossible. This is the import of the Master Argument's second premise according to which the impossible does not logically follow from the possible.⁵

As a typical application of the principle Aristotle gives that of the incommensurability of the diagonal of the square with its side: "All who establish an argument *per impossibile* syllogistically infer that which is false and prove the hypothetical conclusion when something impossible results from the supposition of its contradictory; for example that the diagonal of the square has no common measure with the side, since if one supposes it to have a common measure then the odd numbers are equal to the even ones. One infers syllogistically that the odd are equal to the even numbers and proves hypothetically the incommensurability of the diagonal, since a falsehood results from its negation".⁶ Thus if p signifies that there is a common measure between the side and diagonal of the square, q that one and the same number is at once both even and odd, then given that if p is true q is necessarily true as well, from the impossibility of q the impossibility of p is inferred, *i.e.* the incommensurability of the side and the diagonal of the square.

From this principle of the possible realization of the possible one can draw yet another principle whose intuitive significance is perhaps clearer. Namely, it is equivalent to say of a thing that it is possible and that the conjunction of the possibility of that thing with its realization is possible.⁷

At this point there is need for a note of caution. It happens, both in the case of Aristotle's reasoning and in that of the Master Argument that the realization of the possible under consideration produces a contradictory consequence only by virtue of an as yet non-explicit hypothesis. It is thus the conjunction of the realization of the possible with that hypothesis that produces the contradiction. That contradiction justifies the conclusion that there is incompatibility between the realization of the possible and the hypothesis but in no way does it jus-

⁵This is Prior's premise B: $L(p \supset q) \supset (\sim Mq \supset \sim Mp)$, deducible in the weak system T (see p. 7, note 10).

⁶*An.Pr.*, I, 23, 41^a23-30. This example is used by Aristotle in the *De Caelo*, I 11 281^a7.

⁷The principle in question here is: $Mp \equiv M(p \cdot Mp)$. Since T contains $L(p \supset q) \supset (Lp \supset Lq)$ and the rule of necessitation, it is normal. Hence given $p \supset Mp$ (Hughes & Cresswell TH1), it follows that $p \equiv p \cdot Mp$, and since T is normal, $Mp \equiv M(p \cdot Mp)$ (Note of B. Graham).

tify the conclusion that there is incompatibility between the possible in question itself (possibly non-realized) and the hypothesis.⁸

This latter illegitimate conclusion would be stronger than the premise. It would result in a sophism in the distribution of the modalities. Consequently, whenever a reasoning by realization is employed to demonstrate not the intrinsic impossibility of a thing, but rather the logical incompatibility between the possibility of one thing and the reality of another, simple application of the principle that the impossible does not logically follow from the possible is not enough. What must in fact be demonstrated here is that a contradictory consequence necessarily follows, not from the conjunction of the realization of the possible and the hypothesis, but from the conjunction of simply the possible and the hypothesis. To this end a specific principle is required.

Such a principle indeed has been clearly stated by Leibniz. To Antoine who in his Dialogue with Laurent Valla sets forth "the philosopher's rule" that "whatever is possible may be considered as existent",⁹ Laurent replies: "Philosophers' rules are no oracles for me. This one in particular is not at all exact. It often happens that two contradictories are both possible; can they also both exist?" He thus denounces the sophism of distribution which would have it that the real world, in so far as it excludes the possible worlds, would also render them impossible by the same token.¹⁰

⁸Having recourse to the preceding thesis, let us posit p as existent. The condition of validity of this position is that p be compatible with Mp . Suppose further that the hypothesis of the argument imposes $\sim p$. Clearly $\sim p$ excludes p but not Mp . In other words:

$$\sim p.M(p.Mp)$$

is compatible, for that conjunction is equivalent to:

$$\sim p.Mp.MMp.$$

In order for the position that p is existent to enter into contradiction with the hypothesis, this latter would have to impose $\sim Mp$. Then:

$$\sim Mp.M(p.Mp)$$

is indeed incompatible, since that conjunction is equivalent to:

$$\sim Mp.Mp.MMp.$$

⁹Leibniz, Gerhardt, 1978, p. 359; Jalabert, 1962, p. 371. It is important to insist here on the word 'may' which has as consequence the impossibility of applying the rule in the case where the *impossibility* of the thing would have already been posited.

¹⁰Laurent warns Antoine that the philosophers' rule is false if one makes it say that:

$$(1) (Mp.M \sim p) \supset M(p. \sim p).$$

From (1), by contraposition, comes:

That is why Aristotle inserted in the very expression of the principle of the possible realization of the possible the clause having to do with the principle of conditional necessity (1. 10). It will be seen below, however, that the modalities figuring in this latter principle, instead of belonging to pure modal logic, are affected by temporal indices. If one were then to continue interpreting the principle of the possible realization of the possible in terms of pure modal logic an inconsistency would creep into the Aristotelian passage, for the second principle of the *De Caelo* text would then have to be expressed as the conjunction of non-homogeneous terms, the first comporting “pure” modal expressions, the second temporally indexed modal expressions.

2.4 The principle of possible realization of the possible as principle of synchronic contraction of the possible and diachronic expansion of the necessary.

Coming back to Aristotle’s text, just what is it that he enjoins us to actualize the potentiality of in line 11? It is something which the principle of conservation of modality (lines 8-9) assures us will have retained the capacity of not existing. He makes it clear in lines 9 and 10 that this capacity cannot be actualized while the thing in question is in actuality (namely, now, since we know from experience that the world exists, and in the future when, by hypothesis, the world will exist sempiternally) but, consequently, last year and in the past. The possibility of not existing Aristotle speaks of then is not a simple logical or atemporal possibility. It is a possibility subject to a double temporal index, since as a possibility it will persist throughout all the future but because of circumstances and admitted hypotheses its realization can take place only last year and in the past. Such doubly indexed modalities are homogeneous with the principle of conditional necessity, as we shall see, and also with the principle of conservation of the modalities (for what sense would it make to speak of the *conservation* of pure and atemporal modalities?).

If we follow Aristotle’s text to the letter, then the principle of possible realization of the possible signifies that if it is possible at a given moment (that we shall fix as *now* for perspicuity’s sake) that something

$$(2) \sim M(p, \sim p) \supset \sim (Mp.M \sim p)$$

and by virtue of non-contradiction and *modus ponens*:

$$(3) \sim (Mp.M \sim p)$$

which would have as consequence the strictest form of *logical determinism*.

should be realized or actualized at time t , then there is an instant, t_1 , on the interval $|N(i.e. \text{'now'}) - t| \leq e$ situated either in the past or in the future (possibly coincident with the now) such that it is possible at t_1 that that thing should be realized at t_1 . The principle synchronically contracts a possibility posited diachronically over the interval onto an instant of the interval.

By contraposition, if at every instant, t_1 , of an interval $|N - t| \leq e$, where either t precedes N or N is identical with t or precedes t , it is necessary that p at t_1 , then it is necessary at N that p at t .

It might be objected that this proposed formulation is entirely inadequate for expressing the possible realization of the possible. To synchronically contract a possible by saying that it is possible at t_1 that p at t_1 is surely not to realize that possible or to say that p at t_1 . But we have forewarned that Aristotle did not separate the clause of synchronic contraction from the clause of conditional necessity. Should it turn out that this latter clause signifies or implies that a contracted possibility is none other than a realized one, all the requisites of the interpretation will have been satisfied. The aim of the following section will be to show that this is so.¹¹

Aristotle exemplifies his principle now by cases with no relation to time, now by dated events. Thus in *de Caelo* 281^b10-20 he argues about the incommensurability of the diagonal, but also about the possibility of standing and sitting. For a given agent, it is impossible at a given moment to be at any time both sitting and standing. And it is clearly having in mind such doubly indexed modalities that he says in the chapter of *Metaphysics* where he refutes the Megarics (Θ ,3,1047^a23-25): "A thing is capable of doing something if there will be nothing impossible in its having the actuality of that of which it is said to have the capacity".

Therefore, if it is now possible that p at t ($M_N p_t$), the contraction of the possible at t ($M_t p_t$) logically excludes the contracted impossible ($\sim M_t p_t$). Such an impossible would precisely obtain, were we to deny B.

¹¹The principle of possible realization of the possible, when interpreted in conformity with the *De Caelo* text in terms of doubly temporally indexed modalities, may be expressed in the following manner:

$$(t)\{\sim L_N \sim p_t \supset (\exists t_1)\{M_{t_1} p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)\}\}.$$

What is the relation between the law of modal logic

$$(a) \quad L(p \supset q) \supset (Mp \supset Mq)$$

and the second axiom, B, of Epictetus?

Then it would be true to say that:

$$(\exists t) \sim L_N \sim p_t \cdot (t_1)(N \leq t_1 \leq t \supset \sim M_{t_1} p_{t_1}).$$

When $t = N$, a contradiction results. When N is indeterminate, at all t_1 such that $N \leq t_1 \leq t, \sim M_{t_1} p_{t_1}$ would obtain, destroying the

Let us show on what conditions B can be deduced from (a):

1. $\vdash p_t \supset (\exists t_1)[p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)]$
 Existential generalization and conjunction of a truism)
2. $\vdash L\{p_t \supset (\exists t_1)[p_{t_1} \cdot (t \leq t_1 < N \vee N < t_1 < t)]\},$
 (1, $\vdash \alpha \supset \vdash L\alpha$).
3. $\vdash L\{p_t \supset (\exists t_1)[p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)]\} \supset$
 $\{Mp_t \supset M(\exists t_1)[p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)]\}$
 (a).
4. $\vdash Mp_t \supset M(\exists t_1)[p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)]$
 (3, 2, detachment).
5. $\vdash M_N p_t \supset Mp_t.$
6. $\vdash M_N p_t \supset M(\exists t_1)[p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)]$
 (5, 4, Syll.).
7. $\vdash M(\exists t_1)[p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)] \supset$
 $(\exists t_1)M[p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)]$
 (7, BF).
8. $\vdash M_N p_t \supset (\exists t_1)M[p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)]$
 (7, 6, Syll.).
9. $\vdash (\exists t_1)[M_{t_1} p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)] =_{Df}$
 $(\exists t_1)M[p_{t_1} \cdot t \leq t_1 < N \vee N \leq t_1 \leq t]$
10. $\vdash M_N p_t \supset (\exists t_1)[M_{t_1} p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)]$
 (8, 9, Syll.).
11. $\vdash \sim L_N \sim p_t \supset (\exists t_1)[M_{t_1} p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)]$
 ($\vdash \sim L_N \sim p_t \equiv M_N p_t$, 10)

There are four statements in this deduction that merit justification: 1, 5, 7 and 9.

The validity of the first statement stems from the fact that if p is the case at t then there is a t_1 falling between t inclusively and N or N inclusively and t inclusively.

The fifth statement is a characteristic axiom of the logic of the temporal modalities. If it is possible now that p at t then it is *a fortiori* logically possible that p at t , since the synchronic possibility puts added strictures on the logical possibility.

Statement 7 is a consequence of the Barcan formula. That formula is legitimate when the variable is interpreted substitutionally. And in fact a substitutional interpretation is sufficient when the values of the variable are the instants of time.

The definition 9 is that of synchronic possibility. To say that it is possible at t_1 that p at t_1 , with t_1 a function of t and of N , is to say that it is logically possible that p be the case at t_1 with t_1 a function of t and of N and with the logical possibility bearing on the conjunction of the realization of p at the instant t_1 and the defined function of t_1 and t .

The negative form given to the possible will be explained when we come to Chrysippus' theory. (5.2, pp. 108-113)

potentiality of p at any such instant. Finally if we assign a possibility with a fixed date, $t = t_1$, at this date it will be impossible to contract the possible.

An objection could be derived from this last case. "Suppose, it would be said, that I can now do something at a given time tomorrow. It may happen that this possibility will be maintained during some time and us say from now on until $t_2 < t_1$, and that at t_2 an event be produced which makes impossible that p at t_1 . Therefore it may be now possible that p at t_1 and this possibility may persist until t_2 without that p be possible at t_1 , in contradiction with B". But Diodorus as well as Aristotle exclude such a purely verbal interpretation of modality. In the same book of *Metaphysics* (Θ , 5, 1048^a16-21), Aristotle requires that having a potency does not happen without given conditions that distinguish a genuine potency from a rhetorical one: "To add, he says, the qualification 'if nothing external prevents it' is no further necessary; for it has the potency on the terms on which this is a potency of acting, and it is not in all circumstances but on certain conditions, among which will be the exclusion of external hindrances, for they are barred by some of the qualifications of our definition".

2.5 The principle of conditional necessity.

In line 9 and the parenthesis of line 10 Aristotle uses, without formulating it, the principle of conditional necessity. Which are the cases, he asks, in which the consequence drawn from the principle of conservation of modal status (here applied to the negative possible) is invalid? He answers that a thing which has not been but then comes to exist will have later also the capacity of not existing, though not that of not existing while it exists (that which is does indeed exist in actuality). What is excluded by the actual existence of a contingent thing, then, is not only that thing's inexistence, but its very capacity of not existing, while it exists. The first exclusion is guaranteed by the principle of non contradiction applied to temporal things, since it is impossible for one and the same thing to be and not to be at the same time. The second exclusion is stronger. It prevents a negative capacity's coexisting, in so far as the time of the event governed by that capacity is concerned, with an affirmative actuality; just as, symmetrically, it would prevent an affirmative capacity's coexisting with a negative actuality. Thus the actuality of p excludes the capacity of not- p while p and the actuality of not- p excludes the capacity of p while not- p .

It would be futile to try to deduce such a principle from the principles of the modal logics accepted today. We shall limit ourselves

to pointing this out in the case of reasoning by possible realization.¹² Suppose the negative possibility of an event and the hypothesis of that event's existence actualize the possibility. By virtue of the principle of contradiction an impossible consequence results necessarily from that negative realization joined to the posited actuality of the positive realization. But it would be giving in to the sophism of the distribution of the modalities to conclude that the possibility of the non-realization of an event is incompatible with its realization. The logical incompatibility is in fact only between the non-realization and realization.

Since this consequence is unquestionably authorized by the passage however, one of two things must be the case. Either Aristotle fell victim to the illusion we have just censured, or he took this incompatibility of the actuality and the contrary capacity to be a specific principle. To decide the issue let us examine the consequences of this posited incompatibility.

By the reciprocal definitions of the modalities, if p is necessary then it is not possible that not- p . The necessity of a thing, then, excludes the contrary potentiality of that thing. To maintain that the actuality of a thing excludes the contrary potentiality of that thing is thus to assimilate actuality and necessity in that respect. But it will be seen that in Chapter IX of *De Interpretatione* Aristotle does explicitly assimilate actuality and necessity when he defines conditional necessity. Simple necessity is the prerogative either of the insensible beings or of the sempiternal substances, which always are in actuality. Contingent substances sometimes are in actuality and sometimes are not. In so far as, and while, they are in actuality they behave as necessary substances. Only that necessity is temporally conditioned by the actuality, and ceases with it. In short, if there are differences between the eternal actuality of God, the sempiternal actuality of celestial things and the temporal actuality of sublunar things, these differences are not differences of degree and the actuality of a contingent thing has, for a finite time, the same necessity as that belonging to the actuality of an eternal or sempiternal thing.

¹²The sophism of the distribution of the modalities may be written thus:

$$* \sim M(\exists t)(p_t \sim p_t) \supset \sim M(\exists t)(p_t \cdot M_t \sim p_t).$$

The principle of conditional necessity is written:

$$\sim (p_t \cdot M_t \sim p_t) \quad \text{or} \quad \sim (\sim p_t \cdot M_t p_t);$$

that is to say:

$$M_t \sim p_t \supset \sim p_t \quad \text{or} \quad M_t p_t \supset p_t.$$

Does the principle of conditional necessity provoke a collapse of the modalities? It does in that, *while p*, one can conclude the necessity of *p* from *p*. But that conclusion can in no way be detached from its condition,¹³ which limits the extent of the collapse.

The principle of conditional necessity seems to contradict numerous passages where Aristotle speaks of the possibility for a thing to be in a state opposed to that in which it is in actuality.¹⁴ For example, Socrates, who is sitting, might be standing. The contradiction disappears if, as we have proposed, a distinction is made between the logical modalities and the real or temporal modalities, with the validity of the principle of conditional necessity limited to these latter. A temporal modality will be affected by a double temporal index: one situating the modality itself in time, the other situating the event with which the modality has to do. Suppose it possible at time t_1 (in the sense of “real” possibility) that a certain event should take place at time t_2 . Suppose further that at time t_1 the event in question is not taking place. What the principle of conditional necessity tells us under such conditions is that time t_1 cannot be identical with time t_2 . On the other hand, there is nothing contradictory in maintaining that it is possible at t_1 that the event non-realized at t_1 be realized at any time t_2 distinct from t_1 .¹⁵

2.6 The irrevocability of the past or the principle of the impossibility of realizing the possible in the past.

At the end of the second chapter of the sixth book of the *Nicomachean Ethics*, Aristotle develops the following assertion (1139^a 13-14): “No one deliberates about things which are not susceptible of being other than they are”. And he winds up by saying: “concerning the past there is no longer any choice. That is why no one chooses that Illium should have been ravaged; for no one deliberates about that which has been, but only about what will be and what is possible (*ἐνδεχόμενον*); and indeed what has been cannot not have been. This is why Agathon

¹³19^a 22-23.

¹⁴Especially *Metaphysics* Θ, 3 (1047^a 20-29, for example).

¹⁵The logical product of the principle of synchronic contraction of the possible and the principle of conditional necessity has as consequence, in conformity with the *De Caelo* passage, the realization of the contracted possible:

$$\begin{aligned} & ((t)\{M_N p_t \supset (\exists t_1)[M_{t_1} p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)]\} \cdot (M_t p_t \supset p_t)) \supset \\ & (t)\{M_N P_t \supset (\exists t_1)[p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t_1 \leq t)]\}. \end{aligned}$$

is right to say: 'God Himself is hindered in this one thing: to make undone things that have once been done'." (1139^b5-11). In short, there is such a thing as deliberation. Deliberation has only to do with what can be contingently. Nothing of what is past can be contingently. The past is not among the objects of a possible deliberation. The same consequence is drawn in the sentence of lines 13 and 14 of the *De Caelo* passage: "There is no potentiality of the past but only of the present and the future". For if the past is irrevocable it is not an object of deliberation. Not being an object of deliberation, there is no (contingent) potentiality with respect to it, all potentiality is regarding the present or the future. From the fact that the contingent potentiality of opposites regards the present or the future, the irrevocability of the past follows by contraposition: this irrevocability residing in the disjunction of the 'necessity' of the affirmative member or the 'necessity' of the negative member.

This disjunction of retrograde modalities could in no way be justified by arguing that if something has taken place then it is not logically possible that it should not have taken place. The logical impossibility in question would be atemporal, even though having to do with an event dated in the past. But once endowed with its two temporal indices, this necessity, unlike real possibility, will be seen to take on two distinct developments. Its natural movement will be in the same direction as the natural movement of real possibility: *i.e.*, the direction going from past to future. But to this movement there will be added another, retrograde and specific, no longer applying to things, but to our relation to things. It is in virtue of this relation that the necessity of the past will be spoken of. Let us then give each of the modalities a double temporal index. Let us say that it is possible, in the sense of a 'real' possible, at time t_1 that a certain event should take place at time t_2 . What Aristotle asserts is that t_2 can not be anterior to t_1 . What is specific to irrevocability will be sufficiently distinguished from logical necessity by the assignment of a temporal index proper to the modality; and it will be distinguished from real necessity by the exceptional order of the indices relative to the modality and to the event.

Lines 6-7, 12 and 16 of the *De Caelo* passage add yet a further determination to the principle of the irrevocability of the past. There it is said that if one were to admit that there is a (contingent) potentiality of the past one would be led to say of the now that it is last year, and of the last year that it is now. Suppose in fact that we were to deliberate about the fall of Troy. There would result the possibility both that Troy should have fallen and that Troy should not have fallen, and since such a possibility is open only to present action, the past time of the event

would have to come to be identified with the present and the present with the past. In this way it would be denied that the past occupies a different time from the present.

This text confirms the existence of a double temporal index of the modalities, once they are used to characterize substances. The principle of the impossibility of realizing a possible in the past illustrates the constraints affecting this double index just as the principle of conditional necessity does. To say that a possible, which is forward looking, has to do with the past, is to completely upset the order of time. It comes down to postulating that the index of the possible at t_1 , which must, by definition, be equal to or anterior to the index t_2 of the event whose possibility is in question, can, at the same time, be posterior to it¹⁶.

¹⁶This is attested to by the following Commentary of Simplicius: In L. *De Caelo* I, 12, Ad 283^b6, 1894, pp. 355-356: "Having shown that the proposition saying that a generated thing is indestructible implies that that thing has the capacity of contraries' belonging to it at the same time, he goes on immediately to raise the objection that can be brought against this line of reasoning. That which is both generated and indestructible has the capacity of not being in the direction of the past, since it was non-existent before being, and the capacity of being in the direction of the future, since it is supposed indestructible. It is nevertheless not at the same time that it will have the capacity of being and of not being, so that it won't have the contraries in actuality either. But disposing of this objection, he says that all potentiality is in the direction of present or future time. For above all else we call possible the things which are not yet but are capable of becoming, differing from existing things in that they will be but are not yet. If it is not true then as regards anything *to say now that it is last year or that it is not last year* (the two lessons exist = ἀμφοτέρως γὰρ γράφεται); for it is not true to say now that the time of last year is, nor of any event that took place last year that it is now, but neither was it true to say last year that now is a past of the time which took place at the end of the year finished. In effect it is impossible to interchange the times.

If this is then true, it is impossible that something which at some moment does not exist should later be sempiternal, i.e., that that which was generated should continue to be indestructible for the rest of time. Indeed, since that which afterwards is was first inexistent it will also have, once having attained to being, the possibility of not being, though not that of not being then when it has already attained to being: indeed, at that moment it is supposed to be in actuality. It is thus necessary that such an entity should have the potentiality of last year and in the past. This is absurd, since there is no potentiality of that which has happened, but only of that which is and of that which will be. Aristotle says even more clearly moreover: 'let that of which it has the capacity exist in actuality: it will then be true to say now regarding that which has the capacity now of not existing, not only that it has last year the capacity of not existing, but even that *it does not exist last year*'. What is even more absurd, the now itself is in the fact of not existing last year. For last year will be now. Now is in effect supposed to have the capacity of not existing last year. It is thus clear that the following reading is the more consequent, namely, 'that it does not exist last year'. And it is in fact by these

Such a derangement of time leads to a physical impossibility and it comes down to the same logically to say either that when true, the

words that he concludes: 'it will thus be true to say now that it does not exist last year'. He nevertheless immediately applies the fact of existing last year to that which is ungenerated though destructible. But it is right to say that there is no potentiality of that which has happened: for the whole past is necessary (*τὸ γὰρ παρεληλυθὸς πᾶν ἀναγκαῖον*) and is said to be neither possible nor contingent (*καὶ οὔτε δυνατόν οὔτε ἐνδεχόμενον λέγεται*)".

The interpretation of premise A in the logic of the real modalities must fulfill the following three conditions:

- a) it must account for Epictetus' text saying that everything past and true is necessary,
- b) it must express the relation between deliberation and thus possibility *ad opposita*, on the one hand, and the orientation of the possible towards the future, on the other hand,
- c) it must take into account the Aristotelian text saying that a possible oriented towards the past would overturn the order of time.

Let us define what is now contingent as that which is such that it is possible now that p take place at t and that p not take place at t :

$$C_N p_t =_{Df} M_N p_t \cdot M_N \sim p_t$$

(Definition of the contingent object of deliberation).

a) In leaving aside mention of the truth, which is implicit in the formulas, the first condition can be written:

$$(1) p_t \cdot t < N \supset L_N p_t$$

and, by symmetry:

$$(2) \sim p_t \cdot t < N \supset L_N \sim p_t.$$

The logical product of the two has as consequence:

$$(3) t < N \supset (L_N p_t \vee L_N \sim p_t).$$

In effect,

$$\vdash [(P \cdot Q \supset R) \cdot (\sim P \cdot Q \supset S)] \supset [Q \supset (R \vee S)],$$

with Sb: p_t/P , $t < N/Q$, $L_N p_t/R$, $L_N \sim p_t/S$, 2 x Modus Ponens, (1), (2).

$$(4) (L_N p_t \vee L_N \sim p_t) \equiv (\sim L_N p_t \cdot \sim L_N \sim p_t) \equiv$$

$$\sim (M_N \sim p_t \cdot M_N p_t) \equiv \sim (M_N p_t \cdot M_N \sim p_t) \equiv C_N p_t.$$

$$(5) t < N \supset C_N p_t.$$

b) The second condition is written:

$$(6) C_N p_t \supset N \cong t.$$

The contrapositive of (6) is none other than (5). Consequently, to say that what is past and true is necessary (Diodorus according to Epictetus) comes down to the same as saying that there is no contingent but of the present or the future (Aristotle).

c) The two expressions (5) and (6) are equivalent to:

$$(7) \sim (C_N p_t \cdot t < N)$$

which is none other than the second term of the logical conjunction constituting the principle of the irrevocability of the past in the *De Caelo*. This principle, henceforth 'A', contains however another term as well:

$$(8) (t)[(C_N p_t \cdot t < N) \equiv (t < N \cdot N = t)].$$

To realize the contingent in the past is to completely upset the order of time in saying that today is last year. In virtue of (7) the first member of equivalence (8) is false, thus so is the second.

$$(9) (t)[t < N \supset \sim (N = t)]$$

past is necessary, as in the Master Argument's first premise, or that it is impossible to realize a contingent in the past without upsetting the order of time, as in the *De Caelo*, or again, that such a realization leads to positing that time has two opposite directions, as in the *Nicomachean Ethics*.

The following objection will come to mind. According to the principle of conservation of modal status every substance possesses an existence which is either immaterial and thus atemporal and absolutely necessary, or sensible but sempiternal and thus necessary although present in time, or, again, material, generable and destructible, and thus contingent. But a contingent substance sometimes is and sometimes is not. In virtue of the principle of conditional necessity, when it is, it is necessary while it is. When it is not, it is not with an equal necessity and for the same reason. Then too, once it has been, it is no longer possible that it be or it is no longer possible that it not be, in virtue of the "necessity" of the past. The modality of the existence of such a substance is thus subject to change, contrary to the principle of conservation of modal status.

But what must be distinguished here is the modal status of a substance and the modality which expresses this status. The modal status of a necessary sensible substance is expressed by the modality *necessary*, but because that modality *necessary* expresses the essence, it must be posited sempiternally. Due to the principle of conditional necessity, the modal status of a contingent sensible substance is expressed at one time by conditional necessity, at another by conditional impossibility; but these modal changes, relative as they are to a duration, simply translate the immutability of the modal status characteristic of

In other words, the present is not eternal. Aristotle thus rejects, explicitly in (8), and implicitly in (1) and (2) or in (6), the cyclical representation of time.

Intuitively, (8) expresses the case of the *De Caelo*: the negative possible or the inexistence of the world resulting from creation is realized in the past, whereas the positive possible resulting from the sempiternality of the world is realized in the future; by symmetry, (9) expresses the case of the Master Argument. These two statements are paradoxical because the two "pieces" of the contingent take opposite temporal directions.

They become contradictory when the two pieces of the contingent are asserted simultaneously. This is the case in the *De Caelo*, since, from the sempiternality of the world results its positive possibility. It is the case in the Master Argument, since, from the present and future non-realization of the possible results its negative possibility.

(A) $M_N p_t \cdot (t < N \cdot M_N \sim p_t) \supset (M_N p_t \cdot M_N \sim p_t)$
 (simplification of the product) $\supset \sim (t < N)$ (by (4))

(A') $M_N \sim p_t \cdot (t < N \cdot M_N p_t) \supset \sim (t < N)$.

This is the true sense of the 'necessity' of the past.

contingency, a thing being contingent precisely if sometimes it is and sometimes it is not.

2.7 The principle of the subsistence of a possible that is not to be realized.

At lines 9 and 10 of the *De Caelo* passage Aristotle asserts that, because of the conservation of the modalities, this created world, which was consequently non-existent in the past, conserves and will always conserve the potentiality of not existing, but that since it does exist and, by hypothesis, always will, this potentiality of not existing will, because of conditional necessity, never be actualized. (That is why the only means of actualization of this possibility to be found will be to attempt to actualize it in the past).

The Master Argument's third premise will simply substitute for the consideration of a negative possible, which by hypothesis will never be actualized, that of a positive possible which, also by hypothesis, will never be actualized either.

This third premise deserves further scrutiny, albeit because of only a minor difficulty. In translating the Greek text as saying that the possible is that which is not presently true and never will be so, one might mean to say that no possible will ever be realized.

A possible that becomes actualized would falsify such a clause, which is unjustified even as regards the possibles *in utraque*.

If one is tempted to accept this extreme interpretation, it is that its negation is maintained by Diodorus when he says that that which will never be actualized is impossible.

As regards the relations of the possible to actualization—inevitably future, since there is no potentiality of the past—there are three conceptions that arise *a priori*. 1) One can, with Diodorus, treat the possibles and the actualized futures (taking future here in the broad sense which includes the present) as equivalent. This same thesis has been mistakenly ascribed to Aristotle because of a confusion between virtues which verify and chances which falsify it.¹⁷ 2) One can, with Leibniz, hold

¹⁷Possibles *ad unum* are possibles which result from active irrational potencies (fire can but heat). Possibles *in opposita* are possibles which result either from rational potencies containing a deliberation (the doctor can give a remedy or a poison to the patient) or from passive irrational potencies (air can be heated or cooled) (in *de Int.*, 22^b36-23^a5). Let me summarize Cajetan (in *St. Thomas*, Fretté, 1875, 88-89; Oesterle, 1962, 217-221). These two sorts of possible are each defined with respect to that which is susceptible of movement. The possible *ad unum*, however, can, by equivocation, apply to the immobile beings of mathematics and of logic. For that extended class of possibles (*ad unum*), the thing is said possible because it is in act. Contrarywise, for the possibles *in opposita*, the thing is said possible

that the futures constitute a proper part of the possibles: they are the events yet to take place of this world which was chosen by God, along with the past and the present, as being the best. By possible, then, is to be understood any idea in the mind of God. Those possibles that are realized are realized by virtue of a "conditional necessity" which is itself beyond the pale of chance and contingency and is thus fit to retrograde, since it serves only to mark that which, in Creation, distinguishes the best from the non-contradictory. 3) One can again, with Aristotle, distinguish two sorts of futures. Some have virtues that could not not be realized. The others are contingent possibles. Among these latter, some will be realized, but there remain others that will never be realized. The range of possibles, then, is by no means exhausted by the futures that will be realized. Diodorus wants to show that no non-realized future is possible. But he need not for that suppose that his adversary holds that all possibles are not and will not be; the fact that Aristotle admits of the non-realization of some possibles is already enough.

This is clearly the way Bayle understood the Master Argument's third proposition. "The very famous dispute of possible things and impossible things owed its inception to the doctrine of the Stoics regarding fate. The question was whether, among the things which never have been and will never be, there are possibles, or if all that is not, all that never has been, all that never will be, was impossible".¹⁸ And at

because it can be actualized (in *de Int.*, 23^a6-23^a15). As for the natural (irrational) possible *ad unum*, Cajetan (in *St. Thomas*, 1875, vol. 22, p. 89; Oesterle, 1962, p. 219), in referring to the *Metaphysics* (Θ, 2, 1046^a36 *sq.*) remarks that, besides their inapplicability to opposites, the active potency which is their *raison d'être* enters necessarily into operation as soon as the subject is present and impediments are removed. Heat necessarily heats a material which is present, once all insulation is removed.

To keep the original sense of the possible *ad unum*, that of a virtuality which develops necessarily, once all impediments are dropped, let us call it a virtue. To speak of the virtue of *p* is to say that it is possible that *p ad unum*. On the other hand, let us call a possible *in opposita* chance. To speak of the chance of *p* is to say that it is possible contingently that *p*.

Must we grant the axiom of necessity for virtue, while refusing it for chance (23^a15-16)? If it is necessary that a material substance perish then it is possible *ad unum* that it perish. This is in contrast to chance. If Socrates can be seated and not be seated, *in opposita*, then there is no necessity of either one.

It must be added that a possible *ad unum* develops its actualization only on the supposition of the existence of its subject. But that existence itself remains contingent and has to do with possibles *ad opposita*. Once born, it is necessary that Socrates die. That Socrates be born is contingent.

¹⁸Leibniz, Gerhardt, 1978, p. 212; Jalabert, 1962, p. 223.

the end of the passage¹⁹ he goes on to add: "I think that the Stoics took it upon themselves to give a greater extension to possible things than to future things in order to soften the awful and odious consequences drawn from their dogma of fatalism". Leibniz too gives reasons, and different from Aristotle's, as to "why one is mistaken or at least speaks very incorrectly when one says that there is no possible but that which is or what God has chosen; this is the error of Diodorus the Stoic in Cicero, and of Abélard, Wyclef and Hobbes among the Christians".²⁰

Cicero's text is indeed definite. In the *De Fato* (VII, 13) he interprets this third premise as: "Some events which will not take place are possible", when "Diodorus says 'That alone (*solum*) is possible which is either true or will be true; whatsoever (*quicquid*) will not be is impossible'".

The Master Argument's third premise will then be accorded the same sense as the corresponding *De Caelo* premise—the sense of a particular proposition: namely, there is a possible that will not be realized. It is precisely because this premise is a particular that we are permitted to reason on an illustrative case. In doing this, we will be justified in construing the premise as a conjunction where it is affirmed that a same event is possible which neither is nor will be realized.²¹

2.8 Reconstruction of the *De Caelo* demonstration.

What is meant to be shown in the *De Caelo* is an incompatibility in the Platonic cosmological hypothesis according to which the world, although engendered, is imperishable. And this is to be done by taking into account only the principles of the conservation of modal status, the possible realization of the possible coupled with conditional necessity, and the impossibility of realizing the possible in the past.

In virtue of the conservation of modal status, what has been inexistent will conserve in the future the possibility of not existing. Such is the case with the world. If the world has been engendered and is from now on sempiternal, it is from now on sempiternal while conserving in the future the possibility of not existing.

¹⁹Leibniz, Gerhardt, 1978, p. 215; Jalabert, 1962, p. 225.

²⁰Leibniz, Gerhardt, 1978, p. 442, §22; Jalabert, 1962, p. 448, §22.

²¹The *De Caelo* subsistence principle will thus be written:

$$(\exists p)[(\exists t)(\sim L_N p_t \cdot N \cong t) \cdot (t)(N \cong t \supset p_t)]$$

whereas the third premise of the Master Argument will result from the above by simple substitution of $\sim p$ for p :

$$(\exists p)[(\exists t)(\sim L_N \sim p_t \cdot N \cong t) \cdot (t)(N \cong t \supset \sim p_t)].$$

Examining the consistency of the Platonic hypothesis, then, comes down to examining whether it is possible for the world to exist sempiternally simultaneously with its conserving sempiternally the potentiality of not existing.

According to the principle of the possible realization of the possible, a thing's (here the world's) sempiternal capacity of not existing should be able to be realized. It can and will be able not to be at every future instant of time from "creation" on. Consider such an instant, say t_1 . Given that a possible is situated by a double temporal index, t_1 and t_2 , fix t_1 , leaving t_2 arbitrary. In other words, the possible realization of the inexistence of the world signifies that at t_1 it is possible that the world should become inexistent at t_2 .

Instant t_1 determines three cases of the universe, that is to say, three possible worlds. Either t_2 , which situates the inexistence of the world in time, is anterior to, identical with, or posterior to t_1 . This is an exhaustive distribution of the universe.

Because of the impossibility of realizing a possible in the past, the situation is physically contradictory if t_2 is anterior to t_1 . Again the situation is contradictory on the hypothesis of the identity of t_2 and t_1 , and this by virtue of the principle of non-contradiction, since it is supposed that the world exists at t_1 . But the possibility $M_{t_1} \sim p_{t_1}$ entails $\sim p_{t_1}$, by conditional necessity, and this is the contradictory of p_{t_1} . For there to be contradiction if t_2 is posterior to t_1 , it suffices to consider the interval $t_1 t_2$. We posit that it is possible at t_1 that the world not exist at t_2 . By the principle of contraction of the possible, there is then a t on the interval $t_1 t_2$ at which it is possible that the world not exist at t . But because of conditional necessity this possibility is realized. By hypothesis, however, the world exists at t . There is thus a contradiction for we have simultaneously p_t and $\sim p_t$. If no case of the universe representing the disjunction of all cases is possible, then the universe itself is not possible.

Thus the realization of the possible, viz. of the possible inexistence of the world, produces an impossible consequence. It is thus itself impossible. In short, granted the principles of the possible realization of the possible, conditional necessity, and the impossibility of realizing the possible in the past, it is impossible to affirm at the same time that the world exists and will always exist and that its inexistence should become act.

In a second argument Aristotle demonstrates in symmetrical fashion that an unengendered world that will perish is impossible. One gets from the first to the second demonstration in changing the affirmations into negations. Let us only point out that in this case the principle of

the impossibility of realizing the possible in the past takes on a positive form: it is impossible to realize in the past, not now the inexistence but rather the existence of the world.

2.9 Reconstruction of the Master Argument.

There are only two points of difference between the Master Argument and the *De Caelo* passage considered. First, the Master Argument is simpler, as it makes no use of the principle of conservation of modal status. Secondly, the third premise of the Master Argument speaks of a possible that will not be realized, whereas the Platonic hypothesis specified that this possible was the destructibility of the world.

To fix our ideas, let us suppose the Master Argument's possible to be situated at the present moment.

Then the event of which it is the possibility will be realized either in the past, at present, or in the future.

In virtue of the principle of the impossibility of realizing a possible in the past (premise A), the first case of the universe is excluded as being physically contradictory.

Realize the possible then at present. According to the principle of the possible realization of the possible, it is now possible that p now. But, by virtue of conditional necessity, it is the case that p now. In the third premise, however, it is supposed that not- p now. Thus the second case of the universe is excluded by virtue of the principle of non-contradiction.

Realize then the possible in the future. By the principle of contraction of the possible there is an instant t , on the interval between the present and that future, at which it is possible at t that this possible be realized at t . Thus, granted conditional necessity, there is an instant t on the interval between the present and that future where p is the case at t . But it had been supposed that the realization will never take place. The third case of the universe is excluded by virtue of the principle of non-contradiction.

With the three cases that exhaust the universe excluded as impossible, the universe, disjunction of these three cases, is itself contradictory. Therefore, since the realization of the possible necessarily produces an impossible consequence, there is an incompatibility between a possible that will never be realized and that realization.

Thus a possible that will never be realized entails necessarily an impossible consequence, if, along with the principle of conditional necessity and that of the possible realization of the possible, one admits the principle of the irrevocability of the past. Diodorus turns against

Aristotle the argument that Aristotle himself had developed against Plato. Aristotle had combatted the creation and destructibility of the universe. Diodorus combats the Aristotelian conception of a potency that does not become actualized, that is to say, the representation of future alternatives as required by the Stagirite to serve as a basis for natural contingency and human freedom.

Against the validity of the Master Argument it has been alleged that it uses simultaneously two different concepts of necessity: one logical, the other real. But this objection turns out, in fact, to be futile.

From the conjunction of the two premises A ("necessity" of the past) and C (subsistence of a possible that does not become actualized) together with the principle B of the possible realization of the possible (that is to say, of the synchronic contraction of the diachronic possible), it follows that for some instant t , identical with or posterior to now, it is possible at t that the event subject to this possibility be realized at t , even though this event is not realized at t . But by conditional necessity it follows that at t the event subject to this possibility is realized at t , although C prohibits, by hypothesis, that that event be realized at t . Thus we arrive at a conjunction of two contradictory propositions without its mattering in the least that non-logical possibilities occur in each of them. The same remark can be made about the impossibility of realizing the possible in the past. Even if A expresses no more than a physical impossibility—that of reversing time—once it is admitted, it does, for the past, preclude the possibility of contracting the possible into the past and thus logically bars one of the three alternatives constituting the universe of discourse. The conjunction of C, B, A and of hypothetical necessity has thus a logical impossibility as consequence. Therefore it is not logically possible itself. Q.E.D.

2.10 Sketch of a formal reconstruction of the Aristotelian reasoning at *De Caelo*, I, 283^b6-17.

Axioms

The sign ' \leq ' is taken here as designating a relation which is simply antisymmetric, reflexive, transitive and connected (a simple ordering

relation).

- D $C_{t_0}p_t =_{Df} \sim L_{t_0} \sim p_t \cdot \sim L_{t_0}p_t$.
 (a) $(t_o)[(\exists t')(t' < t_o \cdot \sim p_{t'}) \supset (t)(t_o \leq t \cdot \sim L_{t_0}p_t)]$
 A_A $(t)(t_o)\{[(C_{t_0}p_t \cdot t < t_o) \equiv (t < t_o \cdot t_o = t)] \cdot \sim$
 $(C_{t_0}p_t \cdot t < t_o)\}$
 B_A $(t)(t_o)\{\sim L_{t_0}p_t \supset (\exists t_1)[M_{t_1} \sim p_{t_1} \cdot$
 $(t \leq t_1 < t_o \vee t_o \leq t_1 \leq t)]\}$
 C_A $(t_o)(\exists p)[(\exists t')(t_o \leq t' \cdot \sim L_{t_0}p_{t'}) \cdot (t)(t_o \leq t \supset p_t)]$
 NH_A $(t)(M_t p_t \supset p_t)$
 * $(t)(p_t \supset \sim L_{t_0} \sim p_t)$

*Demonstration*²²

1. $(t_o)(t)[t_o = t \supset t_o = t \vee t_o < t]$
 $(\vdash P \supset P \vee Q)$.
2. $(t_o)(t)[(C_{t_0}p_t \cdot t < t_o) \equiv (t < t_o \cdot t_o \leq t)]$
 $(A_A, \text{Simplification, 1, Syll.})$.
3. $(t_o)(t) \sim (t < t_o \cdot t_o \leq t)$
 $(A_A, \text{Simplification, 2, } \vdash (\sim P \cdot P \equiv Q) \supset \sim Q)$.
4. $(t_o)(t)\{(C_{t_0}p_t \cdot t_o \leq t) \supset (\sim L_{t_0} \sim p_t \cdot (\exists t_1)[M_{t_1} \sim p_{t_1} \cdot$
 $((t_o \leq t \cdot t \leq t_1 < t_o) \vee (t_o \leq t \cdot t_o \leq t_1 \leq t))]\}$
 (Add $\sim L_o \sim p_t \cdot t_o \leq t$ in the premise and in the
 consequent of B_A).
5. $(t_o)(t)[(C_{t_0}p_t \cdot t_o \leq t) \supset (\exists t_1)(M_{t_1} \sim p_{t_1} \cdot t_o \leq t_1 \leq t)]$
 (4, Simplification, 3).
6. $(t_o)(t)(C_{t_0}p_t \cdot t_o \leq t) \supset (\exists t_1)(\sim p_{t_1} \cdot t_o \leq t_1 \leq t)$
 (5, $NH_A, \vdash [(P \supset Q \cdot R) \cdot (Q \supset S)] \supset [P \supset (S \cdot R)]$).

²²I thank Mr. Jean Mosconi for his help.

7. $(t_o)(t)[(C_{t_o}p_t \cdot t_o \leq t) \supset (\exists t_1)(\sim p_{t_1} \cdot t_o \leq t_1)]$
(6, Simplification).
8. $(t_o)(\exists t)(C_{t_o}p_t \cdot t_o \leq t) \supset (\exists t_1)(\sim p_{t_1} \cdot t_o \leq t_1)$
(7)
9. $(t_o)(\exists p)\{(\exists t')(t' < t_o \cdot \sim p_{t'}) \supset [(\exists t)(t_o \leq t \cdot \sim L_{t_o}p_t) \cdot (t)(t_o \leq t \supset p_t)]\}$
(*(a)* with simplification, $C_A, \vdash [(P \supset Q) \cdot Q \cdot R] \supset [P \supset (Q \cdot R)]$)
10. $(t)(p_t \supset \sim L_{t_o} \sim p_t)$
(*)
11. $(t_o)(\exists p)\{(\exists t')(t' < t_o \cdot \sim p_{t'}) \supset [(\exists t)(t_o \leq t \cdot C_{t_o}p_t) \cdot (t)(t_o \leq t \supset p_t)]\}$
(9, 10, $\vdash \{[P \supset \{(\exists x)(Fx \cdot Gx) \cdot (x)(Gx \supset Hx)]\} \cdot (x)(Hx \supset Jx)\} \supset \{P \supset [(\exists x)(Fx \cdot Jx \cdot Gx) \cdot (x)(Gx \supset Hx)]\}$)
12. $(t_o)(\exists p)\{(\exists t')(t' < t_o \cdot \sim p_{t'}) \supset [(\exists t_1)(t_o \leq t_1 \cdot \sim p_{t_1}) \cdot (t)(t_o \leq t \supset p_t)]\}$
(11, 8, $\vdash \{(\exists p)[P \supset (Q \cdot R)] \cdot (Q \supset S)\} \supset (\exists p)[P \supset (S \cdot R)]$)
13. $(t_o)(\exists p)(\exists t)(\exists t_1)(t \leq t_o \leq t_1 \cdot \sim p_t \cdot \sim p_{t_1} \cdot p_{t_1})$
(12)
14. $\sim ((a) \cdot A_A \cdot B_A \cdot C_A \cdot NH_A)$.

2.11 Sketch of a formal reconstruction of the Master argument.

Axioms

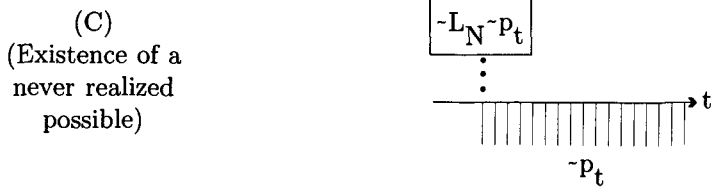
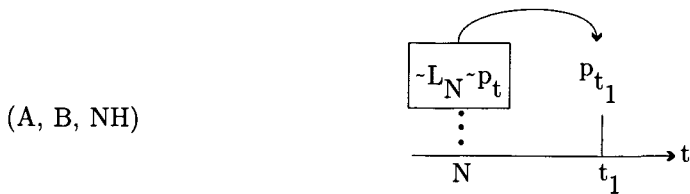
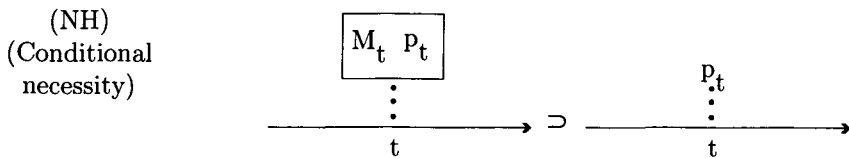
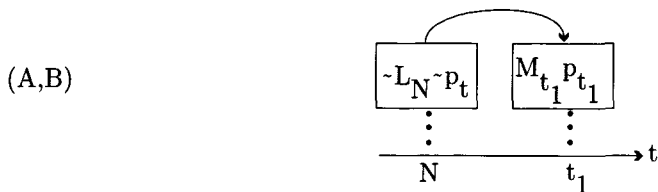
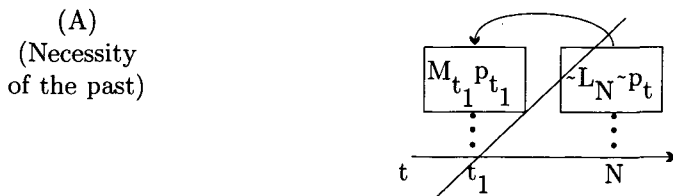
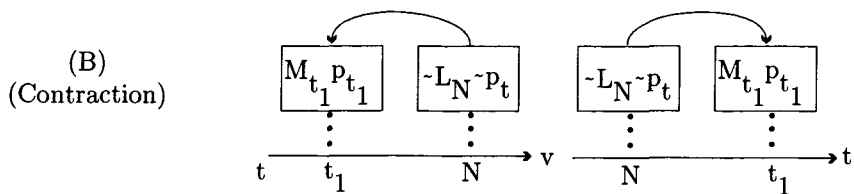
The sign ' \leq ' is to be taken here in the same sense as in the preceding paragraph.

- | | |
|-----------|--|
| <i>D</i> | $C_N p_t =_{Df} \sim L_N \sim p_t \cdot \sim L_N p_t$. |
| <i>A</i> | $(t) \{[(C_N p_t \cdot t < N \equiv (t < N \cdot N = t)) \cdot \sim (C_N p_t \cdot t < N)]\}$ |
| <i>B</i> | $(t) \{\sim L_N \sim p_t \supset (\exists t_1)[M_{t_1} p_{t_1} \cdot (t \leq t_1 < N \vee N \leq t \leq t_1)]\}$ |
| <i>C</i> | $(\exists p)[(\exists t')(\sim L_N \sim p_{t'} \cdot N \leq t') \cdot (t)(N \leq t \supset \sim p_t)]$ |
| <i>NH</i> | $(t)(M_t p_t \supset p_t)$ |
| * | $(t)(p_t \supset \sim L_N \sim p_t)$ |

Demonstration

1. $(t)[N = t \supset N = t \vee N < t \supset N \leq t$
 $(\vdash P \supset P \vee Q)$
2. $(t)[(C_N p_t \cdot t < N) \equiv (t < N \cdot N \leq t)]$
 $(A, \text{Simplification, 1, Syll.})$
3. $(t) \sim (t < N \cdot N \leq t)$
 $(A, \text{Simplification, 2, } \vdash (\sim P \cdot P \equiv Q) \supset \sim Q)$.
4. $(t)\{(C_N p_t \cdot N \leq t) \supset (\sim L_N p_t \cdot (\exists t_1)[M_{t_1} p_{t_1} \cdot$
 $((N \leq t \cdot t \leq t_1 < N) \vee (N \leq t \cdot N \leq t_1 \leq t)))]\}$
 $(\text{Add } \sim L_N p_t \cdot N \leq t \text{ in the premise and}$
 $\text{in the consequence of } B)$.
5. $(t)[(C_N p_t \cdot N \leq t) \supset (\exists t_1)(M_{t_1} p_{t_1} \cdot N \leq t_1 \leq t)]$
 $(4, \text{Simplification, 3})$.
6. $(t)[(C_N p_t \cdot N \leq t) \supset (\exists t_1)(p_{t_1} \cdot N \leq t_1 \leq t)]$
 $(5, NH, \vdash [(P \supset Q \cdot R) \cdot (Q \supset S)] \supset [P \supset (S \cdot R)])$.
7. $(t)[(C_N p_t \cdot N \leq t) \supset (\exists t_1)(p_{t_1} \cdot N \leq t_1)]$
 $(6, \text{Simplification})$.
8. $(\exists t)(C_N p_t \cdot N \leq t) \supset (\exists t_1)(p_{t_1} \cdot N \leq t_1)$
 (7)
9. $(t)(\sim p_t \supset \sim L_N p_t)$
 $(*, Sb \sim p_t/p_t)$.
10. $(\exists p)[(\exists t')(C_N p_{t'} \cdot N \leq t') \cdot (t)(N \leq t \supset \sim p_t)]$
 $(C, 9, \vdash \{(\exists x)(Fx \cdot Gx) \cdot (x)(Gx \supset Hx) \cdot (x)(Hx \supset$
 $Jx)\} \supset (\exists x)(Fx \cdot Jx \cdot Gx) \cdot (x)(Gx \supset Hx))$.
11. $(\exists p)[(\exists t_1)(p_{t_1} \cdot N \leq t_1) \cdot (t)(N \leq t \supset \sim p_t)]$
 $(\vdash [(\exists P)(P \cdot Q) \cdot (P \supset R)] \supset (\exists P)(R \cdot Q), 10, 8)$.
12. $(\exists p)(\exists t_1)(p_{t_1} \cdot N \leq t_1 \cdot \sim p_{t_1})$
 (11)
13. $\sim (A \cdot B \cdot C \cdot NH)$.

The following diagram will help the reader get a firmer grip of the demonstration.



Part II

Systems of Necessity: The Megarians and the Stoics

A System of Logical Fatalism: Diodorus Cronus.

This part of the book will be consecrated to an analysis of the different responses which Epictetus tells us the Ancients made to the Master Argument.

By means of a purely combinatory consideration Epictetus' text deduces the possible responses. It amounts to the rejection of one of the three explicit premises of the argument, the other two being maintained. What we find thus laid out is an *a priori* system of the history of philosophy.

	Admitting	one refuses
1.	A and B	C
2.	B and C	A
3.	C and A	B

The first response is that of Diodorus, the second that of Cleanthes, the third that of Chrysippus. The history of the Megarian and Stoic philosophies is to be found encompassed in this combinatorial arrangement.

3.1 Diodorus' Solution.

If premise C of the Master Argument means that there exists a possible such that it neither is now nor ever will be realized, Diodorus' solution, which consists in denying that premise, must come down to affirming that no possible is such that it is not now and never will be realized.

Cicero's testimony confirms this interpretation. "For Diodorus then the only possible is that which is true or will be true".¹

Premise C, a particular proposition, related the modality to temporal determinations, without reducing it to them. It is this reduction that Diodorus will undertake. In spelling out that the only possible is that which is or will be true, Cicero furnishes the condition that permits defining the modality in temporal terms. Indeed, that that which is possible is or will be true stands as a necessary condition for something's being possible. Then again, since alone that is possible which either is or will be true, being true now or in the future constitutes the sufficient condition for something's being possible. When Boethius² writes that Diodorus defined the possible as that which either is or will be, he explicitly draws out the consequence resulting from the negation of C. For Diodorus the modalities have simply a derivative status. The words expressing them are nothing but abbreviated ways of introducing complex temporal determinations; that is to say, they are logically equivalent to conjunctions or disjunctions of such determinations.

Thanks to the interdefinability of the modalities, the definition in terms of temporal determinations just laid down for the case of possibility can be extended to them all. The impossible is that which neither is true nor will be so. Since, according to Diodorus, every declaration is true or false, the preceding definition could also be expressed by saying that the impossible is that which is false and will never be true. The necessary is that which is impossible not to be. Thus the necessary is that which is true and never will be false. Finally, it is possible that something not be when it is not necessary that it be. Consequently, the possibly not is that which is false or will be so. These definitions are attested to by Boethius.³ "Diodorus defines the possible as that which is or will be, the impossible as that which, being false, will not be true, the necessary as that which, being true, will not be false, and the non-necessary as that which either is or will be false".

Diodorus' solution has two qualities in its favor.

First, it affords a means of defining the modalities and thus of eliminating them as well.⁴ This renders modal sentences transparent

¹Cicero, *De Fato*, IX (17); Bréhier, 1962, p. 479; see above, 2.7, p. 31.

²Boethius, 1880, p. 234.

³Boethius, 1880, p. 234.

⁴Mrs. Kneale points out that Diodorus' is the only non-circular definition of the modalities in antiquity (1962, p. 125). There is nothing to prove however, that the Stoics and Peripatetics thought of themselves as defining the modalities (Mignucci, 1978, p. 334, note 43). It is possible to interpret their circular definitions as simply so many stipulations of the conditions that must be met by the modalities in their relations to temporal determinations.

in translating them into general statements in which the bound variable is time. The laws of modal logic are thus reduced to the syllogistic of categorical propositions.

Secondly, the definition formally prevents any collapse of the modal expressions that would reduce the possible to the actual and the actual to the necessary. This in no way contradicts what has just been said. The quantifiers serving to eliminate the modalities occur uneliminably in the *definiens*. The modal declarations considered are singular. They are translated into general temporal declarations, *i.e.* quantified declarations; and these, in turn, are not reducible to singular non-modal declarations.

If the possible is said to be that which is or will be, the actual cannot be derived from the possible, for one member of a disjunction (that which is) does not follow logically from the disjunction. Likewise, if the necessary is said to be that which is true and will not be false, the necessary cannot be derived from the actual, for a conjunction does not follow logically from one of its conjuncts (that which is true). It is right then to underline, as some have⁵, the formal capacity of the Diodorean system to preserve that which is specific to the modalities. The possible and the non-necessary, which are subcontraries, may both be true at the same time. The predicate *contingent* can thus be formally introduced in the Diodorean system. The contingent is that which is possible and non-necessary, *i.e.* the logical conjunction of that which is or will be and that which is not or will not be. As a result⁶ of this definition the contingent is that which is not and will be or that which is and will not be or that which will be and will not be.

The logical square of the modalities, according to Diodorus, sets out an arrangement of two opposing categories. In the first place, the necessary and the impossible contain two conjoined clauses. The first of these fixes the present truth-value of the proposition about which the modality is said to be. The second stipulates the exclusion, for all future time, of the contradictory truth-value. Such a definition could be compared to a complete induction in the mathematical sense of the term. Assignment of a predicate to the first ordinal number would correspond to the assignment of present truth-value, whereas the

⁵Blanché, 1965; Boudot, 1973, p. 440 and p. 444: to hold that possibility implies necessity "would be to affirm that if a statement like 'Socrates is standing' is true now or will be so at some moment of the future, then it is true and always will be so".

⁶ $(p \vee Fp) \cdot (\sim p \vee F \sim p) \equiv$
 $(p \cdot \sim p) \vee (p \cdot F \sim p) \vee (\sim p \cdot Fp) \vee (Fp \cdot \sim Fp) \equiv$
 $(p \cdot F \sim p) \vee (\sim p \cdot Fp) \vee (Fp \cdot F \sim p).$

inductive clause would be extended from the non-continuous domain of numbers to the contiguous domain of the instants of time. The definitions in question lay it down that a declaration is necessary or impossible when its truth-value does not change. The second modal category, on the other hand, is compatible with a change of truth-value though, given the sub-alternation relations between the modalities of the two categories, it doesn't imply one. The contingent, defined as a triple disjunction as stated above, specifies an inevitable truth-value change for itself. For if the contingent is that which, being, will not be or that which, not being, will be or that which will be and will not be (this last excluding that something both will be and will not be at the same moment), all three cases of the universe that realize contingency imply a change of truth-value for the declarations expressing it.

3.2 Two possible interpretations as regards the object of the Diodorean modalities: nominalism and realism.

Up to this point our account has remained neutral. The term "declaration" was used to designate that about which the Diodorean modalities were supposed to be, without inquiring into the nature of that object. But now such an inquiry is in order.

In scrutinizing the Diodorean definitions it will first be shown that if the declarations do lend themselves to an interpretation as sentences, they lend themselves equally well to an interpretation as propositions. What will remain to be shown then is why the truth conditions set down by Diodorus have given rise to yet another interpretation on which the declarations would be construed as propositional functions.

To begin with, Diodorus defines the modalities in assigning the truth-value conditions. Since it is these values alone that are determinant and since all the operations permitted in the linguistic constructions are extensional, there is no need to retain distinctions that make no difference whatsoever as regards these operations. An interpretation in terms of sentences here would come down to the same as one in terms of propositions. We shall not insist on their distinguishing characteristic, since they are in fact equivalent from the point of view of the definitions at hand. We shall be further ahead remaining non-committal in their regard, combining the two for the moment under the heading of *statement*⁷.

⁷A proposition is a class or sentences having the same meaning. The logical status of this notion has fallen under fire because of the difficulties involved in identifying propositions. This particular question does not have to be dealt with here.

In the second place, once this subsidiary question is taken care of, the Diodorean definition still leaves a doubt. In eliminating modality in

As a matter of fact, propositions have an essential occurrence only within opaque contexts of discourse, i.e. in contexts liable to change their truth-value in the case of substitution of expressions having the same denotation but not the same sense, and particularly of sentences having the same truth-value but not belonging to the same proposition. The modal operators are among these opaque contexts. For example, granted the identity:

Scott = the author of Waverley

and the modal logical truth

necessarily Scott is identical with Scott

it is doubtful that the following sentence can be held true:

necessarily the author of Waverley is identical with Scott.

But Diodorus, in defining the modalities the way he does, does not overstep the bounds or the means of expression characteristic of first order predicate logic. That is a logic comprising only the connectives (*and*, *or*, *not*) and quantifiers over only individuals and instants of time, which in turn guarantee its extensionality. All opaque contexts are excluded.

Under these conditions it is only by virtue of their truth-values that sentences enter into reasoning. Since sentences having the same meaning, have a *fortiori* the same truth value, any proposition could be replaced by one or the sentences belonging to it as to a synonymy class.

Why not then just settle for talk about sentences rather than insisting on the neutral term *statement*?

In the first place, Sextus Empiricus (*Adversus Mathematicos*, VIII, 11-13, and 132-139) speaks of the disagreement among the dogmatic schools on the subject of truth. Some placed truth and falsity in the thing signified, others in the sound, and others in the movement of the intellect. The first then distinguish the signifier or sentence, the reference or existing thing, and the signified or "proposition"; and it is to the proposition that they attribute a truth-value. This is the stance taken by the Stoics. The Epicureans do not admit or meanings (the signified) between the signifier and the thing. It is to the sentence that they attribute truth-value. Sextus does not specifically mention the adherents of the third doctrine, which we would call "psychologist" today. It can be seen that Aristotle doesn't hesitate to insist on the importance of beliefs. In any case, accordingly as one considers an individual movement of the intellect or a class of such movements, one falls back upon the distinction between sentence and proposition, psychologically interpreted. We do not know to which of these parties Diodorus belonged. It is possible that it should have been to the Stoic one.

In the second place, in its original formulation, the Master Argument does use modal operators; and it can be taken as refuting Aristotle only on the supposition that such operators have propositions as argument. Even if the Diodorean definitions free us from making that supposition, it is still a good idea to carry over a trace of it in the polemical means of expression Diodorus used.

It is important for the sequel (especially 3.5) that propositions, in the sense here adopted, are temporally indexed. For example, the proposition that I shall be in Paris on January 14, 1995, as the class of sentences like 'Ich werde am 14. Januar 1995 in Paris sein', is at the future as are all the sentences of the equivalence class. Therefore the truth-value shifts are the same for the proposition and for these

favor of a quantification over time, is the intention to turn the modality into a quantification that would fix the satisfaction conditions for a propositional function?⁸ In this case every modal proposition would be a particular or a universal general proposition. But is it also possible to construe quantification as being over the truth-values of complete statements, whether general or singular, and whether they be taken to be sentences or propositions?

In constructing models for the two sorts of interpretation it will be noticed that different consequences arise.

Consider the strong modalities first.

Suppose that necessity applies to a propositional function of which the argument is time. For such a function to be true now and never false in the future it is necessary and sufficient that it be true at the present moment and at all future moments. The Diodorean predicate 'true now and never false in the future', translated in terms of quantification over time, will have as consequence the temporal determination of the propositional function and its ineluctable transformation into a proposition about a permanent state of affairs, at least from the present moment on. The existence of the world on the Platonic hypothesis would be necessary, for instance. On the other hand, necessity would have to be refused to past and finished occurrences not only of such events as naval battles but even of such events as eclipses, the very model of necessary events for the Ancients. Interpreted in terms of propositional functions, the *necessary* signifies either a formal implication ('snow is white (at all times)'), or a singular proposition sempiternally true from an initial moment (now) on.

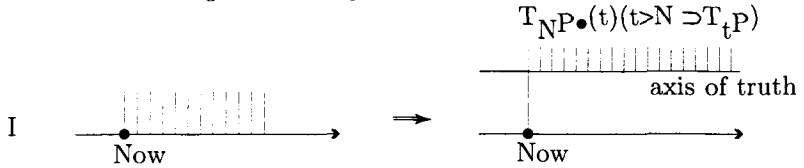
sentences. -A completely new situation would arise, were we to admit 'eternal dated propositions' such as 'My being in Paris on January 14, 1995 is a fact', where the word 'is' is given an intemporal meaning. Such an eternal proposition would not admit of truth-value changes. In the following there will never be question of such propositions.

⁸A propositional function is derived from a singular proposition by the transformation of a constant into a variable. For example, from 'Peter is running' the function 'x is running' can be derived; or again, from 'It is raining now' the function 'It is raining at time t' can be derived. A propositional function is different from a proposition in that it has no truth-value. The corresponding concept for the function is that of satisfaction. For example, 'x is running' is satisfied by individuals such as Peter and 'it is raining at time t' is satisfied by moments such as 'now'.

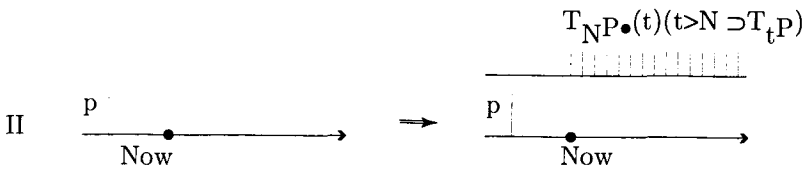
One more thing must be made clear. The propositional functions that could be the objects of the Diodorean definitions are of the form $f(N, t)$, where N standing for *Nunc* designates a fixed term 'now', and t is variable with respect to it. It would be possible to construct propositional functions of two variables, with the position of N not fixed on the time axis. But since the texts of Diodorus eliminate the interpretation in terms of propositional functions, we shall have no recourse to such constructions.

But now apply necessity to a statement. *Necessity* signifies the stability of the truth of that statement from now on. The necessity of the propositional function entails that of the statement. But the statement can be necessary without it's being the same for the corresponding propositional function. Take the statement 'There was a naval battle at Salamis', which is of the temporally indefinite kind with a determinate subject. It was false before and during the battle, for then the battle had not yet taken place. But as soon as the battle was over it became necessary, for then it was true and has never since nor will ever become false that the battle has taken place. On this interpretation the movement of truth has nothing to do, properly speaking, with the time of the event about which the statement is made.

This difference might be diagrammed roughly as follows. Let horizontal lines directed from left to right represent the temporal and 'parallel' truth-axes respectively, with the point on them representing the designated moment *now*. With a perpendicular raised on the time-axis representing the occurrence of a verifying event, on the truth-axis the truth of a statement, the necessary interpreted in terms of propositional functions, gives rise to just one state of affairs:



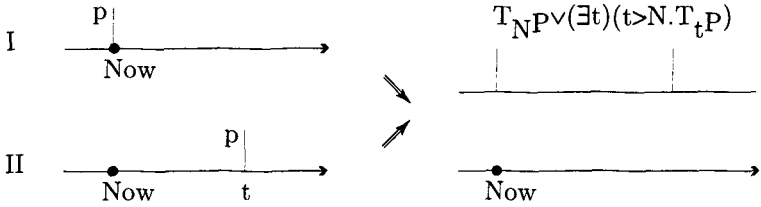
Interpreted in terms of statement, the necessary gives rise to two states of affairs. The first is the same as that given above, to which is added:



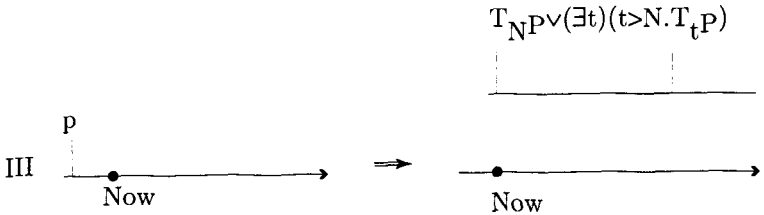
where the 'T' in the formulas is to be taken as 'it is true that'. Analogous schemata, symmetrical for the false, would model the sense of the two interpretations for the impossible.

The situation is similar with respect to the weak modalities. Interpreted in terms of propositional functions, *the possible* signifies that it is true or that it will be true that a certain event should occur: in

other words, that a certain event is occurring or will occur. Two states of affairs correspond to this first interpretation:



Any model in which the realization of the verifying event would precede the *now* is excluded. The interpretation in terms of statement however is not subject to this limitation. Whatever the statement is, thus whatever the real time at which its verifying event is seen to be realized, that statement is possible if it is true now or at some ulterior moment of time. But any genuine past proposition fits in precisely with this condition. There are consequently three states of affairs corresponding to the interpretation of the possible in terms of statement. The first two are identical with those given above, to which is added:



Analogous schemata, symmetric for the false, would model the sense of the two interpretations for the possibly-not.

It is seen then, in the case of both the strong and the weak modalities, that if a declaration is necessary, impossible, possible or possibly-not on the interpretation in terms of propositional function then it is *a fortiori* necessary, impossible, possible or possibly-not on the interpretation in terms of statement. The inverse implication does not hold.

In the two interpretations can be found the trace, so to speak, that the two modal usages have left in the very definition that eliminates them. The interpretation in terms of propositional function corresponds to the modalities *de re*, which, since they attach to the properties of things, fall within the quantifier's scope. The interpretation in terms of statement corresponds to the modalities *de dicto*,

where the object is a complete *dictum*. The first will be referred to here as *realist*, the second *nominalist*, where these terms here shall simply be taken as designating the two modal usages mentioned.⁹

Before examining the respective claims to validity made by the two interpretations in the face of the Diodorean definitions of modality, we shall make a detour through the Diodorean notion of implication because of the deceptive light it could throw on the issue.

⁹Take the following definition:

$$MX =_{Df} (\exists t)(t \geq N \cdot X_t)$$

If M is a quantifier then X is a propositional function $f(t)$. But it is in nowise necessary that this be the case: X may be taken as designating a complete statement.

The first interpretation is that of Mates (1961) who regards the objects of the Diodorean modalities as propositional functions (p. 37). Independently of any historical considerations, Russell appears to have expressly adopted such a conception of the modalities (von Wright, 1979c, pp. 232-233).

The second interpretation is that of Boudot (1973, p. 442).

The two may be written as follows:

1. Modalities defined in terms of propositional functions:

$$M_N p_t = (\exists t)(t \geq N \cdot p_t)$$

$$M_N \sim p_t = (\exists t)(t \geq N \cdot \sim p_t)$$

$$L_N p_t = (t)(t \geq N \supset p_t)$$

$$\sim M_N p_t = (t)(t \geq N \supset \sim p_t)$$

2. In terms of statement:

$$M_N p = T_N p \vee (\exists t)(t > N \cdot T_t p)$$

$$M_N \sim p = T_N \sim p \vee (\exists t)(t > N \cdot T_t \sim p)$$

$$L_N p = T_N p \cdot (t)(t > N \supset T_t p)$$

$$\sim M_N p = T_N \sim p \cdot (t)(t > N \supset T_t \sim p)$$

Commentators often juxtapose the two interpretations without distinguishing them. Sedley, for instance (note 139, p. 116) has the following to say: "necessity belongs only to propositions which can be stated truly both now and at all future times. These must include, above all, eternal and analytic truths, but also certain statements about the past (as in premise (1) of the Master Argument). By contrast, a proposition is possible if it will express a truth now or at *some* future time". (It is difficult to see how only *certain* statements about the past would be necessary).

Boudot reproaches Mates for conceiving the propositions (of the object language itself) as if they contained a temporal variable. "For Diodorus", he adds, "the statement is 'It is day' and not 'It is day at time t'. The date, if it must figure in the expression, determines not the statement but the moment of its assertion; and the temporal variables belong to the metalanguage, not to the object language". Once granted that it is sentences that the modalities govern, the temporal determinations of truth and falsity will be taken as belonging to the metalanguage which will be carefully distinguished from the object language. The neutrality of the statement, grouping both sentences and propositions, doesn't commit us to the same extent. Excepting this last nuance, our presentation here owes much to the clear and penetrating paper of Boudot.

3.3 The meaning of Diodorean implication.

It would be possible for partisans of the realist interpretation to invoke the Diodorean theory of implication, arguing that, since it is a theory of formal implication, it does indeed govern propositional functions and not propositions.

We shall examine the nature of this implication then and the false appearances that have beguiled the partisans of propositional functions.

Philo¹⁰ admitted that a conditional is true if and only if the antecedent is not true when the consequent is false. That is precisely the conception of material implication taken up by the moderns. As for Diodorus, according to Sextus' account, "he says that a conditional is true whenever it neither was nor is nor will be possible for the antecedent to be true and the consequent to be false, which is incompatible with the thesis of Philo. For according to Philo, a conditional such as 'if it is day then I am conversing' is true when it is day and I am conversing for, in that case, its antecedent 'it is day' is true and its consequent 'I am conversing' is true. But according to Diodorus it is false. For it is possible for its antecedent 'it is day' to be true and for its consequent 'I am conversing' to be at the same time false, namely when I have become silent. And it was possible for the antecedent to be true and for the consequent to be false, for, before conversing, the antecedent 'it is day' was true, but the consequent 'I am conversing' was false".¹¹

Unless the criterion of Diodorean implication is to be taken as circular, the modalities figuring in it must be spelled out in terms of temporal determinations. We can say that a proposition p implies a proposition q *à la Diodorus* if and only if it never has been the case and it is not and never will be the case that p should be true and q false.

But just what kind of formal implication is this? The two interpretations demand a hearing once again.

Sextus gives us two cases of conditionals, valid according to Diodorus, of which the first belongs to an inconclusive syllogism. Consider first the Epicurean argument: 'If motion exists, the void exists'. As it always begins with a falsehood and ends with a falsehood—Diodorus denies both motion and the void—, it is in itself true; but as the minor premise 'But motion exists' is false in turn, no conclusion can be

¹⁰Sextus Empiricus, M, VIII, 113.

¹¹Sextus Empiricus, M, VIII, 115; Mates, 1961, p. 98.

drawn.¹² The second example is: 'If there exist no indivisible elements, then indivisible elements exist'. It begins with the false and ends with the true, and that disposition always holds.¹³

We could, to begin with, construe the expressions 'motion exists', 'the void exists', 'indivisible elements exist' not as statements in themselves but as propositional functions. The Epicurean conditional would then signify that for any time *t* whatsoever, if motion exists at that time then the void exists at that time. The complete and exact form of the expressions 'motion exists', 'the void exists', 'indivisible elements exist' would be rendered by 'motion exists at time *t*', 'the void exists at time *t*' and 'indivisible elements exist at time *t*'. Diodorean implication then would not be different from a universal proposition in terms of times.¹⁴ As the binding of the variable would be over two propositional functions, the implication would express nothing other than a law of physics in the modern sense of the term. Its model would be a proposition of the type 'Snow is white' that would be rendered in temporal terms as 'if something is snow at time *t*, it is white at the same time *t*'. Consequently, since the truth-values are fixed simultaneously for the antecedent and consequent of the conditional, it would be excluded that, at one and the same time, the antecedent could be true when the consequent is false. And this is in conformity with Diodorus' requisite.

The specious character of this interpretation becomes obvious as soon as we consider what Diodorean implication was meant to do. Sextus has conserved two of Diodorus' arguments intended to block objections made to him when he said that there is nothing which *is* in motion but only something that *has been* in motion.¹⁵ It was objected that "if preterites are true, it is impossible that their presents should be false and they must be true; and similarly, the preterites must be false when the presents are false".¹⁶ Among Diodorus' replies, two have to do with the question of the role played by the movement of

¹²Sextus Empiricus, M, VIII, 330-333 (Bury, II, pp. 412-413; Döring, fr. 143, p. 44).

¹³Sextus Empiricus, PH, II, 110-111 (Bury, I, pp. 222-223; Döring, fr. 141, p. 43).

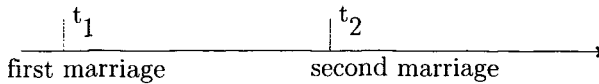
¹⁴Mates, 1961, pp. 45-46.

¹⁵Sextus Empiricus M.X.86 (Bury, III, pp. 252-253; Döring, 1972, fr. 123, p. 35). As pointed out by Sedley (1977, pp. 85-86), the Diodorean passage here echoes that of Aristotle showing that if the theory of indivisibles applied to time is true, then there will have been motion without there being motion (*Physics*, Z, 231^b 21-232^a 17, 240^b 8-241^a 6). If we go along with Sedley in thinking that Diodorus is replying to Aristotle here, this correspondence suggests that Diodorus takes the Aristotelian implication as holding and that, since he does admit the antecedent (temporal indivisibles) as well, he is contesting the absurdity of the consequent, which Aristotle had simply rejected.

¹⁶Sextus, M, X, 91 (Bury, III, pp. 255-256; Döring, 1972, p. 35).

truth. To begin with,¹⁷ he says that preterites may be true when their presents are false. The past-tense proposition 'these men married' may be true at one and the same time when the corresponding present-tense proposition 'these men are marrying' has never been true. It is sufficient for this to be so that the men in question should have married at different moments of the past. The correspondence would require that they should have married simultaneously. The second argument¹⁸ is of the same sort. The past-tense proposition 'Helen had three husbands' may be true, because Helen was married three times, without it's ever having been the case that the present-tense proposition 'Helen has three husbands' is true, since the three marriages in question were successive.

Independently of whether these arguments are sophistical, as Sextus maintains,¹⁹ they prove that Diodorus does systematically use complete temporal statements as antecedents and consequents of the conditional. Formal implication over time has nothing to do with the time proper of the propositions but only with that of their truth values. The following schema illustrates why the conditional of the second example is invalid. It could only be valid if the antecedent was never true when the consequent is false.



The antecedent is the past-tense proposition about the conjunction of the marriage at time t_1 and the marriage at time t_2 . It is true immediately after the accomplishment of time t_2 and will never be false. The consequent, on the other hand, is never true; it says that there was a moment of the past at which both marriages were simultaneous.²⁰ To say that there has been a first marriage at time t_1 and a second at time t_2 is to utter authentic statements, and not to speak in terms of propositional functions as the first interpretation would have had it. Formal implication has to do with the truth, which varies with the time of these statements. Formal implication here is nominalist.

What is more, Diodorean implication, as opposed to a formal implication that would directly involve propositional functions, must be

¹⁷Sextus, M, X, 97 (Bury, III, pp. 258-259; Döring, 1972, pp. 35-36).

¹⁸Sextus Empiricus, M, X, 98 (Bury, III, pp. 258-259; Döring, 1972, p. 36).

¹⁹Sextus Empiricus, M, X, 99-100 (Bury, III, pp. 260-261; Döring, 1972, p. 36).

²⁰Formally, the falsified implication is the following:

$$T_N P(p \cdot q) \supset (\exists t)(t < N \cdot T_t(p \cdot q))$$

able to connect an antecedent and a consequent situated at two different moments of time. This is the case when one tries to bring into correspondence the present truth of a past motion with the past truth of that motion as present.

It is owing to this temporal peculiarity of formal implication that Diodorus distinguishes himself from the ancient Eleatics in leaving some room for motion,²¹ quite like he left room for the possible defined in terms of time, thereby distinguishing himself from the ancient Megarians. To do this, he had to admit a succession of states while rejecting a change of state. But to admit the first and reject the second comes down to denying that the implication of the second by the first is true, since to the truth of a past motion there is no corresponding truth of a present motion that one could have apprehended in its passage.

The Diodorean definition of implication is closely tied to the conception of indivisible instants of time—a conception mentioned by Sextus *ad nauseam*.²² The clearest instance of this conception is probably that of the wall, collapsed without ever collapsing. "If the wall collapses it collapses either while the stones are touching and fitted together, or when they are disjoined. But it is neither while they are touching and fitted together, nor when they are disjoined that the wall collapses; therefore the wall doesn't collapse". Sextus' commentary is clear. "Two times are conceived of, that in which the stones touch and are fitted together and that in which they are disjoined, and, aside from these no middle time can be conceived. If the wall collapses then, it must be in one or the other of these times", but that cannot be in any case.²³ The present, conceived of by Aristotle as a breadthless mobile limit, disappears. If it is different states of the world that are observed, and it is, then it is impossible to apprehend or to posit an instantaneous change of state in being. Time is made up of indivisible parts. One can then say that there has been motion without being committed to admit of present motion, *i.e.* instantaneous motion.

In defining implication in the way he does, what Diodorus is really doing is attacking the Aristotelian definition of motion as the fulfillment of what is potential as potential. According to Aristotle, potentiality is present as privation in the moving thing. In denying the infinite divisibility of time, Diodorus is rejecting this kind of potential-

²¹Sextus, M, X, 48 (Bury, III, pp. 234-235; Döring, 1972, fr. 122, p. 34).

²²Sextus, PH, II, 242 (Bury, I, pp. 312-313, and the retort of doctor Herophilus); PH, II, 245 (Bury, I, pp. 314-315; Döring, 1972, fr. 127, p. 37); PH, III, 71 (Bury, I, pp. 376-377; Döring, fr. 124, p. 36); M, X, 87-88 (Bury, III, pp. 252-253; Döring, fr. 123, p. 35); M, X, 142-143 (Bury, III, pp. 282-283; Döring, fr. 125, p. 36).

²³M, X, 347-349 (Bury, III, pp. 378-379; Döring, fr. 126, p. 37).

ity. The future does not haunt the limit-present in the guise of a need. It will be, or else it already is. There is time, but no becoming. There is the generated and the destroyed, but no generation or destruction. From the truth of the fact that there has been motion, it cannot be concluded that it has been true that there was motion then—or, again, that it is true that there was a passage. The whole demonstration hinges on an implication of which the antecedent says that there has been motion and the consequent that there has been passage. Those are both complete statements: the first true, the second false. The formal implication that fails to establish a connection between them has to do with their truth-values and not with their content.

3.4 Diodorean nominalism.

In his definitions of the modalities, the conditions that Diodorus imposes on the truth-values have to do with their behavior over time. Given what they are presently, what they will be is assigned. This behavior is thus relative to the ‘egocentric particular’ *now*. But the temporal reference of a declaration is itself susceptible of three different degrees of precision. The time may be indefinite: ‘I’ll undertake this task one day’. Or the time may be linked to the present moment of the declaration by the indication of a distance fixed by recourse to a calendar: ‘I’ll undertake this task in the second week of next month’. Or, finally: the date of the event is fixed in relation to a chronology within which is situated the experience of the speaker himself: ‘I’ll undertake this task on January 1, 1995’. The reference to time gives rise in the first case to a grammatical tense, in the second to what is called a pseudo-date,²⁴ in the third to a date. It is only the dated declaration that doesn’t depend, apparently at least, on the assignment of now. That is what has made it possible to think of it as ‘eternal’.²⁵

²⁴To use Rescher’s expression, 1971.

²⁵Accordingly as dated declarations are made before, during or after the dated event described, some languages maintain a reference to the time of speaking in the verb of what is said. In such languages, this recourse to conjugation is seen as encompassing the variations necessary for establishing the synonymy of statements in virtue of the rule: accordingly as a statement about a dated event is made before, during or after that event, the statement must be in the future, present or past tense. The logical status of the declaration though is not affected by these necessary grammatical modifications.

A date can only be determined with respect to a point of origin. The question will still arise whether there isn’t ultimately need of an “egocentric particular” for fixing the point of origin. We won’t go into this question here, as its solution would have no bearing on the interpretation of the Diodorean definitions.

The plan the inquiry must follow lies traced before us. We must determine the respective merits of the two interpretations distinguished in section 3.2, first for the case of indefinite declarations, then for the case of dates and pseudo-dates. It is the first case that presents the major difficulties.

In cases where the modality governs an apparent present or a subjunctive, as in English and French, one will be tempted to think that the realist interpretation is imperative: that the modal expression is derived from a propositional function and that the movement of truth will have then determined the time of the event. Instead of indicating the simultaneity of the event with the time of utterance, this present and this subjunctive will be taken as indicating the simultaneity of the event with the time induced on the variable of the propositional function by the truth predicates or directly by a grammatical tense.²⁶ Quoting the Diodorean definition of the possible, Alexander of Aphrodisias does away even with the predicates of truth and falsity, assigning the time directly from a non-temporal verbal form. "According to Diodorus it is possible for me to be in Corinth in the case where I am in Corinth or in the case where I will go to Corinth. But if I were never to be in Corinth, that would not have been possible. And to become a grammarian is possible for a child, if it becomes one one day".²⁷ There is no room here for ambiguity. There is a possible when the thing presently is or will be realized.²⁸ Boethius also reports the Diodorean definition of possibility in purely objective terms on two different occasions,²⁹ without any mention of the truth-predicate. That

²⁶To designate the object of the modalities the definitions employ a neuter pronoun (Cicero and Boethius use *quod*, Plutarch *ὅπερ* Alexander *ὅ*). When any precision is furnished it is by way of translation into a substantified infinitive (*τὸ γενέσθαι*, says Alexander). In short, one is tempted to think that where the French and English employ the subjunctive ('il est possible que j'aille à Paris', 'it is possible that I should go to Paris'), the Ancients use expressions that exclude a determinate tense, consequently appealing to propositional functions determined by the truth predicates, for want of a direct translation in terms of grammatical tense.

²⁷1883, p. 184.

²⁸Plutarch (*De Stoic. rep.* 46, 1055 D-E; Döring, 1972, fr. 134, p. 40; in *Plutarch's Moralia*, 1976, XIII, 2, p. 589), Philoponus (1905, 169, 17-21; Döring, 1972, fr. 136, p. 41), Simplicius (1907, 195, 31-196, 24. Döring, 1972, fr. 137, p. 41) all record the Diodorean definition of the possible in objective terms of that which is or will be. Cicero, in the *De Fato* (6, 12, 7; Döring, 1972, fr. 132 A, p. 39) links the two definitions. Diodorus, he says, "says that that only can happen which is true or will be true, and all that is future, he says that it is necessary that it happen, and all that is not future, he denies that it may happen". The time that counts for characterizing the modality is that of the event, not that of truth.

²⁹Boethius, 1880, 234, 10-235, 9 and 412, 8-21 (Döring, 1971; fr. 138-139, pp. 42-43). The commentary of Boethius leaves no doubt as to his interpretation of the

is possible which is or will be, says the definition. The possible then assigns the present or the future to an indefinite declaration in the apparent present, which is nothing but a propositional function.

Those who would settle for this semblance, however, would be making a double mistake. Historically, they would be taking over arguments of the commentators close to Aristotelian realism to interpret Diodorus, whom we saw staunchly refusing to admit the notion of real potentiality. Logically, they would be forgetting that a realist possible justifies a nominalist possible *a fortiori*. Take the statement 'I am now or will be in Corinth'. To say that that statement is possible is to say that it is true now or that it will be true that I am now or that I will be in Corinth. The truth conditions on the statement are the same as the satisfaction conditions that were on the propositional function.

To decide between these interpretations two conditions must be met.

1. Cases must be found in which the realist interpretation errs by defect or, what comes down to the same thing, cases where the nominalist interpretation errs by excess. The discovery of such cases is a necessary condition for distinguishing between the two interpretations. It would not be a sufficient condition however if one could impute an equivocal usage to Diodorus.³⁰

2. This supposed ambiguity, which moreover contradicts the reputation Diodorus enjoyed,³¹ loses all verisimilitude if we show that it upsets the relations of the 'logical square of opposition' that the Diodorean definitions of the modalities lead us to expect.³²

matter. "While there are two principal parts of the possible: one which is said of that which, though non-existent, can nonetheless exist, the other which is predicated of that which already is something in act and not just potentially, this sort of possible that already is in act gives rise in turn to two further sorts of possible: one which, though existent, is not necessary, the other which, existing, further renders this possible necessary. And it was not only the subtlety of an Aristotle that recognized this fact. Diodorus too defined the possible as that which is or will be. Thus Aristotle holds possible that which Diodorus calls 'future', which, while it doesn't exist, can nonetheless come to be. As for what Diodorus called 'present', it is what Aristotle interprets as being possible, which is said possible precisely because it already is in act". Whatever the validity of the comparison Boethius makes between Diodorus and Aristotle—for in speaking of a possible that is not but can be, Aristotle is not automatically talking about what will be—there is one thing that is sure: Boethius does take Diodorus to hold that the possible is that which is or will be in act.

³⁰ According to Mates (1961, p. 39), the attribution of necessity in the Master Argument's first premise requires taking the word 'necessary' in another sense than that used in the definitions of the modalities, or of the weak modalities at least.

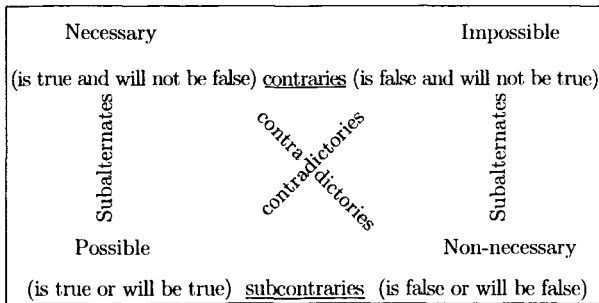
³¹ Sedley, p. 103. See above ch. I, note 22, p. 10.

³²

It is easy to show that the realist theory is in the wrong. If we begin with the function 'there is a naval battle at Salamis at time *t*', the Diodorean definitions fail to assign its modal status. Since such a function is satisfied neither at present nor in the future, it would be impossible at most. The nominalist interpretation, on the other hand, is legitimate, since the statement 'there was a naval battle at Salamis' is true now and will never be false and is thus necessary.

The realist interpreter is thus reduced to accusing Diodorus of ambiguity. But once interpreted in terms of statement, the necessity of the past takes its place alongside the so-called definitions in terms of propositional functions, with the result that the possible no longer follows logically from the necessary. 'There was a naval battle at Salamis' is surely a statement that is true and will always be true: but it cannot be said to express the present or future act of an event.³³

At this point we should examine how declarations about the future fare with respect to the two interpretations. Let us first derive 'Fabius will perish at sea' from the propositional function 'Fabius will perish at sea at *t*'. If the statement derived from such a function is true then, since the death of Fabius represents simply a unique event, the statement will not be necessary. It will therefore be contingent. If the statement is false it will always be false, and is thus impossible. From the nominalist point of view on the other hand, where what we begin with is the complete statement 'Fabius will perish at sea', if the



Square of Modal Opposition according to Diodorus.

(Kneale, 1962, p. 125). Of two contradictories one is true, the other false. Two contraries cannot both be true, but can both be false at the same time. Two subcontraries cannot both be false, but can both be true at the same time. There is a relation of descending implication between subalternates: the necessary (impossible) is possible (non-necessary). A realist expositor like Mates agrees that the laws of the logical square must be respected (p. 37).

³³See next page.

statement turns out to be true it will become false once the event is accomplished, since from that moment on the death of Fabius at sea will take place no more. If the statement is false it will always be false and is therefore impossible. Both interpretations are seen to agree on the two cases: impossibility in case of falsehood, contingency in case of truth. Nevertheless, the contingency for the realist is based on the unicity of the event, whereas for the nominalist it is based on the changing truth-value. Both interpretations will confuse the case of the eclipse with that of the death of Fabius in the same treatment. But the nominalist confusion, based simply on the movement of truth, is due to a more superficial reason and is therefore more acceptable than the realist confusion that puts the two events on a par.

From the realist point of view 'There was a naval battle at Salamis' is an assertion which *stricto sensu* is verified neither now nor in the future:



It is therefore impossible and entails a negative possibility. From the nominalist point of view it is necessary and entails a positive possibility, given that its truth-value doesn't change any more after the event.³⁴

³⁴The nominalist interpretation may be written as follows:

$$\begin{aligned}
 (\exists t)(t < N \cdot p_t) &\supset T_N[(\exists t)(t < N \cdot p_t)]. \\
 &\quad (t')(t' > N \supset T_{t'}[(\exists t)(t < N \cdot p_t)]) \\
 &\equiv L_{nom}(\exists t)(t < N \cdot p_t) \\
 &\supset M_{nom}(\exists t)(t < N \cdot p_t) \\
 &\equiv T_N[(\exists t)(t < N \cdot p_t)] \\
 &\quad \vee (\exists t')(t' > N \cdot T_{t'}[(\exists t)(t < N \cdot p_t)]).
 \end{aligned}$$

(The symbols ' L_{nom} ' and ' M_{nom} ' are here taken as designating nominalist necessity and possibility respectively, as ' L_{real} ' and ' M_{real} ' will be taken as designating the corresponding notions on the realist interpretation).

If, on the realist interpretation:

$$M_{real} p \equiv p_N \vee (\exists t')(t' > N \cdot p_{t'}),$$

the formula expressing that p is past is substituted for p :

$$M_{real} p \equiv [(\exists t)(t < N \cdot p_t)]_N \vee (\exists t')(t' > N \cdot [(\exists t)(t < N \cdot p_t)]_{t'})$$

the resulting formula will illustrate the Aristotelian *De Caelo* passage: it will be true to say now that next year is last year. All semblance of paradox vanishes when the invalidity of the conditional

$$*L_{nom} \supset M_{real}$$

is brought to light.

The realist can, with Aristotle, demonstrate the 'necessity' of the past in his language. He will posit:

$$t < N \supset \sim M p_t \vee \sim M \sim p_t,$$

The realist interpretation seems to fare better once we come to consider dates or pseudo-dates. Even though the propositional function is satisfied without any indetermination as soon as the Now is fixed, it remains contingent because of the fact that it is not satisfied either always or never. There is a price to be paid for this universal contingency though, since the complete determination of the dated or pseudo-dated occurrence of an event—an eclipse, for instance—will not prevent it's being contingent. On the nominalist interpretation all dated and pseudo-dated statements are, on the contrary, necessary or impossible. If I say that it is day now or that it will rain tomorrow and my statements are true, they cannot become false, and if they are false, they cannot become true.³⁵ Here, even the statements that common sense would take as contingent ('It will rain tomorrow', 'It will rain in Paris on May 20, 2000') are transformed by nominalism into necessary or impossible ones.

Should we then, in the case of dates and pseudo-dates, follow the middle-of-the-road lead of common sense, giving equal weight to the two interpretations that entail complementary paradoxes? To do so would be to forget that Diodorus challenges the position of common sense regarding the paradoxes, and that there is no balance to be had

and if the propositional function was satisfied at t , then

$$\sim M \sim p_t.$$

The fact will remain however that this necessity is not in conformity with realism, for it does not entail realist possibility. On the contrary, it logically excludes it.

³⁵The real present is a pseudo-date since it asserts the simultaneity of the utterance with the event. To say that the statement 'It is now day' will be false tonight would be to render variable both the now and the distance separating the event from the now. The synonymy class of the *statement* 'It is now day' includes all sentences of the same form—in English or in another language translation—and uttered during the lapse of time fixed by the chosen unit of measure of the present (the hour, for instance). The sentences that will belong to the same class five hours hence are: 'Five hours ago it was day'. These sentences are true and will never become false (in so far as they incorporate the appropriate compensation with respect to the unit chosen). To overstep the bounds of the chosen unit, though, in saying that the statement 'It is now day' will be false in five hours, would be to employ the statement ambiguously and to violate its strict identity conditions which require that the sentences be appropriately modified.

Those who speak in terms of sentences will have to be content with saying that a sentence is tacitly bound to its moment of utterance. To demonstrate the necessary or impossible character of dated sentences they will use a formal detour. With ' m ' designating the number of units in question, they will posit:

$$P_{in} \text{ in the present tense} \equiv P_m F_m p.$$

Admitting the principle

$$P_m p \supset LP_m p,$$

the conclusion Lp immediately follows (Boudot, 1973, pp. 450-451).

between realism's false contingencies and nominalism's shocking necessities. That necessity should attach to present and future-tense propositions, once unambiguously dated or pseudo-dated, is something that will appear philosophically legitimate if, along with Diodorus, we reduce contingency to the indetermination of the temporal occurrence of events. Far from striking a balance, as common sense would have it, the nominalist and realist paradoxes in fact add up to a corroboration of the nominalist interpretation, since Diodorus' principles oblige acceptance of the former and rejection of the latter.

Finally, there are all those passages of Epictetus and Cicero in which Diodorus explicitly applies truth-values to closed statements.³⁶ There is no way of avoiding the nominalist interpretation here. The same is so when truth-values are attributed to particular ('There are indivisible parts') and universal ('There is no motion') general propositions, of which we know nothing, moreover, beyond the mere examples recorded by Sextus.³⁷ There can be no doubt. Only nominalism conforms to the letter and the spirit of Diodorus.

It must still be explained however why the realist interpretation is so attractive wherever there is question of a genuine 'possible' and why it is only the indefinite character of propositions that blocks necessitarianism.

In the absence of textual evidence there will be something arbitrary about all our conjectures. But the dispute over Diodorus' necessitarianism will amply show that we could not do without such conjectures.

3.5 Diodorus' necessitarianism.

The Ancients, like the classical philosophers, thought that in denying the Master Argument's third premise Diodorus was implicitly committing himself to necessitarianism.

The nominalist interpretation, on the other hand, which has the merit of showing that Diodorus stood out against the early Megarians, the enemies of all possibility,³⁸ maintains that as long as the propositions involved are neither dated nor pseudo-dated Diodorus is in no

³⁶Cicero, *De Fato*, VI, 12; IX, 17.

³⁷Sextus Empiricus, PH, II, 242 (Bury, I, pp. 312-313); Sextus Empiricus, PH, II, 110-111 (Bury, I, pp. 222-223; Döring, 1972, fr. 141, p. 43).

³⁸Aristotle, *Metaphysics*, Θ, 3, 1046^b29-32: "There are those philosophers, the Megarians, for example, who hold that there is no potency except in act, and that where there is no act there is no potency. Thus he who is not building has not the power to build; but only he who builds has, at the moment when he is building (ἄταν οἰκοδομῆ).")

way committed to necessitarianism. He must admit to it however once there is question of these sorts of proposition.

Let us examine the arguments of both parties.

That of the Ancients would be given the following form.³⁹ The Diodorean definition of the possible is to be taken to mean that "only that is possible which either already is the case or will be the case. The consequence of this definition is that all future events are necessary. For, since their contrary is not already present and will not be, their contrary is not possible. But that of which the contrary is not possible must itself be necessary. Therefore, all future events would be necessary".

It is only the detour by way of the absurd that renders the reasoning acceptable here. Supposing a *unique* possible true now or at some moment of the future, its negation is false now and at *all* moments of the future. Remaining false, it is therefore impossible; and in virtue of the interdefinability of the modalities, the possible with which we began is necessary.

This is surely the way we must take Cicero's comments on Diodorus. "If, he says, addressing himself to Chrysippus, you sanction the prophecies of the diviners, you will reckon among impossible things events falsely predicted as to the future, and as to veracious predictions of future events that are to occur, you will maintain that these are necessary. But this is precisely Diodorus' view, completely opposed to your own". And in giving the example of a necessary causal connection he is careful to put the antecedent in the past ('You were born at the rising of the dogstar') in order to render it necessary in conformity with the Master Argument's premise A, and careful too to give the consequent a negative form ('You will not die at sea') to avoid the possibility of a truth-value shift.⁴⁰ The negative of the consequent here connects the objective necessity of the event (not dying at sea) with the grammatical necessity of the statement (you will not die at sea), since this statement is and will remain true.⁴¹ But Cicero takes great care in commenting on Diodorus' chapter, "Possibles", to say that "nothing happens that was not necessary; whatever can happen either is now or will be; propositions about the future can no more, from true, become false than can propositions about the past; but as regards the latter, the impossibility of changing them is apparent, and

³⁹M. Frede, 1974, p. 115.

⁴⁰Cicero, *De Fato*, VII, 13, 14; Bréhier, 1962, p. 478.

⁴¹The same holds true for the example discussed in *De Fato*, VI, 12; Bréhier, 1962, p. 477.

since it is sometimes not apparent as regards propositions about the future it just doesn't seem to be there".⁴²

Putting it this way, Cicero takes no pains to show that a proposition about the future has a constant truth-value even though it be future, whereas a proposition about the past has a constant truth-value because it is past. He simply shows that a difference of degree in the specification of the temporal determination of the future makes no difference of degree as to the necessity of the proposition. He thus takes no account of the grammatical form of the future which is the cause of truth-value instability. When a Bayle or a Leibniz takes up the question again they spontaneously adopt a point of view that excludes from the outset both ambiguity of designation and grammatical cavils. They don't even discuss the matter and identify Diodorus' philosophy with the system of most absolute necessity. There is only a nuance, evanescent at that, to distinguish the immobilism of the Ancient Megarians from the logical fatalism of Diodorus, for they both agree in recognizing in each essence "in some sense, a universe complete and closed upon itself, which contains from all eternity all that will be in the course of its duration: it is fated, said Diodorus, either that you reap or that you not reap; it cannot be but the one or the other, between the two contradictories there is no middle; therefore, necessarily, *whatever happens*, either you will reap or you will not reap".⁴³

Nominalism's response to this accusation is to argue that it is based on a sophism. If the possible is interpreted in terms of statement, every future, true or false, will undergo a truth-value change and the realist's detour by way of the false is illusory. Granted, every possible *will be* necessary, since it will be transformed into a past. But it *is not* necessary. A possible at moment $t(t \geq N)$ has always been possible, since at every moment of the past it has always been true that it will be. It can be concluded from this that if something is possible it has been necessary that it be possible,⁴⁴ but not that it is necessary that

⁴²Cicero, *De Fato*, IX, 17; Bréhier, 1962, p. 479.

⁴³Robin, 1944, p. 120, who points out the connection between the Reaping Argument and the Idle Argument. The Reaping Argument is set out by Ammonius (1961, pp. 251-252) as follows: "If you are going to reap, it is not that you are going perhaps to reap, perhaps not, but that you are going to reap in any case (*modis omnibus*); consequently, it is of necessity that you are either going to reap or not going to reap". And Ammonius comments: "What is eliminated then is the 'perhaps' itself, to the extent that it obtains neither according to the opposition between what it is to reap in the future and what it is not to reap in the future, since necessarily one of the two will occur, nor according to the consequent referred to either one of the two suppositions".

⁴⁴The Diodorean system belongs to what the moderns call the S_4 modal system.

it be possible, nor *a fortiori* that it is necessary. Retrogradation of the truth does not, therefore, change the possible into the necessary. As to the discreteness of time, if something is necessary at time t , it allows us to say at time $t + 1$ that it always will be true. But as that is a future truth no conclusion can be drawn from it as regards time $t - 1$ and it may be that that thing was not the case at $t - 1$. Necessity, therefore, does not retrograde.

We're left with an argument however that proves too much and not enough.

It proves too much, for all possibles that will be realized must be seen as contingent.

It proves too little, for once a statement is temporally determinate it becomes necessary.

These two flaws seem moreover to be related to a self-same defect of the Diodorean doctrine regarding contingency. For according to Diodorus, a statement that changes truth-value is contingent. But there are two possible interpretations of such a change. When it is said that the closed proposition 'Sometimes it rains' is contingent, what is meant is that simultaneously (it is true now or it will be true in the future) and (it is false now or it will be false in the future) that it is raining now. The contingency then is due to the succession of occurrences of showers and occurrences of dry spells. The realist interpretation is also legitimate in this case since the satisfaction conditions of the propositional function 'It's raining at t ' will have the same consequences as the truth conditions of the statement 'It sometimes rains'. But consider the statement 'Fabius will perish at sea'. Here again realists and nominalists agree in holding it to be contingent, but this time for entirely different reasons. The realist will invoke the unicity of the verifying event, but will deny that the proposition will have gone from being true to being false. The nominalist will keep track only of the grammatical tense shift to attribute a truth-value change to the proposition. The nominalist interpretation thus resorts to a verbal quibble to preserve by excess the contingency of all indefinite futures, and it is also this verbal recourse that explains why all determinate futures are turned into necessities. The prestiges of the realist interpretation are understandable. The truth-value shift of statements seems too feeble a ground upon which to found freedom.

There would be a way out however if, in examining what is positive in the Diodorean conception and what it is that distinguishes it from that of the Ancient Megarians, we could make sense of how and why it is that the nominalist interpretation, the only one historically and log-

ically admissible, ends up with arguments that are at once insufficient and excessive.

It will be said that the only difference that allows our distinguishing Diodorus from the Ancient Megarians is that he deploys over the succession of time the necessary events that the others concentrate in a timeless eternity, thereby allowing human ignorance to ludicrously break the dike of fatalism. It will be added that grammatical contingency has to do simply with the uncertainty of the time at which an event will occur, which if dated or pseudo-dated would become necessary. Real contingency, on the other hand, should reflect a hesitation due to Nature itself. The Diodorean system thus makes modality a subjective property of our knowledge. If this contingency sometimes passes for real contingency it is because the realist interpretation, also illegitimate moreover, in part parallels the nominalist one and so gives rise to the illusion of a modality that would partially simulate the contingency of things.

But let us take this described difference to its term. To elucidate the meaning of a contingency reduced to temporal indetermination let us consider a series of 'sophisms' to which Diodorus had recourse. They are 'the Horned Argument', 'The Veiled Argument' and 'the Sorites'. It will be seen that all these sophisms have to do with the problem of decision.

How is one to answer the question 'Have you lost your horns'? The answer is neither yes nor no, although Diodorus accepted the principles of bivalence and excluded middle.⁴⁵ What can be said to the following argument? 'You say that it is impossible to know and not to know the same thing. Well, you know your father. But if I show you a man with his head veiled you will say that you do not know him. So if it turns out that the man with his head veiled is your father, you will both know and not know your father'.⁴⁶ How is one to answer the Sorites?⁴⁷ A property P will be called 'predominant' if, given a body composed of n indivisible particles, $P(m)$ and $m > n/2$. Every predominant property must be considered as applying to the body as a whole. Let a body consisting of three indivisible particles have two of these moving and one stationary. According to the principle of predominance then, the body as a whole is counted as moving. Adding a new stationary particle to the body of three already moving, the new composite of particles

⁴⁵Sextus, M.X., 112-117, Döring, fr. 129; Sedley, 1977, p. 102.

⁴⁶Sextus, M, X, 112-117, Döring, fr. 129; Sedley, 1977, p. 102.

⁴⁷Sedley, 1977, p. 95.

will also be moving; and continuing in like manner any number of stationary particles whatsoever will be moved by two moving ones.

In these three cases Diodorus is exposing a typical nominalist sophism. 'To have lost one's horns', 'to know someone', 'to be predominant' are not predicates that give rise to well-formed sentences, that is, sentences capable of verifying the laws of logic. 'To have lost' should be analyzed into the conjunction 'to have had and have ceased to possess', of which each conjunct may be independently falsified. 'To know x ' as it is used here should be expanded into 'to know what x looks like', which may be falsified without falsifying 'to know x '. The Sorites arises from our treating any composite body, including the one we are reasoning about, in terms of 'predominance', as a 'black box' and forgetting to account for the process of attribution. But let us penetrate into the composite. Let us postulate that for any number n of particles we shall attribute P to the whole only if $m > n/2$. The Sorites vanishes for the predicate 'predominant' must be analyzed into 'predominant given the internal composition', and this predicate may be falsified (as in the case of the Sorites once $m = 2$ and $n > 4$) without falsifying the absolute predicate 'predominant'.

These syntactical precautions are obviously imperative when the infinite comes into play. There are two complementary arguments of Euclid and Diodorus that attest to the fact. The first is 'the Elusive Argument', important for its critical consequences regarding the perfect wise man of the Stoics.⁴⁸ This is in a sense the antique sketch, though negative here, of the notion of a chain used by Dedekind to demonstrate the axiom of infinity starting from the knowing Subject.⁴⁹ Since an act of reflection can always apply to any number whatsoever of cognitive acts, self-knowledge yields a given infinite. From this line of reasoning the Megarians conclude that one reflection always escapes self-knowledge. Self-knowledge therefore is ever-elusive. A statement, therefore, must be finite. On the other hand, it is possible that Diodorus proved atomism in applying his definition of the possible to division. Since no division is nor will be carried out to infinity, how could such a division be considered possible and what sense does it make to speak of infinite divisibility?⁵⁰

In warning us against the false appearances of incomplete statements, Diodorus shows us by way of contrast what a genuine statement is. It is a finite statement whose truth-value we are able, in principle,

⁴⁸Sedley, 1977, p. 84.

⁴⁹Vuillemin, 1962, p. 297, note 2.

⁵⁰Sedley, 1977, p. 88.

to decide. But let us go back to the Master Argument's final premise, to that possible that neither is nor ever will be realized. The statement positing it is not a finite and decidable one and is therefore to be rejected as syntactically deficient.

It was thus wrong of Cicero to ignore the degree of precision of the temporal determination of statements. The whole question of freedom resides in that degree. There is what is decided, what is decidable and what is undecidable. We cannot change what is decided. Dates and pseudo-dates will entail necessity then, and they will be its domain. Nor can we on the other hand be misled by what is undecidable and chimerical. And when in order to avoid these we define the possible as that which is or will be true, we must not press the existential character of the statement any more than taking it against Diodorus' intention in supposing that the occurrence of the event could be deferred indefinitely, which would only be a back-handed way of reinstating the rejected premise. The indetermination required for freedom is not the indefinite and that is why the truth-value change of indefinite futures is guaranteed.

Is this indetermination a result of our ignorance or of irresolution proper to Nature? We do not know. Whenever the Master Argument's third premise has been challenged it has been so in the name of subjective requirements characteristic of the supposed limitations of our cognitive faculty. But does Diodorus rest the weight of indetermination on our faculty for knowing, as others, probably inspired by him, will do? He loosens the vice-grip of fatalism through indetermination. If he refuses to open it further as the third premise seems to do, it is surely that he rejects speculation and accepts only finite statements. It is not for all that that he makes indetermination a subjective property of our faculty of knowing. His critical nominalism brings him close to a philosophy of inquiry. He remains compatible nevertheless with a dogmatic conception of indetermination.

Eternal Return and Cyclical Time: Cleanthes' Solution.

Cleanthes, in rejecting the Master Argument's first premise, denied that the past was necessary. Beyond this one certitude all is conjecture.

We shall examine, and subsequently reject, two of these conjectures, each ascribing to Cleanthes a logical solution of the Master Argument.

The first, of nominalist inspiration, sees in Cleanthes a precursor of Ockham. On the one hand, he would have contested, not the necessity of the past in general, but the necessity of propositions having the grammatical form of the past without being about the past. Calling this first into question has moreover the added advantage of extending the benefit of contingency to dated and pseudo-dated propositions—something Diodorus had managed to assure only for the case of certain indefinite propositions. Fatalism is avoided by eliminating modality *de re* in favor of modality *de dicto*. Should one wish to rid this first conjecture, however, of its imputation of ambiguity to Diodorus' Master Argument, it would be necessary to extend the principle, throwing doubt not on the necessity of the past, *i.e.* the Master Argument's first premise but rather on the principle of conditional necessity, *i.e.* the premise the argument borrows from the paradigm of the *De Caelo*. But in doing this one deliberately parts ways with Cleanthes.

The second conjecture, exemplified by Leibniz, reverses the first. If Cleanthes doubts of the necessity of the past, it is that he would refuse to accord it any other than a conditional necessity. The first premise loses all force but what would accrue to it from that additional premise.

The second conjecture, although historically more plausible than the first, will have to be rejected as well. It is acceptable neither historically, as we shall see, nor logically, in that it deprives itself of the very possibility of distinguishing that which specifically characterizes

the 'necessity' of the past. The conjecture to be advanced and espoused here is founded rather on physics than on logic. In so far as the modality of the past is concerned it results in the notions of eternal return and cyclical time.

4.1 **First conjecture. Necessity of the past *secundum vocem* and *secundum rem*: Ockham's conception on Prior's hypothetical reconstruction. Modality *de dicto* and modality *de re*.**

The first interpretation of Cleanthe's thesis to be examined may be summarized in the following three arguments.

I. The validity of the Master Argument does not depend, as it does in the interpretation advanced in this book, on conditional necessity but on the adjunction to premises A, B and C (as interpreted by Prior) of two additional premises. The one of these additional premises posits the retrogradation of truth, the other the discontinuity of time. What leads to the incompatibility of the premises, once granted the discontinuity of time, is the interplay of the modalities and time which is governed solely by the grammatical form of the sentences involved. That is why this interpretation can be called 'nominalist'. In this perspective, it is possible to get out of the fix by denying premise C, as Diodorus Cronus did. Necessitarianism is then avoided, but only in so far as the sentences reasoned about are indefinite ones. As soon as pseudo-dates (yesterday, today, tomorrow) or dates are brought in, necessitarianism is back to claim the field. To challenge necessitarianism at its very root then, it is that axiom which is its veritable reason for being that must be denied. The axiom in question is axiom A and what warrants casting doubt upon it is precisely its ambiguous grammatical form. Denying A then is surely in conformity with the spirit of nominalism. It is precisely the step that Ockham took.

II. The Ockhamist modal system allows for the explication and formal expression of the nominalist intuitions regarding the logic of indefinite, pseudo-dated and dated sentences. Possible ambiguities that creep into modal and temporal sentences are brought to light.

III. The Master Argument again. In distinguishing modality *de dicto* from modality *de re*, the confusion of which had led to the illusion of necessitarianism, Diodorus' challenge will have been met.

I. The nominalist reconstruction of the Master Argument¹ will conclude that, limited to the temporal logic of indefinite propositions, the Diodorean definition of the modalities does preserve the distinction between possible and necessary. It is only the step to a logic of pseudo-dates or, *a fortiori*, to one of dates, that causes the distinction to collapse and renders necessitarianism inevitable. If this can be the case it must be that there is something only tentative about the Diodorean modal distinctions. And they are indeed bound to a meager and very particular system of temporal expression. But "the sense of a system of temporal logic may remain hidden as long as that system incorporates only a part of the temporal determinations of discourse".² The introduction of pseudo-dates, and then of dates, will lift the veil and bring out into the open that which seemed dubious in Diodorus' principles only because of a lack of thoroughness due to the system's incomplete formulation.

What renders Diodorus' principles dubious is the confusion that crops up between the temporal form and the temporal scope of the formulas. No one has done better at exposing that confusion than Ockham has. The third postulate that he posits in order to solve the problem of predestination, God's foreknowledge and future contingents, does in effect distinguish the word from the thing.

"Some propositions are about the present as regards both their wording and their subject matter (*secundum vocem et secundum rem*). Where such [propositions] are concerned, it is universally true that every true proposition about the present has [corresponding to it] a necessary one about the past -e.g., 'Socrates is seated', 'Socrates is walking', 'Socrates is just', and the like. Other propositions are about the present as regards their wording only and are equivalently about the future, since their truth depends on the truth of propositions about the future. Where such [propositions] are concerned, the rule that every true proposition about the present has [corresponding to it] a necessary proposition about the past is not true. And this is not remarkable, since there are true propositions about the past and about the future that have no true [proposition] about the present [corresponding to them]. For example, 'what is white was black' and 'what is white will be black' are true while their [corresponding proposition] about the present—'what is white is black'—is false".³ In treating of

¹Prior, 1967; Boudot, 1973.

²Boudot, 1973, p. 463

³Ockham, 1945, pp. 12-13; Ockham, 1969, pp.46-47. As the translators point out this last example does not justify the conclusion that there is no necessary proposition corresponding to such a proposition as 'Peter is predestinate' which,

the question as to whether 'A was predestinate' is a necessary proposition Ockham makes a distinction between propositions about the past with respect both to form and to subject matter (*secundum vocem et secundum rem*), which are necessary, and those which, though having a past form, are really about the future. The proposition 'A was predestinate' is equivalent to the proposition 'God *will grant* A eternal life'. "...that proposition which is in the present tense in such a way that it is nonetheless equivalent to a future-tense proposition, and the truth of which depends on the truth of a future-tense proposition, has nothing of the necessity of the past; what is more, even though put in the past tense, it has the same contingency as its corresponding present-tense proposition".⁴

Ockham's solution, then, comes down to flushing out ambiguities in the determination of real time which are concealed beneath the apparent determination of grammatical time. In the *Treatise on Predestination* Ockham presents the following objection: "To every proposition about the present that is true at some time corresponds a necessary proposition about the past. For example, if 'Socrates is seated' is true, 'Socrates was seated' will be necessary forever after. But suppose 'Peter is predestinate' is now true. In that case 'Peter was predestinate' will always be necessary. Then I ask whether or not he can be damned. If he can be, suppose that he is. Then 'Peter is reprobate' is true of the present. Thus 'Peter was reprobate' will always be necessary of the past. In this way 'Peter was predestinate' and 'Peter was reprobate' would be true at one and the same time." Ockham replies to the objection: "I maintain that the major premise is false, for to that proposition which is about the present in such a way that it is nevertheless equivalent to a proposition about the future, and the truth of which depends on the truth of a proposition about the future, there does not correspond a necessary proposition about the past. On the contrary, the past-tense proposition is contingent, just as is the present-tense one corresponding to it. All propositions having to do with predesti-

although in the present tense, is really about the future. Ockham's point is that the truth of this last proposition is something that has to be established by what will have come to pass in the future, which makes it like the proposition 'Socrates will be seated' and unlike 'Socrates is seated' whose truth is already established at present.

Due to the necessities of quotation the distinction between *proposition* and *sentence* has largely been set aside in section 3.1. For most occurrences of the word *proposition* it would be more suitable to read the word *sentence* as is natural for a nominalist system.

⁴Ockham, 1945, p. 13; Boudot, 1973, pp. 452-453; Prior, 1967, p. 121; Ockham, 1969, p. 38.

nation and reprobation are of this sort, for they are all equivalently about the future, even if grammatically they are in the present or the past. Therefore 'Peter was predestinate' is contingent just as is 'Peter is predestinate'.⁵

II. The entire difficulty lies in giving formal expression to that intuition. Consider first a logic of pseudo-dates. To this effect we shall construct an 'Ockhamist structure' representing a plurality of possible evolutions of a possible world. An 'Ockhamist model' is obtained by the assignment of truth-values to the propositional variables of a structure. Since a structure is made up of a unique past, but of several branches for the future, the evaluation of a variable will be relative to a particular route. In such a model,⁶ a) the logical pre-determination of the future is satisfied, just as it is in the metaphysics of Leibniz; b) the distinction between propositions about the past and those simply formally in the past can be read off the formulas themselves; c) propositions about the past are necessary; and d) the entailment of necessitarianism is blocked. It will be possible to distinguish between a proposition about the past, even if couched in a grammatical future ('In two days I shall have arrived four days ago'), and a proposition about the future ('There will be a conflict in ten years') even if it occurs in a grammatical past ('Three years ago there was bound to be a conflict in thirteen years').⁷ In 'Ockham's system' thus conceived it is legitimate, on the basis of p , to conclude to the necessity n units of time ago that it will be true, n units of time later, that p ; the necessity here being *de dicto*. By contrast, it is not allowed to infer from p that n units of time ago it was the case that necessarily n units of time later it will be true that p . This would amount to a *de re* predication of necessity.⁸ Predetermination then is seen to be purely logical. From among all future propositions, equally determined from the logical point of view, we could distinguish the proper subset of propositions which are, in addition, causally true and determined in that the present or past causes of their truth already exist. In any case, there is nothing that obliges taking this subset as coextensive with the set of all futures.⁹

⁵Ockham, 1945, pp. 5-6; 1969, p. 38.

⁶Boudot, 1973, p. 456.

⁷Propositions about predestination and reprobation are of this last sort (Ockham, 1969, p. 38).

⁸The formula $p \supset LP_n F_n p$ is thus legitimate, but not the formula $p \supset P_n LF_n p$.

⁹Boudot, 1973, pp. 460-462. It is seen here that predetermination in the sense of the Ockhamist system is weaker than Leibnizian predetermination which includes, rather than excludes, causal necessity (Leibniz, Gerhardt, VI, p. 27 and pp. 123-124; Jalabert, 1962, p. 38 and p. 130). What is called predetermination in Ockham's sense corresponds simply to foreknowledge and futurity, whereas

If we turn now to a system of dates properly speaking, expressed according to fairly natural conditions, and adjoin it to an Ockhamist model, we run the risk of necessitarianism again. The necessitarian thesis depends on two principles: the one maintaining that the truth of a dated sentence is omnitemporal; the other defining the necessary as that which is sempiternally true. The first, which does no more than express the purely logical predetermination of futures, is plausible. But the second, in going from the logical predetermination to a real necessitation, conceals just that confusion denounced by Ockham of past-tense sentence with sentence about the past. It will suffice to escape determinism, then, to admit the validity only of the implication from necessity to sempiternality, refusing the converse implication.¹⁰

III. What of the Master Argument then?

As will have been noted, in order to explain Ockham's position we have tentatively accepted Prior's reconstruction of the Master Argument: reconstruction already rejected for the reasons given in Chapter 1. We shall thus begin by pointing out the virtues of Ockham's solution on the hypothesis of that reconstruction. But it is only after having abandoned that hypothesis that it will be possible to make a truly critical examination of Ockham's principles in their capacity to resolve the Master Argument. That examination, moreover, will lead us to abandon the philosophy of Ockham for that of Duns Scotus.

Leibnizian predetermination requires more, since it nullifies the choice of a middle science.

—It is easily seen in comparing the Aristotelian and Ockhamist notions of the determinateness of a thing, which serves as basis for the truth-value of the corresponding proposition, that the sempiternality of the true does not entail necessity. According to Aristotle (Adams and Kretzmann, in Ockham, 1969, p. 14), "x's being A at t_m is determinate at t_n just in case it is settled by something actual in the past or present relative to t_n that x is (was, will be) A at t_m ". Determinateness of the truth-value then implies the existence of a causal chain going back as far as it does. For Ockham, on the other hand "x's being A at t_m is determinate at t_n just in case it is settled by something actual at some time or times that x is (was, will be) A at t_m ". The truth-value of the proposition 'x will be A at t_m ' can thus be fixed at t_n chosen arbitrarily far back: thus from all eternity. The determinateness of the fact 'x will be A at t_m ' does not entail the existence of a corresponding eternal causal chain. It could be, for instance, that the determinateness of x's being A at t_m goes back only to t_m itself or to an instant close to t_m . Predetermination then turns out to be compatible with contingency. In other words, Ockham's predetermination is logical, having to do only with the truth-value of the proposition and without in any way prejudging of the determination of the thing. Leibnizian predetermination on the other hand—adhering, on this point, to the Aristotelian realism of truth—links the logical assignation of truth-value to the physical assignation of cause.

¹⁰Boudot, 1973, pp. 468-472.

Ockham's solution consists in accusing the ambiguity of the premise attaching necessity to indefinite or pseudo-dated past-tense propositions. (In the system of corresponding propositions in the logic of dates it is possible to formulate the Master Argument but easy to refute it.) For the first premise, asserting the necessity of the past, is not universally applicable. To suppose it so is to suppose that to the truth-value of a past-tense proposition there corresponds the already given actuality of a verifying or falsifying event, even if, in spite of its grammatical form, that proposition is really about the future. This comes down to substituting, right from the start, a linear and unique series of causes for the junctions of contrary possibles. It amounts to settling an ontological question by grammar.¹¹

Of the nominalist solutions that can be given to the problem of future contingents, Ockham's is the most elegant. It preserves all the laws of the logic of sentences and quantification, of modality and tense (with the one exception of that dubious 'law': 'that which is always true is necessary'). The amendment it makes of Diodorus, in order to avoid his conclusion, concerns only the formulation of the first premise, for no doubt is cast on the necessity of sentences which truly are about the past. The first amendment has to do with a metaphysical subrepre-

¹¹ "It is easily verified in Ockhamist semantics that Diodorus' conclusion is invalid. From the truth of ' $\sim \alpha$ and $F \sim \alpha$ ' we cannot conclude ' $\sim M\alpha$ ' (or ' $L \sim \alpha$ ' which is equivalent to it). To prove this suppose α to be ' Fp ' and that two routes branch out from a point x . At every point of the first route ' p ' is false, whereas it is true at certain points of the second. The prima-facie assignment at x according to the first route gives the true as value to ' $\sim p$ and $\sim Fp$ ', while ' $L \sim Fp$ ' gets the false as value. Is this to say that the Diodorean argumentation was somehow defective? Not at all; it is the reasoning's premises that must be called into question. The formula expressing that from the possible the impossible cannot follow is a law of Ockhamist semantics. Likewise, among the additional premises we have introduced, ' $\alpha \supset \sim P \sim F\alpha$ ' is valid and, subject to the supposition that the structures are discrete, that is to say that every point in these structures has an immediate predecessor, ' $(\sim \alpha$ and $\sim F\alpha) \supset P \sim F\alpha$ ' is valid as well. It is naturally the formula schema ' $P\alpha \supset LP\alpha$ ', by which the necessity of the past was expressed in Diodorean logic, that is now invalidated. *It could be justified only if we restricted ourselves to the consideration of linear Ockhamist structures.* But in that type of structure the schema ' $\alpha \supset L\alpha$ ' that expresses necessitarianism is equally valid". (Boudot, 1973, pp. 462-463).

It is seen through this quotation that for Ockham the Master Argument's first premise, understood as signifying ' $Pp \supset LPp$ ', is not universally admissible. Necessity, taken *de dicto*, applies to a grammatical form in itself ambiguous, since it is not known *a priori* whether the event about which the sentence is has already taken place or not. -The realist interpretation we have proposed of the first premise as signifying the irrevocability of the past excludes, on the contrary, any formal ambiguity in the sentences. It consequently renders such an Ockhamist solution inoperative.

tion, the second with a grammatical one. They are in conformity with the maxim of economy proper to Ockham but are characteristic of all nominalism as well.

Nominalism applied to the modalities posits as principle that they are properties of discourse, not of things. This principle, moreover, is general, as can be seen from its application to the problem of universals. Either language is separable from ontology and the laws governing the entities of language commit us to nothing with respect to being: any liberties taken then with the laws of language will be without danger, since these laws govern only the use of signs. Or, if there is a suspicion that such-and-such linguistic usage does commit us with respect to being, all the laws having to do with those usages, and whose mixed character would allow the surreptitious extension of simple linguistic conventions to ontological commitments, will be eliminated as dubious. As for the modalities then, either it will be posited that the modal laws have to do only with the manipulation of certain symbols, or those that seem to have a mixed—viz. a metaphysically dangerous—character will be proscribed.

But just what is a modality that has to do only with discourse? It is a modality whose scope contains a complete sentence, an expression of discourse. When the expression contains a quantifier, whether explicitly or implicitly—as, for instance, when ‘*p* was’ is taken as equivalent to ‘it has not always been the case that not-*p*’—the modality is *de dicto* if the modal operator precedes the quantifier. Consider on the other hand an expression in which one of these quantifiers has been ‘exported’ and placed in front of the modal sign. This latter sign will then govern an expression containing a free individual variable. This is the case that arises, for instance, when it is said that there is an *x* such that it is possible (necessary) that *x* has a certain property. Two interpretations seem to be open here.¹² Either exportation of the quantifier will be given a weak sense in which modality and predicate are still taken separately. The modal operator will govern a propositional function then, rather than a proposition. Or the predicate could be taken as argument of the modal operator, in which case the two together would henceforth be treated as a single predicate. The reading then would no longer be ‘it is possible (necessary) that *x* has a certain property’ but ‘*x* has a certain possible-(necessary-) property’.

The Scholastics had a clear vision of the distinction between modality *de re* and modality *de dicto*. Buridan, for example, in his commen-

¹²Hughes and Cresswell, 1972, pp. 183-184, footnote 131, which suggests distinguishing between $L(\varphi x)$ and $[L\varphi]x$.

tary on the *De Caelo*,¹³ exposes the ambiguity of the proposition: 'All that is destructible will be destroyed necessarily', which can be taken in a 'composite' or in a 'divided' sense. In the composite sense it means that for any x whatsoever, if x is subject to destruction, then it is necessary that x will be destroyed (*Omne corruptibile de necessitate corrumpetur*). In the divided sense it means that it is necessary that, for any x whatsoever, if x is subject to destruction then x will be destroyed (*De necessitate omne corruptibile corrumpetur*). The first sense is *de re*, the second *de dicto*.

This last distinction appears most clearly when the modal calculus is extended to include identity. Accordingly as the modality is or is not treated as a possible property of objects, this adjunction leads to two sorts of entirely different systems: necessary identity systems and contingent identity systems. In the first, every identity is necessary, as is every difference. This necessity stems from the fact that, if the identity of two things entails that a property which appertains to the one appertains to the other, then, identity with itself, appertaining necessarily to the one, is communicated necessarily to the other.¹⁴ In this case, necessity is treated just as any property whatsoever. Of course some well known paradoxes follow from this conception. If that man who is sitting is Socrates, it seems difficult to admit that it is necessary that that man who is sitting be Socrates.¹⁵ Contingent identity systems avoid these paradoxes in restricting the substitution of propositional functions containing identities to those not falling within the scope of a modal operator.¹⁶ In these systems, possible-property and necessary-property do not count as genuine properties.¹⁷ Identity is defined only within a given world, and may not obtain in another. Contingent identities are thus preserved, but only at the expense of evacuating the truly logical content from the very notion of identity. These last systems are extremely nominalistic with respect to the modalities which play a harmless role in the realm of discourse. At the same time, it becomes no longer possible to identify an individual, and the entire edifice of quantification theory is threatened.

¹³Buridan, 1942, p. 118.

¹⁴Hughes and Cresswell, 1972, p. 189 *sq.*

$I_1 : x = x, I_2 : x = y \supset [\varphi(x) \supset \varphi(y)]$ and $L(x = x)$.

(1) $x = y \supset [L(x = x) \supset L(x = y)]$, substitution in $I_2 : x = z/\varphi(z)$.

(2) $L(x = x) \supset [(x = y) \supset L(x = y)]$, permutation in (1).

(3) $x = y \supset L(x = y)$, modus ponens applied to (2).

¹⁵*Id.*, *ibid.*, p. 191.

¹⁶*Id.*, *ibid.*, p. 195: cf. I'2.

¹⁷*Id.*, *ibid.*, pp. 199-200, ft.n. 151.

When modality and predicate together form a new predicate *sui generis*, the modality is said to be *de re*. It introduces distinctions in the universe between those properties that happen to belong to things by chance, those that are inherent in them and essential, and those that, as possibilities, are attached to their development. In this usage modality goes from the sign to the signified. It is no longer linguistic, but ontological. That is why nominalism must banish it, in showing that such usage can be eliminated from discourse (as is supposed when in a *de re* context the modality is interpreted as operating from the outside on a propositional function which is its argument) either by means of some general and regular procedure, or by adopting particular measures. We would dispose of a general and regular procedure if we had a modal logic in which all modalities *de re* were automatically eliminable in favor of modalities *de dicto*. To this end von Wright has suggested the adoption of a 'principle of predication' according to which all properties could be exhaustively partitioned into two exclusive categories: *formal* properties, whose belonging to an object is always necessary or impossible, and *material* properties, whose belonging to an object is always contingent.¹⁸ But, on the one hand, even the adjunction of the 'principle of predication' to the most classical modal logic¹⁹ falls short of providing for the elimination of the modalities *de re* and it is an open question as to whether there is an effective method allowing for the construction of a formula of elimination for all the modalities *de re*.²⁰ On the other hand, just what would the principle of predication come down to? Its point is evidently to separate the truths of reason from the truths of fact; but this is accomplished by reducing the former to simple reports of linguistic transformations, the latter alone being susceptible of expressing any empirical content. The question might be raised however as to whether such a transparent language, thus freed from the opacities favoring the confusion of ontology and grammar, would still have a real power of communicating ideas.

To be effective the nominalist must formulate particular rules of exclusion. Ockham's method consists precisely in doing this. Implications in which the consequent involves a modality of necessity *de dicto* are accepted;²¹ those same implications where the modality would figure *de re* are refused.²² Necessity is escaped, then, either through

¹⁸ von Wright, 1951, pp. 26-28; Hughes and Cresswell, *op. cit.*, pp. 184-188: $(x)(L\varphi x \vee L \sim \varphi x) \vee (x)(M\varphi x \vee M \sim \varphi x)$.

¹⁹ S₅.

²⁰ Hughes and Cresswell conjecture that there is no such method (*op. cit.*, p. 187).

²¹ For instance: $p \supset LP_n F_n p$.

²² $p \supset P_n L F_n p$.

excluding Diodorus' first premise or through systematizing, within the context of a richer and thus more distinct system than that of Diodorus, the distinction between the two modal usages. In the one case it is the universality of the necessity of the past that is denied; in the other, with respect to what has taken place, it is accepted that it is necessary that it has taken place, without its being accepted for all that that it was necessary that *it should* take place. What is called into question then is not the first premise properly speaking; it is only its formal expression in which a distinction is drawn either between grammatical and real necessity or between necessity of the thing and necessity of the sentence.

4.2 Inadequacy of Ockham's solution. Incrimination of the principle of conditional necessity: John Duns Scotus.

Ockham's nominalism does succeed in avoiding necessitarianism. The question remains as to whether this is so for the reason alleged.

That reason is said to be the ambiguity of the past tense in a sentence. But Diodorus would surely have responded by purging his first premise of any ambiguity himself. Could he, in fact, have done this? Yes, he could have, on the condition of considering propositions having a determinate subject instead of sentences. For such propositions the distinction between past and future is fixed absolutely by the now. From there on the necessity of the past becomes an ineluctable truth, even on an Ockhamist interpretation. Ockham himself, in commenting on Aristotle's *Ethics* (*Eth. Nic.* vi. 2, 1139^b5-11) where it is said that "in this alone is God deprived: to make undone things that have been done", says that "if some assertoric [non-modal] proposition merely about the present that is not equivalent to one about the future is true now, so that it is true of the present, then it will always be true of the past. For if the proposition 'this thing is'—some thing or other having been indicated—is true, then 'this thing was' will be true forever after, nor can God in His absolute power bring it about that this proposition be false".²³

If one accepts Prior's interpretation, the reason why the Master Argument doesn't go through has to do with the ambiguity of the sentence about which one reasons. A sentence about the future (third premise) is transformed (by means of Prior's second additional premise) into a past-tense sentence and then, thanks to the first premise, its modality is fixed. The remainder of the argument serves only to deduce the im-

²³Ockham, 1945, p. 4; 1969, p. 36.

possibility of what had been proposed. Necessitarianism is thus seen to result from the fact that the first premise is applied to a proposition about the future although expressed grammatically in the past.²⁴

There is a twofold line of objection however that does away with this argument.

The first is inherent in Prior's conception itself. Just what is the meaning of saying of something that it neither is the case nor ever will be? The statement is at once both present and future and its future part does no more than continue and repeat its present part. If we represent the model of this future, it excludes, by hypothesis, any branching and is thus linear. In this way the thing is determined now and causally so, not simply logically so. This is also the reason why it can retrograde. The Master Argument's third premise, then, is one of those propositions about the future that Ockham would take to be necessary:²⁵ it is not a future contingent. Granted that, the argument from the formal ambiguity of past-tense propositions no longer applies since the future spoken of here is in reality already given.

Should one reject this first line of objection in denying that the pre-determination of truth entails a causal predetermination here, there will remain an even greater difficulty if one abandons Prior's reconstruction of the Master Argument which was in fact meant to reveal an ambiguity in the argument's first premise. Suppose that the first premise does simply signify the irrevocability of the past due to the fact that a possible cannot be realized in the past. Thus construed it can have to do only with true pasts, and Ockham's maneuver loses its point.

On what condition could there possibly be another interpretation that would invalidate the Master Argument while applying Diodorus' first premise only to genuine pasts? The proposition saying that p is not true now nor ever will be true in the future is true now. On Ockham's principles then its truth can retrograde without ambiguity.²⁶ Is the Master Argument irresistible? It will be unless one holds that the realization of the possible in the past leads to a proposition which is certainly false but not impossible. The non-realization of the possible

²⁴

$$\{[(\sim p \quad \sim Fp) \quad \supset \underline{P} \sim Fp] \quad \supset \sim M \sim \underline{P} \sim Fp\} \supset \sim Mp$$

3 5 1 4 and 2

The letters accused of ambiguity have been underlined; the numbers refer to Prior's premises (1967, pp. 32-33).

²⁵Ockham, 1945, note 6.

²⁶*Si haec propositio sit modo vera: Haec res est, quacumque re demonstrata, semper postea erit haec vera: haec res fuit* (Ockham, 1945, p. 4; quoted by Prior, 1967, p. 35).

would not then entail its non-realizability. That apodeictic incompatibility that is required before the second Diodorean premise can be applied would hence disappear. The Ockhamist model of branching time is excluded in reality because it has been posited that the possible will not be realized. Different possible worlds are not dismissed for all that. In short, what would be challenged here is no longer the validity of the first premise, nor that of any of the Master Argument's explicit premises for that matter, but the validity of conditional necessity, which is required for transforming non-realization in the future into non-realizability.

Ockham's interpreters that we have referred to assert that, to avoid determinism in a system of dates, Ockham refused to admit that what is always true is necessary. They justify this refusal, which allows going from the necessary to the sempiternally true but blocks the inverse route, by limiting the validity of the axiom: if p is true at t , then it is true at t' that it is necessary that p be true at t . The restriction they adjoin is: $t' \geq t$. Abandoning this last condition would result in necessitarianism. Compelling predetermination would then be the rule. But this is precisely because one would have taken the formal past, t' , of the retrograded statement of p , for its real past, t .²⁷ Denying the passage from the sempiternally true to the always true in the logic of dates comes down once again to eradicating the ambiguity of grammar and reality in the first premise. Yet these same interpreters pride themselves on having saved the principle of conditional necessity for Ockham. And indeed, positing $t' = t$, the preceding axiom reads 'if p is true at t , then it is true at t that it is necessary that p at t ', which is simply another expression of the principle of conditional necessity recognized as legitimate.

But does this argument, designed to render the logical predetermination of a dated event compatible with the real contingency of that event, apply in effect to the Master Argument? Not really, because the third premise precludes the realization of a certain possible for all time going from now to the most distant future. What is at stake then is not the logical predetermination of a future event, but the continuous determination of a sempiternal event—as was required by both the *Timaeus* model and the *De Caelo* interpretation. But the sense of the contested passage from the sempiternal to the necessary changes radically when it is no longer applied to a transitory event but to a sempiternal duration (in so far as the future is concerned).

²⁷Boudot, 1973, pp. 471-472.

Suppose that conditional necessity holds. Since the possible mentioned in the third premise does not exist and continues not to exist for all future time, the posited axiom compels us to say that it is necessary that it should not exist for all this time. And since it is necessary that it not exist during all this time, it is impossible that it exist during all this time. Q.E.D. We do not see then how Ockham can avoid having the Master Argument's third premise reduced to the impossible, if he does admit the principle of conditional necessity.

His interpreters were not mistaken in constructing an Ockhamist model that verifies the principle of conditional necessity. The question is how such a model can still meet Diodorus' challenge.

In the third question of his *Treatise on Predestination* Ockham discusses, and subsequently rejects, the calling into question of the principle of conditional necessity. "How can the contingency of the will, both created and uncreated, be preserved [in the case of its] causing something external? That is, can the will, as naturally prior to the caused act, cause the opposite act at the same instant at which it causes that act, or can it at another, subsequent instant, cause the opposite act or cease from that caused act?"²⁸

Duns Scotus' doctrine maintains that there are two possibilities. It recognizes in the will, alongside an evident capacity for opposite acts or opposite objects in succession, a nonevident capacity for opposites without succession. Ockham himself admits the first capacity. He contests the second.²⁹

²⁸Ockham, 1945, p. 32; 1969, p. 71.

²⁹Ockham, 1945, pp. 32-33; 1969, p. 83. The fundamental text is that of Duns Scotus, Vivès, t. 10, 1843, *In librum primum sententiarum*, D. XXXIX, Q. 1, 628-629. "This freedom is accompanied by an evident capacity for opposites. It is not allowed that there be in it a capacity to will and to not-will at the same time, for this does not exist; but there is in it the capacity to will and then afterwards to not-will, i.e. the capacity for a succession of opposite acts, and this capacity is evident in all changing beings in which opposites succeed one another. There is [in it] too, nevertheless, another capacity that is not evident so and that is outside all succession. For if we posit a created will existing only at one instant and having a certain volition in that instant, then it will have that volition in a non-necessary manner. Proof: if in that instant it did indeed have it in a necessary manner, since there is no cause outside that instant where it would cause, the will, when it would cause, would then simply cause necessarily. Indeed, it is not only the cause that preexisted that instant in which it causes and which then, as preexistent, was able to cause or not cause, that is contingent, for just as that being when it is necessary or contingent, so the cause, when it causes, causes then in a necessary or contingent manner. From which it follows that in that instant it causes this willing in a non-necessary, and thus contingent manner.

There is thus a capacity of the cause, that causes without succession, for its opposite, and that real capacity is the capacity of that which is naturally prior, as first acts, to opposites, which are naturally posterior, as second acts. For the first

Consider first created will. Scotus, criticizing the naturalism of the philosophers, refuses to conceive of the created will as a passive capacity to be moved by a representation. His argument is based essentially on the existence of a volition and the question as to how the negation of an object would have the force to determine the will.³⁰ In short, nothing extraneous to the will is the total cause of volition in the will. Regarding the nature of the things with respect to which our will is free, three different freedoms must be distinguished. In the first place the will is free *ad oppositos actus*, *i.e.* to will or to refuse a same object. Next, it is free *ad opposita objecta*, since it has the power of tending towards opposite objects. Finally, it is free *ad oppositos ef-*

act, considered in that instant where it is naturally prior with respect to the second act, posits this latter in being as its effect in a contingent manner, so that in so much as naturally prior it could equally posit its opposite in being."

³⁰Duns Scotus, Vivès, t. 13, 1843, *In secundum librum Sententiarum*, D. XXV. Q. 1, 199-201.

"A modern doctor (Godfrey) says here that something other than the will is the effective cause of the volition in it and he posits that this other thing is a phantasm. His principal reason is the following: mover and moved must be distinct as to the subject, but in the intellectual part of the soul there is nothing that can be distinct as to subject from the will; hence there is no motive for it there; so there must be something else outside the intellectual part, and that is the phantasm. He proves the first proposition thus: to posit that motor and moved are not distinct as to subject is the same thing as to posit that a thing moves itself, which is impossible. First, because one and the same thing would then be in act and in potency at the same time with regard to itself, because it is said (*Physics*, 3) that as the motor is in act, so the moved is in potency. He says also that that, *viz.* that there cannot be a self-mover, is the principle of metaphysics; to do away with it therefore is to deprive oneself of the veritable principle of research in metaphysics.

The second reason is that matter and efficient cause do not coincide in a numerically single thing (*Physics*, 2); thus neither do agent and patient.

Likewise (*Metaphysics*, 5), the motor is really in relation with the moved, but the relation of the motor with itself is not a real relation. The proof of this is that what are really relatives are really opposites, but opposites cannot be simultaneously proper to a same thing".

"Another opinion is that of a more ancient Doctor (*St. Thomas*) who posits the same conclusion, *viz.* that the will is moved by something else, but posits that that other thing is the object cognized or comprehended".

"Against the conclusion in itself there are reasons of principle, and I advance these in reasoning as follows. A natural agent cannot by itself be the cause of contraries concerning a same passive being..., but it is in the power of our will to have *not willing* and *willing*, which are contraries with regard to a same object. These two things, therefore, cannot result from an agent naturally, thus not from an object either, which is a natural agent. If it is posited then that an object would be the cause of willing, there must nevertheless be another that would be the cause of *not willing*. But that object, other than the will, could not be but a bad object; but since evil is only a privation, it could not be the cause of the posited acts of the *not willing* type. Therefore it must be effectively by the will". Commentary in Gilson, 1952, p. 583.

fectus, i.e. with respect to the opposite effects it can produce.³¹ As regards the manner in which contingency or possibility results from freedom, Scotus distinguishes two capacities. The first results from the first freedom, *ad oppositos actus*. Our will can engage in opposite acts successively, i.e. it can will after having refused or refuse after having willed. On the other hand, it is impossible to will and not will at the same instant: this would be nothing.³² The second capacity is not manifest and excludes succession. "For if we suppose that there is a created will that exists at only one instant, and that at that instant it has this or that volition, then it does not necessarily have it at first. For if at that instant it had the volition necessarily (since it is a cause only at that instant when it caused the volition) then, absolutely, the will, when it caused the volition, would cause it necessarily. For it is not now a contingent cause because it pre-existed before the instant at which it causes and pre-existing then could either cause or not cause. For just as this or that being, when it is, is then either necessary or contingent, so a cause, when it causes, causes then either necessarily or contingently. Therefore whatever this willing causes in that instant, and causes not necessarily, it causes contingently. Therefore this capacity to cause the opposite of that which it does cause is without succession. And this real capacity is a naturally prior capacity (as of first actuality) for opposites-[opposites] that are naturally posterior (as of second actuality). For first actuality, considered at that instant at which it is, is naturally prior to second actuality. Thus [first actuality] contingently posits [second actuality] in reality as its effect, so that as naturally prior it could equally posit the opposite in reality".³³

³¹Ockham, 1945, p. 33; 1969, p. 82; Gilson, 1952, pp. 586-587.

³²Ockham, 1945, p. 33; 1969, p. 82.

³³Quoted in Ockham, 1969, pp. 82-83. Gilson (1952, p. 587) comments as follows:

"Consider a created will whose existence would last but a single instant. Imagine moreover that in that unique instant it exercises a determinate volition (*hanc volitionem*). Obviously its freedom of choice would be exhausted in that unique act that could be said to fill out the totality of its existence. Yet that unique volition would not be necessary, and here is the proof: if the will exercised that unique volition necessarily, since it would be a cause only in that one instant, it would be a necessary cause, thus a 'nature', that is to say the contrary of a will. It makes no difference here whether it existed in a preceding instant where it would have willed the contrary. It is in the present instant, the only one in which it exists, that it must be considered. But just as in each present instant of whatever duration a being is either contingent or necessary, so also, in the present instant in which it exercises its causality, the cause exercises it either in a contingent or in a necessary manner. Since by hypothesis it is a will that is spoken of, it does not cause in a necessary, but in a contingent, manner. *A will is thus capable of willing the contrary of what it wills and of causing the contrary of what it causes, in the same time in which it wills and causes.* Assuredly, it cannot will or cause contraries simultaneously, but,

Duns Scotus reasons on the false supposition of the instantaneous existence of a created will. But this false supposition implies no contradiction: it constitutes a possible world. In such a world there would be a single act that would exclude its contradictory. If we were reduced simply to the manifest contingency in the succession of opposites, such a world would be necessary because unique. But it was supposed that it was a free, even though instantaneous, will that acted in this world. It must be then that this freedom is not absorbed by its act and that there is a capacity for contraries which is not deployed and which is not deployable in time. This is to affirm logically that an act is compatible with the potentiality of its contrary; and such is the radical contingency *ad extra* presupposed by the freedom of a created will reduced to an instant of life.

Scotus next takes up the case of the divine will. Regarding the things with respect to which it is free, it differs from human will especially in that it is not free *ad oppositos actus*, as God is immutable. As for that kind of contingency that relates it *ad extra*—and it is this radical contingency that characterizes the Christian God, distinguishing Him from the God of the philosophers—,³⁴ we find again what was said of the non-successive contingency for man (successive contingency being excluded by the divine immutability): "...our will, as naturally prior to its act, elicits that act in such a way that it could at one and the same instant elicit its opposite. In the same way the divine will, insofar as volition itself is naturally prior to such an intention (*tendentia*), intends the object contingently in such a way that at the same instant it could intend the opposite object. And this [is] as much by virtue of a logical capacity—*i.e.* the compatibility (*non repugnantia*) of the terms—as by virtue of the real capacity—*i.e.* [the will's being] naturally prior to its act".³⁵

at the same time as it wills and causes the one, it retains its essential aptitude to will and cause the other: *est ergo potentia hujus causae ad oppositum ejus quod causat, sine successione*".

³⁴Gilson, 1952, p. 34, p. 262, p. 363; Duns Scotus, Vivès, t. 4, 1841, *De rerum principio* (Against the necessary production of creatures by God, according to Avicenna), Q. IV, art. I, sect. I, p. 305. "One cannot posit then that the creature, which has the capacity for not-being, issues necessarily from the necessary being, for, even though formally it would not be the necessary being, it would nevertheless have the necessity of being, in virtue of which one opposes to it, in a contradictory manner, the fact that it can not-be".

Q. IV, art. I, sect. III, p. 307. "It is therefore not due to any necessity that God wills any extrinsic thing whatsoever; He does only what He wills and how He wills it; thus He neither makes nor produces anything extrinsic necessarily".

³⁵Ockham, 1969, p. 83; Duns Scotus, Vivès, t. 4, 1841, *De rerum principio*, Q. III, art. III, sect. I, p. 297. "Thus, in the creature, the will is equally called will-

The psychology of volition supposes a logic of propositions in contingent matter.³⁶ Consider the proposition: the will that wills A may

ing, because the same will, by its nature, insofar as it is a free capacity, involves opposites, like to read and not to read, and is free as regards any time difference whatever, because it can will or not will in the divided manner (*divisim*) as regards any now. It is nevertheless constricted by the act of willing, for its act is such that it is one thing to will that it be, it is another thing to will that it not be, and it is one thing to will to do a certain thing tomorrow and not before, another to will the opposite or to do nothing. But it is the contrary with God, as it is the same volitional capacity by which He can will that this be and will that it not be, and that this be tomorrow, and by which He can will that the same thing be before or after. And, in short, it is the same capacity by which He wills or does not will all that He wills or does not will; what is more, it is the same capacity by which He wills that this not be and wills that this be, and before or after or once again: so it is with the same act of willing neither changing nor varying nor repeating itself that He wills that this be and can will the opposite".

P. 299 (same section). "Thus by the same act of willing God wills contradictories, not that they exist together, for this is impossible, but he wills them together; in the same way, it is by the same intuition, or by the same science that He knows that contradictories are not together, but they are known together by the same act of science, which is one and the same act."

Leibniz is on Ockham's side here, rejecting Scotus' subtleties as purely verbal. In his *Essays on the Goodness of God, the Freedom of Man and the Origin of Evil*, second part, §132-XVII (Leibniz, Gerhardt, VI, p. 184; Jalabert, p. 93), he quotes the principle of conditional necessity as it is formulated in Chapter IX of Aristotle's *De interpretatione* and adds: "The nominalists adopted this Aristotelian maxim. Duns Scotus and many other scholastics seem to reject it, but at bottom their distinctions come down to the same thing".

³⁶Gilson, 1952, p. 588; Duns Scotus, Vivès, t. 10, 1843, *In librum primum sententiarum*, pp. 629-630. "...*The will willing A can not-will*, that proposition is false in the sense of composition in that it signifies the possibility of the composition: *the will willing A does not will A*. But it is true in the sense of division in that it signifies the possibility for opposites successively, because the will willing at A can not will at B. But if we also accept a proposition about the possible uniting the extremes in a same instant, like this one: *the will not willing something at A can will it at A*, this last must also be distinguished as to composition and division. And in the composed sense it is false, for it says that it is possible that it [the will] itself be at the same time willing at A and not willing at A. But the divided sense is true; it signifies then that to that will, to which willing at A belongs, there can belong not willing at A, but without this latter having to exist at the same time, because then the willing would not belong to it.

"And to make this second distinction which is more obscure understandable, I say that in the composed sense there is one single categorical proposition, whose subject is: *the will not willing at A*, and the predicate: *willing at A*; and so this predicate is simply attributed possibly to this subject to which it is repugnant, and therefore it is impossible that that which is noted as belonging possibly to itself should belong to itself. In the divided sense there are two categorical propositions, attributing two predicates to the will. In a proposition of class-membership the predicate *not willing A* is attributed to the will and the categorical falls implicitly under composition. But in the other categorical [proposition] on the subject of the possible, it is stated possibly *willing A*. And these two propositions are verified at

not will A. In the composed sense: the will that wills A does not will A is an absurd and impossible proposition. In the divided sense and taken in the order of succession the proposition is acceptable, for the will³⁷ can will a certain object at a moment A and not will it at moment B. But even in a given instant, the proposition remains true in the divided sense; for even though the will that wills A cannot not will it that will that wills A is such that it may not will it. Even willing it at moment A, it is of itself capable of not willing it at moment A. Scotus recognizes that this is a difficult distinction to grasp: *obscurior*, but it is well-founded, for, in the divided sense, it justifies the possibility of two distinct affirmative possibilities: the one saying that the will wills at the moment A, the other that it is possible that the will not will at the moment A. Being about the same time, but not about the same object, they can be simultaneously true; and indeed they are, for it is true that at the very moment at which the will is not willing it can will, and that, willing one object, it could will another.

Ockham refuses Scotus' capacity without succession in arguing that such a possible could not be realized. "This nonevident capacity can be actualized by no capacity, since if it were actualized the will would will something at t_1 and not will it at t_1 , and so contradictories would evidently be true at one and the same time".³⁸ Therefore, "it is inconsistent to say that the divine will as naturally prior posits its effect in reality at t_1 in such a way that it can *not* posit it in reality at the same instant. For there are no such instants of nature as he [Scotus] imagines, nor is there in the first instant of nature such an indifference as regards positing and not positing. Rather, if at some instant it posits its effect in reality, it is impossible by means of any capacity whatever that both the instant occurs and the effect does not occur at

A because they signify that their predicates are attributed to the subject during the same instant. But this is true. For proper to that will in the same instant is *not willing A* together with the possibility of the opposite at A as is signified by the proposition about the possible.

"There is an example of this distinction in the proposition: *all men who are white run*. Being posited that all white men run and no blacks or dark-skinned men, it is true in the composed, and false in the divided sense. In the sense of composition it is a proposition having a unique subject determined by: *who are white*. In the sense of division there are two propositions stating two predicates of the same subject. Likewise for the proposition: *all men who are white are necessarily animals*. In the sense of composition it is false, because the predicate is not proper necessarily to all this subject. But in the sense of division it is true, because two predicates are affirmed of a same subject, one necessarily, the other absolutely without necessity, and both are appropriate, and the two categorical propositions are true".

³⁷Created.

³⁸Ockham, 1945, p. 33; 1969, p. 85.

that instant, just as it is impossible by means of any capacity whatever that contradictories are true at one and the same time".³⁹

Let us now bring the discussion back to its logical significance. Scotus maintains that one can at one and the same instant act and possibly act otherwise than one does.⁴⁰ This means that the act is compatible

³⁹Ockham, 1969, p. 87. Concerning the question of instants of nature cf. Baudry, 1958, p. 128. "*Instans naturae*. To explain the knowledge God has of future contingents Scotus imagines what he calls instants of nature. In his treatise on future contingents (p. 35) Ockham rejects the Scotist theory in a word. '*Non teneo Scotum quia in omnibus illis instantibus naturae erravit*'. The reasons for which he does so are found in E.A., fol. 85 a. When it is said that one thing is prior to another by a priority of nature, it must not be imagined that it exists in an instant in which the second does not exist. For that instant would be either in the soul or outside the soul. It cannot be said that it exists in the soul, for on that hypothesis, if the soul didn't exist that priority of nature wouldn't exist either, and moreover, that instant of nature could be neither a *habitus*, nor a capacity, nor a *species*, nor the act of intellection nor its object. It cannot be said to exist outside the soul, because it would then be either a substance or an accident. Which is impossible. For, consider the priority of nature Socrates has with regard to his whiteness. If one admits that Socrates exists in an instant of nature in which his whiteness does not exist, either that instant of nature is none other than Socrates and it will have to be said that Socrates is in Socrates, or that instant is a part of Socrates and then Socrates will be in a part of Socrates, or that instant of nature is an accident of Socrates and Socrates will be in one of his accidents. On the other hand, it cannot be said that that instant is something exterior to Socrates. It must be held as indubitable, therefore, that a priority of nature no more entails the existence of an instant of nature than the highest honorability or highest perfection entail the existence of instants of perfection and honorability. The instants of nature admitted by Duns Scotus do not exist': '*Indubitanter est tenendum quod non magis sunt instantia naturae quando aliquid est prius alio natura quam sunt instantia honoris et perfectionis quando aliquid est honorabilius et perfectius alio... Universaliter tenendum est quod non sunt aliqua instantia naturae nec originis praeter instantia temporis*".

⁴⁰Duns Scotus, Vivès, t. 4, 1841, *De rerum principio*, Q. IV, art. II, sect. V, p. 315. "But willing and refusing, which are one in God, are names expressing the connection of this same divine willing to different effects in such a way that this connection and relation is really in the creature, but according to reason and our concept in God, just as creator and God express for an eternal thing the new relations which, according to reason, are in God, but, according to the thing, in the creature. Whence it is seen that, from one side, it is true that God wills and at the same time does not will a thing, in so far as that act is identical with God, and connotes no thing.

From another side, they are neither a same thing nor in the same time, but successively, as long as they imply an exterior connotation, in such a way that there is a succession according to the thing in what is connoted, but according to reason and our concept in God".

(Same section, p. 316) "Just as divine intuition sees that the one and the other part of a contradiction in the same time do not exist, but can happen (*fore*), not in conjunction (*conjunctim*) but in division (*divisim*), so does the divine will too, by a single immutable willing, bear on the one and the other extremes of the

with the potentiality of its opposite. Ockham replies that there is incompatibility here, since to realize the contrary possible would be to enter into contradiction with the act. The divine will itself cannot do that. What is clear is that Ockham admits the principle of conditional necessity. The act, *while* it is, excludes the capacity for its opposite whose realization can take place only after it itself will have disappeared. But that successive representation of potentiality, as legitimate as it may be in suggesting something positive in the potentiality and going beyond the simple logical reality or non-contradiction, appears to be ill-founded in Ockham. When he claims to reduce Duns Scotus to the absurd, he is, in reality, simply giving in himself to the sophism of the distribution of the modalities, confusing the real contradiction between act and contrary act with the apparent contradiction between act and contrary possible.⁴¹

contradiction. It wills in willing them simultaneously, not however that they should exist together, but it wills simultaneously that they should be able to happen [here I propose correcting *forte* to *fore*] not in conjunction but in division. And just as divine intuition sees the contingency of causes and the effects that will result contingently from them, so does the divine will will that created causes act in a contingent manner, or, on the contrary, a necessary one, according to the order of causes" (see the end of note 36).

⁴¹Duns Scotus explicitly contests the principle of conditional necessity in a text that highlights his virtuosity as a dialectician: Vivès, t. 10, 1843, *In librum primum sententiarum*, pp. 630-631.

"Aristotle's proposition: *it is necessary for whatever is that it be while it is* can be categorical or hypothetical, as is also the proposition: *it is necessary that an animal run, if a man runs*. As a conditional, this latter gives rise to a distinction accordingly as *necessary* can mean to say the necessity of consequence or of the consequent. In the first sense it is true, in the second false. But the second which is a categorical, signifies that the whole: *should run if a man runs* is predicated of animal with the mode of necessity, and the categorical is true, because the predicate thus determined belongs necessarily to the subject. But it is not a question of the predicate absolutely and hence to conclude from the predicate thus determined to the predicate taken absolutely is the error *secundum quid ad simpliciter*. Likewise I say here that if Aristotle's proposition is accepted as a temporal hypothetical, then the *necessity* notes the necessity of the consequence or that of the consequent. If of the consequence, it is true; if of the consequent it is false. But if taken as a categorical, then what is, while it is, does not determine an implicit composition in what is, but a principal composition signified by the fact that what is *should be*, and then the predicate that *that be when it is* is affirmed of the subject which is with the mode of necessity, and in this way the proposition is true. Thus it doesn't follow that it is necessary that it be, but there is a sophism of *secundum quid ad simpliciter*, as appears in the other case. Hence no true sense of this proposition denotes that being something in the instant where it is *be* simply necessary, but only *be* necessary *secundum quid*, i.e. when it is, and as this is in harmony with the fact that in the instant where it is the thing is simply contingent, it is consequently in harmony also with the fact that in that instant its opposite can belong to it."

In all likelihood neither Scotus nor Ockham knew the Master Argument. It just happened that similar difficulties gave rise to responses that can be compared systematically with those of the Ancients.

Scotus clearly expresses the consequence for freedom of calling conditional necessity into question (*ibid.*, p. 637).

"Just as our will, in so far as naturally prior with respect to its act, produces that act in such a way that it should be able at the same instant to produce the opposite act, so the divine will, in so far as it is itself under a volition, is naturally prior with respect to such a tendency and tends to that object contingently, in such a way that in the same instant it can tend to the opposite object. And this is so from the point of view both of the logically possible (which is the non-contradiction of terms, as was said regarding our will) and of the real capacity, which is prior with respect to its natural act."

In his Commentary in the Vivès edition, Lychet sets down Ockham's objections:

1. There is not such a nonevident capacity for opposites in the same instant, for no capacity, not even an infinite one, could reduce such a capacity to act except by a contradiction in the facts (p. 641). This argument is none other than the sophism of distribution.

Gregory of Rimini answered this first argument in positing first (p. 643) "that whatever is in a given instant can not-be in that instant, so that the proposition stating that it is not can be true in that instant. This can be understood in three ways. a) That this is possible in the composed sense, so that in that instant that thing, at the same time as it is, is not in that instant. Then both the proposition stating that it is and the proposition stating that it is not conjunctively and simultaneously in the same instant will be true. But this sense is truly impossible because it implies a contradiction. b) It can be conceived or understood to be true in the divided sense; and this in two ways. According to the first, the thing that is in a given instant, can already, being posited in being, cease to be, and pass from being to non-being in the same given instant. The proposition stating that it is ceases to be true and from true becomes false, so that in that instant it is false; and, conversely, its opposite ceases to be false and becomes true. But this sense is impossible because such a passage and such a succession are not possible in the instant. According to the second way, it is absolutely and simply possible that this thing not be then and that it not be posited in being by its cause. The proposition stating that it is not in that instant is true. This sense is true and in no way improper."

2. Ockham's second argument against Scotus is based on the nature of the past (p. 641).

"It is generally admitted by philosophers and theologians that God cannot make a non-past of the past without its afterwards being true to say that the past was. Therefore, since by hypothesis 'the will wills at A' is determinately true and, consequently, always will be true afterwards, whereas 'the will does not will this at A' never was true, 'the will did not will this at A' always was impossible."

Gregory of Rimini answers (p. 641). "Suppose that God can not make it happen that the past not be the past (though many theologians contest that supposition). I say that if the will wills something in the instant B, afterwards the proposition 'the will willed this in the instant B' will always be true, so that the consequence 'the will wills this at B' is necessary. Therefore, after B, the proposition 'the will willed this in B' will always be true if it is formed. But I say that the antecedent is contingent and can not-be-true, even in the instant B. And if one posits that it is not true at B, as is possible to do in virtue of the third sense permitted, 'that

Ockham hasn't really called the first premise into question. He has simply removed an ambiguity attaching to one of its modal formulations. He is not then comparable to Cleanthes. His entire thrust is concerned with the distinction between the logical and the real, between predetermination of the true and predetermination of the cause. If we measure the system we can attribute to him by its effectiveness in resolving the Master Argument, Ockham's system remains weak even when taken within the confines of Prior's reconstruction. Ockham adopts the principle of conditional necessity. Applying to a sempiternal event, it confers necessity on its whole duration. The one thing more that could be said in Ockham's favor has to do with the negative character of the sempiternal event in question in the third premise: the exact symmetry postulated between affirmative and negative propositions is dubious for the nominalist. What is more likely is that if Ockham had known the Master Argument he would have taken it after the manner of a Carneades: on the one hand, he would not have been frightened by Diodorean necessitarianism since it concerns discourse only but not reality; on the other, the formal truth of a proposition does not entail a causal consequence, the predetermination of the true is without ontological implication.⁴²

will willed this at B', will not be true after B, nor, consequently, necessary either, no more than its opposite will be impossible."

As for Ockham, it has been maintained that he too contested the Aristotelian principle of conditional necessity. He does indeed write in his Commentary on Chapter IX of the *De interpretatione*: "Sciendum est, quod ista propositio: *Omne quod est quando est necesse est esse*, de virtute sermonis est simpliciter falsa." (Quoted by Boehner in Ockham, 1945, p. 71). But by that Ockham means simply that the absolute or simple necessity, i.e. not qualified by the temporal condition of the *quando* of the thing or the event, must not be accepted. That false interpretation of Aristotle rejected, Ockham accepts the principle in its authentic Aristotelian form: "It is necessary, if a certain thing exists, that it exist then." (*ibid.*). He elaborates: "...But the Philosopher says that this proposition is necessary: whatever is, is, when it is; for this proposition cannot be false. The following proposition is likewise necessary. Whatever was, was when it was. Likewise: whatever will be, will be, when it will be." (*ibid.*, p. 72).

What misled the commentator (*ibid.*, pp. 70-72) is Lukasiewicz' interpretation of Aristotle from which the temporal condition is omitted. The discussion against Scotus proves the point. What is at stake, in the Aristotelian principle, is the possibility of the coexistence *at the same instant* of an act and of the contrary capacity. Ockham, along with Aristotle, denies that possibility that Scotus asserts (cf. below, 9.1, pp. 227-228).

⁴²What Ockham refuses is not the principle of necessity but the inference that goes from truth to necessary truth. His position is reminiscent of that of a Carneades (cf. below, 8.2, pp. 209-210). Ockham does indeed refuse to pass from truth to necessary truth. Boehner interprets this refusal (Ockham, 1945, p. 68) as a rejection of the principle of conditional necessity, in view of *Qu. Ila, B, ad sec.* of the *Trac-*

Duns Scotus, taking up Plato again unawares, casts his doubts on the principle of conditional necessity. The refusal of that necessity, altogether consonant with modern thought, entails no contradiction. Scotus' theory is thus consistent. By this refusal it escapes the Master Argument without having to challenge any of the three explicit premises of Epictetus' text. Therefore it has no relation to Cleanthes solution either.

4.3 Cleanthes again and the second conjecture: the conditional character of the necessity of the past according to Cleanthes; the interpretation of Leibniz.

Those who have interpreted Cleanthes through Ockham have attributed to the former a nominalistic attitude that is rendered highly improbable by what is known of the rest of his doctrine. Not that the erstwhile boxer Cleanthes, whom Diogenes Laertius presents as a lout, would have disdained the dodge. It is just that this dodge supposes an ambiguity in the Master Argument's first premise. Had it been possible, no doubt everyone would have adopted that simple solution instead of affronting the detours and the obscurities of non canonical systems of logic or physics. It must therefore be supposed that in denying the validity of the first premise Cleanthes meant to say not that past-tense propositions not about the past are not necessary, but that propositions about the past *tout court* are not necessary. It is in this strong sense that Leibniz understood his position.

He writes in a passage that could be clearer: "There is a question whether the past is more necessary than the future. Cleanthes was of this persuasion. It is objected that it is necessary *ex hypothesi* that the future happen, like it is necessary *ex hypothesi* that the past should have happened. But there is this difference: it is not at all possible to act on the past state, that's a contradiction; but it is possible to bring about some effect on the future. Nevertheless, the hypothetical necessity of the one and the other is the same; the one cannot be

tatus and of the fourth article (*L et seq.*), whereas it implies only the abandonment, in the sense of Carneades, of the dogmatic definition of truth. The texts alleged say: 1) that from necessary science to the necessity of the known the consequence does not hold (*ibid.*, p. 18); 2) that from the immutability of the truth-value of a statement about a future contingent to its necessity, the consequence does not hold (*ibid.*, p. 29). But this implies only that a proposition can be determinately true without its causal chain being already present for all that.

changed, the other will not be and, with that posited, it cannot be changed either."⁴³

Leibniz seems to say that, after having identified the hypothetical necessity of the past with that of the future, Cleanthes added something to the hypothetical necessity of the past that explained why deliberation could not apply to it. But what is that something else? Isn't it precisely what for Aristotle and Diodorus and nearby all the Ancients made the hypothetical necessity degenerate into a sort of absolute necessity, insisting that it is only a *sort* of absolute necessity because it is solely retrospective and, insofar as the event is concerned, it has only to do with its time, not its mode of production? And what was Leibniz' own position? "Voluntary actions and their consequences", he writes "will not take place whatever we do, whether we will then or not, but because we will do and because we will will to do what leads to them. And this is contained in the forecast and in the predetermination and is even their reason for being. And the necessity of such an event is called conditional, hypothetical, or again, necessity of consequence, because it supposes the will and the other *requisites*; whereas the necessity that destroys morals and that renders punishment unjust and reward superfluous is in the things that will be no matter what we do and what we will to do, and, in a word, in what is essential. And this is what we call an absolute necessity. Nor will it serve at all, as regards what is absolutely necessary, to make prohibitions or commandments, to propose penalties or rewards, to blame or to praise; it will not be any the more or any the less for all that."⁴⁴ Let us compare this passage with that already quoted from the *Nicomachean Ethics* (Z, 2, 1139^b5-11), keeping in mind that voluntary choice and project do not apply to the past.

Aristotle says that the past makes the conditional necessity of the event degenerate into a sort of absolute necessity. Leibniz seems to agree with him on this point, though he himself makes the conditions on conditional necessity more stringent. It no longer suffices to apply it to momentary events, because now there is the additional demand for the *requisites* of voluntary deliberation. Leibniz even gives as his reason for agreeing that it is a contradiction to suppose that we have any effect on the past; but what is possible is that which is not the antecedent of a contradictory consequence. If something is possible we can have an effect on it; but we cannot have an effect on the past; therefore the past is not possible. This is clear. Leibniz concedes that

⁴³Leibniz, Gerhardt, VI, p. 214; Jalabert, p. 226.

⁴⁴Leibniz, Gerhardt, VI, p. 380; Jalabert, p. 383.

the necessity of the past signifies nothing other than the impossibility of realizing the possible in the past. But it is just this conclusion that contradicts Cleanthes and puts Leibniz in agreement with most of his adversaries, among whom Aristotle must be numbered as well.

To situate these Leibnizian intentions let us return to the Master Argument. Leibniz praises Aristotle for having distinguished between conditional and absolute necessity. He even claims to borrow this distinction from him. But he blames Aristotle for his doubts relating to the principle of the excluded middle. But in Chapter IX of the *De Interpretatione* Aristotle combines these doubts with that distinction in order to escape the Master Argument whose three premises, which Diodorus borrowed from him moreover, he admits. In crediting Cleanthes with the distinction between the two sorts of necessity, Leibniz is interpreting it as tantamount to a denial of the first premise, *i.e.* of the absolute necessity of the past. Since he himself, given his theory of possible worlds, admits the existence in the divine understanding of possibles that will never pass over to an empirical existence and since he does not challenge Diodorus' second premise, we can suppose that what he attributes to Cleanthes is a doctrine quite like his own. To think that the past is absolutely necessary is to blot out the distinction between conditional and absolute necessity. Every event, past or future, is determined. But what is determined is in nowise necessary if its existence-conditions, which caused or will cause it to be chosen, depend not solely on the logical principle of non-contradiction, but on the principle of the best, as well. But any event, in so far as it is part of this world, exists on the condition of what Leibniz calls the free choice of the best and what Cleanthes called Divine Providence.

This distinction of two necessities, which for Leibniz and for Cleanthes as interpreted by Leibniz, though not for Aristotle, suffices for avoiding fatalism, is itself amenable to two different forms of expression. On the one hand, in asserting that the past is hypothetically necessary, one might be saying that it is necessary that an event occur if it has occurred; whence it is not possible to conclude that it is simply and absolutely necessary that that event should have occurred. On the other hand, one might posit that if an event has occurred, it was necessary, during the time of its occurrence, that it should occur; the necessity is thus conditional in that it depends on the duration of a past event, whence again the impossibility of concluding that it was simply and absolutely necessary that that event should have occurred.

On either of the two different forms of expression, the first of which is more Leibnizian, the second more Aristotelian, although Aristotle himself does not apply it to the past, conditional necessity entails a

complete symmetry of past and future. For it can be said of a future just as of a past, on the one hand, that it is necessary that it will be if it is, as the past is necessary from the fact that it was, on the other, that it is necessary, while it will be, that it be, as it is necessary that the past should have been while it was. From the point of view of Divine Providence moreover, the principle of the best and the principle of determination apply equally to the one and the other. If Leibniz distinguishes them it is because he appeals to the impossibility of realizing the possible in the past. He consequently superposes on the conditional necessity applying to all existences, past and future, an irrevocability characteristic of the past and appropriate to rendering it necessary, hypothetically too, though in a sense other than the future. It is probable that he attributed the same distinction to Cleanthes.

According to Aristotle, conditional necessity degenerates into a sort of absolute necessity whenever either the limits of an event belong to the past or, although they belong to the future, there is already present a causal chain determining them and allowing us to consider them as already given then.⁴⁵ The separation of the ways comes in between that about which one may still deliberate and the rest, including that about which one could but no longer can deliberate—the irrevocable—and what arises from external necessity and upon which deliberation can have no effect. For the Leibnizian Cleanthes, on the contrary, the past in itself, although removing the event from the province of deliberation, does not plunge it into necessity that is somehow stronger than the conditional variety common to all existents. The irrevocability blocking realization of the possible in the past does not confer brute necessity on that past.⁴⁶

The authority of Leibniz is not to be taken lightly. There are two arguments however that prevent us from accepting his opinion. The first has to do with the internal economy of the Master Argument, the second with the historical likelihood of a Leibnizian Cleanthes.

In the first place, thinking that he is interpreting Cleanthes, Leibniz adopts a peculiar stance regarding the argument's first premise. On the one hand he weakens its modal force in saying that the necessity of the past is conditional, not absolute. On the other hand, he enhances this in refusing to reduce it to the general hypothetical necessity such as he attributes to futures. In doing this he refuses the complete symmetry of past and future. At no point does he clearly spell out just in

⁴⁵This is the case for "tomorrow's eclipse". (*Metaphysics*, K, 8, 1065^a16; cf. below, 6.5, p. 144).

⁴⁶This distinction corresponds to Schuhl's solution of the Master Argument. As von Fritz has pointed out (1962, p. 145) this turns out to be Cleanthes' solution.

what this modal surplus consists that enables distinguishing the necessity of the past from the necessity of the existent in general, without assimilating it to metaphysical necessity for all that. For Leibniz, a complete symmetry of past and future would probably have been seen as entailing the nonsense of a Pierre Damien saying that God could make what was not have been. If this is nonsense, it is because the necessity of the past is specific in just that it is impossible to realize the possible in the past. Time must then be taken to be asymmetrical and linear. But if this is the case what has been gained for solving the Master Argument? The interpretation of the first premise requires no more than the irrevocability of the past. On the other hand, Leibniz does not seem to contest the argument's other premises. He does have the merit of showing the real difference there is between the irrevocability of the past and the necessity of a mathematical deduction. But that is a difference Diodorus can accommodate.

There remains the question of the historical likelihood of the Leibnizian reconstruction. Let us first point out that the distinction between two sorts of necessity was known to the Ancients as an Aristotelian one. Had Cleanthes, the Stoic, taken it over from Aristotle, the antique sources would have noted it. Above all, however, in interpreting the first premise, Leibniz no more denies its validity than had Ockham before him. The premise asserts the necessity of the past: Ockham had clarified the sense of the word *past*, Leibniz clarifies the sense of the word *necessity*. These clarifications purify, they do not contest the premise itself.

The Epictetus passage reports that Cleanthes meant to reject the premise, not to amend it. The conjectures advanced thus far are therefore irrelevant to his project. In particular, we have seen Leibniz defend, without justification, the linearity of time, in order to safeguard the asymmetry of past and future. It is in this way that it remained impossible to realize the possible in the past. But if one is to invalidate the first premise, isn't it just that impossibility that must be questioned?

There remains open then one last possible conjecture that would allow for an interpretation in terms of physics and of the properly Stoic conception of time and would negate the Master Argument's first premise. First the condition of possibility of that interpretation will be examined. It will then be shown that it seems to correspond to Cleanthes' theory.

4.4 Third conjecture: cyclical time and the numerical conception of the identity of beings in eternal return.

There is a passage in the *Problems* of Aristotle that helps us fix the issue. The impossibility of realizing the possible in the past was based on the linearity of time. Realizing the possibility in the past would make last year coincide with now. But there are philosophers who have maintained that absurdity.

“After what manner”, asks the pseudo-Aristotle,⁴⁷ “are we to understand the words *before* and *after*? Are they to be taken in the following manner: Those who lived in the days of the Trojan War are before us, and before them are those who lived earlier still, and so on *ad infinitum*, those who are found to be in the past being always taken to be before the others? Or, if it is true that the Universe has a beginning, a middle and an end; that what by aging has reached its end, should, in virtue of that have come back again to its beginning; if it is true moreover that things which are before are those that are closest to the beginning, what is to prevent our being nearer the beginning [than those who lived in the days of the Trojan War]? If that were so, then we should be before them. Since by its local motion the firmament and every star goes in a circle, why shouldn’t it be the same for the coming into being and decay of every perishable thing, such that that same thing might also come into being and decay again? So they say too that human affairs go in a circle. To think that the men who are being born are numerically the same is absurd; but a better opinion would be expressed in saying that they are preserved as to their form. It could be then that we are before even [the contemporaries of Troy]. The series of events will then be taken to be such that there is an inevitable return to the state which served as point of departure and the taking up again, without discontinuity, of a course which passes again by the same things. Alcmeon said that men are perishable because they are unable to join their end to their beginning. That is well put, if he is taken to have spoken figuratively and one doesn’t insist on taking what he said literally. If the series of events is a circle, and since a circle has neither beginning nor end, we cannot, by virtue of a greater proximity to the beginning, be before those others, nor they before us.”

Suppose then that the Master Argument’s first premise signifies simply the impossibility of realizing the possible in the past. On what condition would that impossibility disappear? It is necessary and suffi-

⁴⁷ *Problems*, XVII, 3, 916^a18-39.

cient 1) that the universe be subject to comings-into-being and decays in such a way as to have a middle, beginning and end, 2) that these generations and decays form a reproductive cycle, 3) that that reproduction be numerical, not simply specific. The first two conditions are easy enough to see. The third is necessary. For if the reproduction of the universe were only specific, we could number each successive universe or, if that succession is from all eternity, we could at least distinguish it. There would be the universe in which Socrates lived and the universe in which Socrates didn't exist. The clock, so to speak, would be extraneous to the universe, since its phases would be discernible. But if it is numerically that the universe repeats itself, its phases become indiscernible and it is the same time that repeats itself. Then, and only then, is there a complete symmetry between past and future. Then, and only then, can one contest the Master Argument's first premise taken in its Aristotelian sense.

Just which schools of the followers of Heraclitus and Pythagoras fall under this Aristotelian opposition of specific and numerical systems is of no interest here. The issue is to know whether this same conflict existed within ancient Stoicism. Zeno, in his treatise on the *Eternity of the World*, which is directed against the Peripatetics, maintains that the eternity of the universe is cyclical. After 365 times 10,800 years, according to the count of Diogenes the Babylonian, the final conflagration purifies the universe and restores time.⁴⁸ Neither Cleanthes nor Chrysippus seem to have departed from Zeno on this part of the doctrine. There is nothing that justifies doubting that ancient Stoicism was in harmony with the periodic palingeneses⁴⁹ of the universe. Panaetius is the first in the Stoic school to question the renewal of the universe at each universal conflagration.⁵⁰

Eusebius, in his *Praeparatio Evangelica*,⁵¹ has the following to say. "Here is what the Stoics have to teach about the conflagration (*ἐκπύρωσις*) of the World: the oldest adherents of that sect thought that all things, at the end of certain extremely long periods of time,

⁴⁸Bréhier, 1962, pp. XVIII-XIX.

⁴⁹Goldschmidt, *op.cit.*, 1977, p. 42.

⁵⁰Cicero, *De Natura Deorum*, XLVI, 118, in Bréhier, p. 450.

⁵¹Quoted by Duhem, 1951, t.I, p. 277. Following the suggestion of Father M. Régnier we have corrected Duhem's translation to read *had some reserve about* in place of *insisted on*. Duhem adds the following passage from Aëtius (Arnim, S.V.F., II, p. 184) "They say that the organization [of the Universe] subsists eternally, that there are certain periodic times at the end of which the same things are all engendered again and in the same way, at the end of which the same disposition and same organization of the World turn out to be safe and sound".

were etherized, that all things disintegrated into a fire resembling ether...

“From this it is evident that Chrysippus did not think of that dispersion as affecting the very existence [of the World], for that is impossible, but as playing the role of a transformation (*μεταβολή*). For those who teach that disintegration of the Universe in a state of fire, that they call conflagration (*ἐκπύρωσις*), do not admit that that destruction of the World that recurs after very long periods of time is a destruction strictly speaking. They use the expression ‘destruction’ (*φθορά*) in the sense of a natural transformation. Indeed, it was in accordance with the Stoic philosophers that the Universe should be transformed into fire, as into its seed (*σπέρμα*), and then that from that fire should be produced again a disposition exactly like the one that existed before. This dogma was admitted by the principal and most ancient philosophers of the sect: Zeno, Cleanthes and Chrysippus. It is said that Zeno [of Tarsus], who was a disciple of this last and his successor at the head of the Stoic school, had some reserve about the conflagration of the Universe...

“Then the common reason reappears to take up again the selfsame course (*ἐπὶ τοσοῦτον*). The common nature, become more ample and full, finally drying out all things and taking them back into itself, is engendered to full existence. It starts its course again according to the rule it followed a first time. It recommences that restoration (*ἀνάστασις*) that accomplishes the Great Year. For, following that Great Year, there comes about this renewal (*ἀποκατάστασις*) [of the World] which starts from a certain state and returns again to the same state. Nature begins again, in the order in which it deployed itself a first time in like manner, to accomplish again, according to the same law, the same series of events. And from an eternity the same periodic cycles ceaselessly recur”.

In such a world view one is led to say of the now, as opposed to Aristotle, that it is last year, and of last year that it is now. The upheaval among physical notions naturally entails an upheaval among modal ones. The possibility of realizing the possible changes radically its sense, once it involves the past as well as the future.

Simplicius reports that the Stoics were divided on the question of whether the Eternal Return reproduced numerically or only specifically identical beings. He also distinguishes two sorts of numerical identity: the first seemingly absolute, the second having to do only with the

essence of the individual. "This question⁵² arises in connection with the palingenesis (παλιγγενεσία) of the Stoics. For they say that, by palingenesis, there will be reborn a man who is the same as me. And they rightly ask if I will then be numerically the same as now, if I will be the same by essential identity (διὰ τὸ τῆ οὐσίᾳ εἶναι ὁ αὐτός), or if I will be different due to my insertion into another Universe from this one." So it all boils down to knowing whether there was agreement in the ancient Stoa to maintain the return of numerical identity and just what kind of identity that was taken to be.

The testimony of Alexander of Aphrodisias allows us to attribute to Chrysippus the thesis of numerical identity understood as having to do only with the individual's essence. "According to the Stoics,⁵³ there must really be a time after the death of Dion at which the separation of body and soul of the one designated by the name Dion takes place. For they think that after the conflagration all things will be regenerated in the world, and numerically the same, so that such-and-such a man in particular (ὁ ἰδίως ποιός) will be once again the same as before, and will so be born in this new world. That is what Chrysippus says in his books *On the Cosmos*...⁵⁴

"They say further, that the particular men born afterwards are affected only by differences, with respect to those having existed beforehand, touching simply on certain extraneous accidents. Such are the differences apt to affect Dion during his life, leaving him nonetheless the same, for they in no way make another man. That he should at first have warts on his face, for instance, and then later not have any more, in no way makes him another man. They say that it is differences of this sort that arise between the particular men of one world and those of another."

There is no doubt that, as long as a difference subsists between the numerical individual and his essential—although proper—⁵⁵ determination, it is impossible to assert the indiscernibility of their total determinations. To have a wart or not, in any case, makes it possible to assign two different moments of a same subject or two different palingenesises of the universe.⁵⁶ Accidental and transitory differences enable

⁵²Simplicius, 1895, *In Aristotelis Physic.*, p. 886; Arnim, S.V.F., II, n. 627, pp. 190-191.

⁵³Alexander of Aphrodisias, *In Aristotelis analyticorum priorum librum I commentarium*, 1883, p. 180; Arnim, S.V.F., n.624, vol. II, pp. 189-190.

⁵⁴Text attested to by the same Alexander, p. 181, 13.

⁵⁵On the theory of τὸ ἰδίως ποιόν, which according to Chrysippus is what characterizes each being in a permanent way, see: Bréhier, 1910, pp. 111-112 and Goldschmidt, 1977, p. 17.

⁵⁶Arnim, S.V.F., II, n. 626, p. 190. Origen says that according to the Stoics "peri-

us to mark off the successive time of two states of a same individual or of two universal palingeneses. Chrysippus could not, therefore, have maintained their absolute numerical identity. Nor could he have rejected the linearity of time. Nor did he venture to reject the Master Argument's first premise.

Extreme prudence is required then in interpreting the Stoic texts. Where they claim the numerical identity of palingeneses, that identity may be imperfect. Tatian's testimony about Zeno does, of course, seem to go further. "Zeno asserts⁵⁷ that after the conflagration the same men will be given to the same deeds, that Anytus and Meletus will again be accusers, that Busiris will begin again killing his guests, that Hercules will again accomplish athletic labors." Is there a perfect numerical identity here? We do know that Cleanthes remained faithful to the Zenonian theses, whereas Chrysippus was an innovator, not hesitating to oppose Cleanthes. It is difficult to grasp these points of difference with exactitude. A text of Nemesius suggests however that there is some connection between the insistence on astronomical periodicity—itsself astrologically interpreted and justifying divination—and the interpretation of palingeneses in terms of absolute numerical identity.⁵⁸ And Cleanthes seems to have been especially interested in astronomy. He constructed an original theory of the Sun which he took to move in its sphere on a spiral path comprised between the two tropics, with all the stars, both fixed and wandering, being transported from East to West.⁵⁹ While for Zeno and the other Stoics the ether was supreme god, Cleanthes attributed that dignity to the Sun.⁶⁰ It was probably for that reason that he accused Aristarchus of impiety. The nurturing warmth of the Sun sustains universal life.⁶¹ That fire is the 'hegemon', self-moved and reasonable.⁶² The stars, beings of fire, move by will and are divine.⁶³ Probably an original development of the theory of the fire artist as Zeno had taught it, Cleanthes' doctrine was well suited to hellenizing the universal sympathy and the sun-cult

odically a universal conflagration takes place and after that conflagration is reborn a disposition of the world exempt from any difference with respect to the disposition that had previously been realized. Many of them attenuated that teaching, saying that the things of one period are subject to a slight difference, an extremely weak difference in relation to the things of the preceding period." It is possible that Chrysippus was the first to attenuate the ancient doctrine.

⁵⁷Tatianus, *Adversus Graecos*, cap. V; Arnim, S.V.F., I, n. 109.

⁵⁸Nemesius, *De Nat. Hom.*, cap. 38, p. 277; Arnim, *ibid.*, I, 109, p. 32.

⁵⁹Tannery, 1912-1950, t. II, pp. 160-162.

⁶⁰Cicero, *Academica*, XLI, Bréhier, p. 246; Cumont, 1909, p. 15 note 3.

⁶¹Cicero, *De Natura Deorum*, IX, 24 sq., Bréhier, p. 417.

⁶²*Ibid.*, XI, 31, p. 419.

⁶³*Ibid.*, XV, 40, p. 422.

that had come from the Chaldeans.⁶⁴ Among all the Stoic doctrines it is this one especially 'that represented the ethereal fire as the primordial principle and held the stars to be the purest manifestation of its power'.⁶⁵

Cleanthes insisted on the astronomic basis of benevolence and fate. His doctrine of the fire principle destined him to philosophically justify the sun-cult of the 'Chaldeans', as his *Hymn to Zeus* shows. It is true that there is no extant text explicitly attributing a perfect numerical interpretation of palingenesis to him, thereby setting him opposite Chrysippus on this point. But all the remaining elements of his doctrine tend in this direction. It was thus within the logic of the system to forgo the linearity of time and to draw from palingenesis what Origen considered to be the only consequence possible: perfect numerical identity.⁶⁶ But if time is symmetric, realizing the possible in the past, and thereby inducing a reversal of time, is no longer absurd. What scandalized Aristotle no longer presents a problem. Challenging the Master Argument's first premise then becomes legitimate.

In short, the Stoics extended to the universe as a whole the cyclical return that had been restricted to the Heaven in Aristotle's system. The *ekpurōsis* realized a kind of synthesis of Heraclitus and Aristotle and answered to Diodorus' negation of motion.

The fatalism of Cleanthes and Antipater⁶⁷ is no longer of a logical, but of a physical, order. The total determination of the sublunar by the supralunar world, in conformity with the astrological dogmas, and the total stability of the system of astronomic equations, in conformity with the dogma of eternal return, bring freedom down to the joyous acceptance of destiny. It is this pantheism that supports the rejection of Diodorus' first premise. As Stobaeus says, "As for Cleanthes, he speaks as follows:... The tension that is in the substance of the universe ceaselessly produces always the same revolution and same arrangement. For just as all the parts of a same individual are born of seeds at the proper times, so too do the parts of the universe, which include the animals as well as the plants, come into being at the proper times. And just as certain seminal reasons of the parts, condensing into a seed, mix and separate again when the parts are born, so do all things

⁶⁴ Cumont, 1929, pp. 160-162.

⁶⁵ Cumont, 1912, p. 69; on Cleanthes' intelligible fire see Cumont, 1909, p. 15 note 2.

⁶⁶ Origen, *Contra Celsum*, V, XX; Arnim, S.V.F., II, n.626, p. 190.

⁶⁷ Who, in the Middle Stoa, maintained the doctrine of the conflagration.

come from one sole being and all condense into one, the revolution having been accomplished, and this in conformity with the order.”⁶⁸

⁶⁸Stobaeus, *Ecl.* I, 17, 3, p. 153, 7 W (Arii Did., fr.38 Diels), Arnim, *S.V.F.*, I, n.497, p. 111.



Freedom as an Element of Fate: Chrysippus.

The uncertainties regarding Chrysippus' solution justify the establishment of certain preliminary 'lemmas'.

Since it has been supposed that the Master Argument's second premise is a thesis of pure modal logic, the discussions have developed within this context. They gave sound results though their context itself may and, according to the present interpretation, must be questioned. This explains why Chrysippus' reform will be analysed as if it concerned pure modal logic before it will be reinterpreted in the light of temporal modal logic.

1. Basing ourselves on the Chrysippean theory of divination, it will first be shown that, if Chrysippus does challenge a thesis of pure modal logic, his doubts have to do with the literal expression of that premise. What he contests is that from the possible, the impossible does not logically follow. He admits, on the other hand, the validity of the affirmative form one is tempted to give of that same premise: from the necessary, the necessary logically follows.

2. The counter-example advanced to falsify the premise in its negative form in no way falsifies its positive form. The question it actually raises is that of the validity of the interdefinability of the modalities.

3. The Stoic definitions of the modalities corroborate the view that Chrysippus adopted a non-standard modal system.

4. This system is indeed related to Prior's 'system Q', which he himself attributes to Chrysippus.

5.1 Were Chrysippus' doubts about the thesis of pure modal logic according to which from the possible the impossible does not follow, they would be about its negative form, not about its positive form.

At first glance, Cicero might be seen as asserting that Chrysippus challenges the Master Argument's second premise in its positive form, that is to say, in that it posits that the necessary logically follows from the necessary.

It is in the *De Fato* (VII, 14) that he analyzes the question of the relation of divination to necessary futures.

"If the following is a true connection, 'If you were born at the rising of the dogstar you will not die at sea' and if the antecedent in the connection, 'you were born at the rising of the dogstar', is necessary (for all true propositions about the past are necessary, as Chrysippus holds, in disagreement with his master Cleanthes, because they are unchangeable and cannot pass from true to false)- if, therefore, the antecedent in the connection is necessary, the consequent becomes necessary too. Although Chrysippus does not think that this principle holds in all cases"¹.

How is this passage to be interpreted?

The conditional is said to be genuine or sound, not simply because it is true, but because it expresses a law of astrology: it therefore expresses a necessity. The antecedent is itself necessary since true in the past. Are we to suppose then that Chrysippus would refuse in such a case to conclude that the consequent 'you will not die at sea' is itself necessary?

If this were so, all science by inference would be impossible. But Chrysippus, it must be remembered, is a dogmatist. Far from refusing the preceding argumentation, he is so convinced of its formal legitimacy and that if its premises were accepted divination would inevitably entail necessity, that he recasts the very form of divination in order to avoid that untoward conclusion. Divination will no longer be conditional, but conjunctive.

"At this point Chrysippus gets nervous and hopes that the Chaldeans and the other diviners will no longer employ expressions of the form 'If anyone was born at the rising of the dogstar he will not die at sea', but will say rather 'It is not the case both that someone was born

¹Bréhier, 1968, p. 478; Rackham, 1942, pp. 208-209.

at the rising of the dogstar and that he will die at sea' " (*De Fato*, VIII, 15).

Just what does this rather odd-seeming transformation come down to? If it is meant to reconcile fate and divination on the one hand and to allow for non-necessary futures on the other, it must evidently be that Chrysippus took this conjunctive form to be weaker than the conditional one for which he had reproached the astrologers. As shown by the passage quoted above (VII, 14), the astrological conditional expresses a necessity: 'it is necessary that if anyone was born at the rising of the dogstar, he will not die at sea'. As the antecedent is itself necessary, since past, positing the necessity of the consequent, which is future, becomes inevitable. It must be then that the change from that conditional to the conjunctive form constitutes a weakening of astrological necessity. Since, for Chrysippus, the doubt cannot affect the necessity attaching to the antecedent, it must be that the conjunctive form weakens the very connection between antecedent and consequent. What is the reason for that weakening in the force of the consequences?

Having distinguished 'sound' implication according to Philo and according to Diodorus,² Sextus goes on to examine a third theory of the conditional, probably corresponding to the Chrysippean thesis. "Those who introduce the notion of connection (*συνάρτησις*) say that the conditional is sound when the contradictory of its consequent is incompatible with its antecedent. According to them, the conditionals mentioned above to illustrate the doctrines of Philo and Diodorus are unsound, but the following is true: 'If it is day, then it is day'."³ The conditional posited by Chrysippus is stronger than that of Diodorus and, *a fortiori*, than that of Philo. It is stronger than the Philonian material implication and stronger than the Diodorean formal implication, for Chrysippus adds a proper modal clause to the conditional. The incompatibility between the negation of the consequent and the truth of the antecedent transforms the conditional into a sort of 'strict' implication,⁴ that is to say, into the statement of a law. Since Chrysippus does not, however, spell out the nature of this incompatibility—is it logical? physical?—it is not possible, without a preliminary inquiry, to determine formally in just what the 'strictness' of that implication consists. It must be added too that the example given by Sextus could be misleading. When it is said that if it is day

²See above, 3.3, p. 50.

³Sextus Empiricus, PH II, 111-112.

⁴A strict implication in Lewis' sense is one for which the conjunction of antecedent and negation of the consequent is not possible. (Hughes and Cresswell, 1972, p. 217).

then it is day, it seems that we could prefix the modal *necessary* to the conditional, taking it to mean that it is necessary that if it is day it should be day. But then it becomes difficult to make sense, on the one hand, of the Ciceronian passage and, on the other, of the very definition given by Sextus, enjoining us to understand the conditional as the incompatibility of the contradictory of the consequent with the antecedent in conjunctive form.

For Chrysippus, if the laws of divination were expressed in conditional form in the astrologer's sense, the conditional would be necessary and not simply true, as it would be if only the Philonian condition were met, or even if one were to add the Diodorean condition, in so far in any case as it is contested that simple permanence of the Philonian hypothetical is enough to assure its necessity. Given a necessary antecedent, the consequent would be necessary as well.⁵

Chrysippus' sound conditional must therefore be formally distinguished both from the conditionals of Philo and of Diodorus, since it is stronger than they are, and from the astrologer's conditional criticized by Chrysippus in the Ciceronian passage, since it must be weaker than that one is. The astrological conditional signifies that it is necessary that if an event *p* occurs the event *q* will occur; the Chrysippean 'conditional' signifies only that it is impossible to verify at once both that *p* occurs and that *q* does not.

But then how are we to explain that these predictions can lose their necessity in taking on a conjunctive form?

One might first look for the reason for that weakening in the fact that Chrysippus conjunction is a truth-function, whereas his conditional and disjunction are not.⁶

But if it were due to the simple fact of taking on the conjunctive form that the astrological argument lost its necessary force, it would have to be because the (negated) conjunction is simply true.

⁵As noted by M. Frede, 1974, p. 88.

⁶M. Frede, 1974, p. 96. An analogous problem is met with in a discussion of Ockham's reported by Prior, 1962, p. 242. Take the conditional 'If A is going to occur, God knows that it is'. If the antecedent is false, the consequent is false and the whole will be true. If the antecedent is true, the consequent is true. In either case, the whole will be true. But if the antecedent is neuter, the consequent will be false; and the whole will be neither true nor false. Taking the conjunctive form, by contrast, 'A is going to occur' and 'God does not know that A is going to occur' cannot both be true together. Therefore it will be true that it is not the case that A is going to occur and God does not know it. In this case the conditional does not follow from the corresponding conjunction.

That discussion nevertheless does not apply to our case where we are dealing with modal and not simply assertoric propositions. But there is a notable analogy in so far as the respective consequences are concerned.

For, in that case, a necessary consequence could not be drawn from an assertoric proposition.⁷ But isn't it clear that that would be to reduce the laws of astrology to simple repeated concomitances, and to make Chrysippus the Hume of astrology? We must therefore determine just what is the nature of the modality affecting conjunction. Cicero too explicitly precluded this first solution in *De Fato*, VI, 12, "... the propositions 'Fabius was born at the rising of the dogstar' and 'Fabius will die at sea' are incompatible, and since it is posited as certain about Fabius that he was born at the rising of the dogstar, the propositions 'Fabius exists' and 'Fabius will die at sea' are also incompatible. Therefore the conjunctive proposition 'Both Fabius exists and Fabius will die at sea' is composed of contradictory propositions, and the event, according to the principle, cannot occur. Therefore the event 'Fabius will die at sea' is of the kind of those which are impossible. Therefore every false proposition about the future states an impossible event".⁸ It is incontestable in this argumentation that the conjunction, being contradictory, is impossible, for to contest that would be to give up astrology and fate. But, in that conjunction, one of the terms is true: namely, that Fabius exists. It must, therefore, be the event designated by the other term that 'cannot occur'. Such is Cicero's conclusion. Chrysippus, though, would have contested such a conclusion. Indeed, from what Cicero expressly says in *De Fato*, VIII, 16, it can be seen that he assumes the right to go from the necessary antecedent-consequent connection to the negation of the sound corresponding conjunctive proposition, and naturally back again from such a negation to such a connection.⁹ If the conjunctive form is meant to save the possibles without destroying fate, it must be this last trans-

⁷M. Frede, 1974, p. 88. Frede points out further that, given $L(p \cdot q)$, the necessity of p and of q would follow by the thesis: $L(p \cdot q) \supset (Lp \cdot Lq)$. That thesis is in fact demonstrable as a theorem of modal logic (Hughes and Cresswell, 1972, T. 3, p. 34) without appeal to the Master Argument's second premise, which is required only for the demonstration of its converse.

⁸Bréhier, 1962, p. 477; Rackham, 1942, pp. 206-207.

⁹If 'Fabius was born at the rising of the dogstar' is designated by ' Pp ' and 'Fabius will die at sea' by ' Fq ', Cicero's reasoning can be expressed as follows, where \supset designates the sound Chrysippean conditional.

$$\begin{aligned}
 Pp \supset \sim Fq &\equiv \sim M(Pp \cdot \sim Fq) \\
 &\equiv \sim M \sim (\sim Pp \vee \sim Fq) \\
 &\equiv \sim M \sim (Pp \supset \sim Fq) \\
 * &\equiv L(Pp \supset \sim Fq) \\
 &\supset LPp \supset L \sim Fq \\
 \text{But } LPp &\quad \text{Therefore } L \sim Fq.
 \end{aligned}$$

The invalid argument step is marked by an asterisk (cf. note 11).

formation that Chrysippus contests, and not the Master Argument's second premise in general, for then no science would be possible.

5.2 Chrysippus' doubt about the interdefinability of the modalities. From the non-possibility of an event's occurrence it cannot be concluded that its opposite is necessary.

This line of reasoning is supported by a seemingly peculiar argument recorded by Alexander of Aphrodisias.¹⁰ Chrysippus constructs a counter-example not, as is often said, to the second premise's positive form, which maintains that the necessary follows from the necessary, but to a negative form maintaining that the possible, and only the possible, follows logically from the possible.¹¹

¹⁰*In an. pr.*, 177, 25-33. "Chrysippus, in maintaining that there is nothing to prevent an impossible's following from a possible, says nothing against the proof given by Aristotle, but tries to show that it isn't so by way of examples which are not correctly constructed. He says that in the conditional proposition 'if Dion is dead, this man is dead', which is true when Dion is designated, the antecedent 'Dion is dead' is possible, for it can be true that Dion dies, while 'this man is dead' is impossible, for after Dion's death the proposition 'this man is dead' destroys itself, since the object designated no longer exists. For the designation concerns something living and is attributed to a living being. Then when Dion is dead, if 'this man' is no longer possible and Dion no longer subsists, so that 'this man is dead' cannot be said of him, then 'this man is dead' is impossible. That proposition would not be impossible if, after Dion's death, it were still possible to attribute 'this man' to precisely that to which was attributed 'this man is dead' in the conditional proposition while Dion was alive. Since that cannot be done it follows that 'this man is dead' is impossible". (Mignucci, 1978, p. 378).

¹¹If we abstract from the temporal indices, it is a question of the formula:

$$(B') * \sim M(p \sim q) \supset (Mp \supset Mq)$$

That formula, rejected by Chrysippus, is equivalent to

$$* \sim M(p \sim q) \supset (\sim Mq \supset \sim Mp)$$

and

$$* \sim M \sim (p \supset q) \supset (Mp \supset Mq).$$

As for the formula:

$$(B'') L(p \supset q) \supset (Mp \supset Mq),$$

which, in spite of appearances, can be taken as another negative form of (B₁):

$$(B_1) \quad L(p \supset q) \supset (Lp \supset Lq),$$

it can be derived from (B₁) in the following manner (Hughes and Cresswell, 1972, T 8, p. 37):

1. $\vdash L(p \supset q) \supset (Lp \supset Lq)$ (axiom) (B₁)
2. $\vdash L(\sim q \supset \sim p) \supset (L \sim q \supset L \sim p)$ (Sb[$\sim q/p, \sim p/q$])
3. $\vdash L(p \supset q) \supset (\sim L \sim p \supset \sim L \sim q)$ ($\vdash q \supset \sim p \equiv p \supset q$, 1, 2, Syll. M.P., and contraposition in second member)
4. $\vdash L(p \supset q) \supset (Mp \supset Mq)$ (interdefinability of M and L).

The conditional given is 'if Dion is dead, this man is dead'. It is not only true¹² but 'sound' as well, for there is incompatibility of negation of consequent and affirmation of antecedent. This conditional that is 'sound' for Chrysippus is *a fortiori* so for Diodorus; it constitutes a 'strict' implication. Further, 'Dion is dead' is possible, for it can be true one day that Dion should be dead. But 'this man is dead' is not possible. Employment of 'this man' presupposes that one can show the object in question, but precisely one can no longer show it once Dion is dead.

We shall examine successively 1) just what Alexander's example involves, 2) how it fits in with the question raised as regards the Master Argument and 3) its connection with the conjunctive expression proper to Chrysippean laws.

1) The argument could be taken as an *ad hominem* one.¹³ In that case, it is the sentence 'This man is dead' that is impossible, because one could never utter or formulate it.

But it is unnecessary to have recourse to the sentence and we can suppose that the argument is formulated in terms of propositions and is taken into account by Chrysippus himself. 'Dion is dead' or 'This man is dead' will be taken then as a 'lecton', that is, as the class of all sentences of the same form, completed by the specification of the circumstances that determine their truth-values. Among the singular propositions, two sub-classes may be distinguished. The simple mean propositions are expressed by sentences of which the subject is a proper

The validity of the demonstration depends at step 4. on the validity of the implication

$$\sim L \sim p \supset Mq,$$

that is to say, on the interdefinability of the modalities.

The sound Chrysippean implication is

$$p \supset q \equiv \sim M(p \sim q),$$

which is equivalent in turn to $\sim M \sim (p \supset q)$, but not to $L(p \supset q)$.

Among the expressions used, it is surely (*B'*) then that Chrysippus contests. The care he takes in refusing to admit the 'conditional' form of the Chaldeans shows that he accepts the validity of (*B*₁). Consequently, either 1) Chrysippus refuses the implication $\sim M \sim \supset L$ in all cases and is driven, from contesting (*B'*), to contest 4 as well, or 2), which seems more probable, he limits the invalidation of the implication to certain cases only, and postulates its validity in particular in formulas beginning with *L*. He would then accept 4 in mathematical arguments, for instance.

¹²M. Frede, 1974, p. 116, says of this implicative statement that it is true, which we must naturally understand in a Chrysippean sense. The statement:

$$(p \supset q) \supset (Mp \supset Mq)$$

can certainly be falsified.

¹³Kneale and Kneale, 1962, p. 127.

name ('Dion is dead'). The simple definite propositions are expressed by sentences in which the proper names have been replaced by egocentric particulars or deictics ('This man is dead'). It is a logical law to derive from a simple mean proposition the corresponding simple definite proposition: 'If Dion is dead, then this man is dead', or 'If it is night, then this day does not exist'. These conditionals must even be sound in Chrysippus' sense, since there is incompatibility between the antecedent and the negation of the consequent. For given that Dion is dead, it cannot be, with 'this man' designating Dion, that 'this man is not dead' should be true.¹⁴ The antecedent, on the other hand, is possible. At the same time, the consequent is not possible. For, in virtue of the Chrysippean definition,¹⁵ that is impossible which is such that, if it is capable of being true, external circumstances prevent its being true. And 'this man is dead' fits that definition exactly. Either, the proposition is capable of being true, which is the case when Dion is living and 'this man' does in fact designate him, but then external circumstances prevent its being true. Or, with Dion dead, the same proposition is no longer capable of being true, since one cannot indicate Dion once he is dead. Once the designated subject has perished, any proposition of deictic form (with 'this one' referring to the subject) turns out to be automatically destroyed,¹⁶ since the indicator there can no longer refer to the subject. That is not simply to say that we would be unable to formulate or express such sentences: it is the propositions themselves that do not exist,¹⁷ for want of corresponding facts.

2) Such an argument is marvelously suited as a response to the Master Argument. Chrysippus, along with Diodorus, admits the validity of the following conditional, based on the first and third premises together with the principle of conditional necessity: 'If there is a possible which is not and never will be realized, given that the past is irrevocable and that a proposition is necessary while it is true, then a contradictory consequence follows'. But Chrysippus, who diverges from

¹⁴With $p =$ 'Dion is dead' and $q =$ 'this man is dead', the truth table would give the conjunction ($p \sim q$) as true only if p is true and q false, which is impossible.

¹⁵See below, 5.3, p. 113 and 5.4.

¹⁶φθείρεσθαι.

¹⁷Prior, 1967, p. 149. Prior is right in drawing a parallel here between Chrysippus and Ryle. It is known that for Chrysippus the proper attitude to take when a sorites was being propounded was to remain silent when the borderline cases arose (Cicero, *Ac. pr.* II, XXIX, 93; Bréhier, 1962, p. 229). The Sceptics interpreted that attitude as one of suspended judgment, seeing in it the failure of dogmatism. But far from being "intended less as a logical solution than as a procedural recommendation" (Sedley, *op.cit.*, p. 91), such a solution arose directly from the fact that in these borderline cases the proposition is destroyed.

Cleanthes and agrees with Diodorus in accepting that conditional, diverges from both Cleanthes and Diodorus when they, remarking that a contradictory consequence is impossible, add that the impossible does not follow from the possible and declare the impossibility of the conditional's antecedent, therefore, from the impossibility of its consequent. For Chrysippus, one can, and must, admit that conditional. But it can happen that, even when the consequent is recognized impossible, the antecedent remains nevertheless possible. Indeed, the antecedent has the form of an existential premise, 'there is a possible of such and such a sort', comparable to a singular proposition of which the subject is a proper name, 'Dion is dead'. Due to the realization of the possible assumed as a means of proof in the Master Argument, the consequent will have the form of a deictic proposition about a certain possible, comparable to the deictic proposition 'this man is dead'. The antecedent 'there is a possible of such and such a sort' or 'Dion is dead' remain possible, even though their deictic ectheses, 'this possible is of such and such a sort' or 'this man is dead', with the demonstrative referring to nothing, be impossible.

There is this difference between the example advanced by Alexander and the solution Chrysippus gives for the Master Argument. In the first case it is a singular proposition having a proper name as subject, in the second an existential proposition that is possible, while the corresponding deictic proposition is impossible. The deictic proposition turns out to be 'destroyed' in the first case because its subject has ceased existing, in the second because its subject does not, and never will, exist. But the inexistence of the subject prohibits neither the formulation nor the existence of a modal proposition. It does, however, prohibit the formulation and existence of a deictic one.

What Chrysippus contests in the second premise, therefore, is the principle according to which, when an antecedent with an in-existent subject is the valid condition of a deictic consequent, the impossibility proper to this second should entail the impossibility of the first. It is not then the second premise's affirmative form that is at issue, but its negative form and thereby the interdefinability of the modalities. For it is the 'destruction' of the deictic propositions that invalidates that definition. Thus for Chrysippus, those conditionals are valid of which the antecedent is true (or even necessary, as in Alexander's example, since it is a proposition true of the past) and the negation of the consequent not true, so that there is a real contradiction between antecedent and denial of consequent. Yet the non-truth of the consequent's negation in all possible worlds is not enough to assure that consequent's truth in any one of them at all, since that would require the formulability

or positive realization of the corresponding proposition in a possible world, and this is precisely what is not given. As regards the consequent, ('this man is dead' or 'this possible is not and never will be realized'), either of two cases may arise. 1) The consequent is capable of being true in the worlds in which the antecedent is false (Dion being living or given a possible of such and such a sort), but then it is prevented from being true by the external circumstances expressed precisely by the antecedent's negation. 2) It is not capable of being true due to the fact that the proposition is destroyed in all the worlds in which the antecedent is true. These two cases being exhaustive, the consequent is impossible. Thus both in Alexander's example and in the Master Argument (in the form 'If there is a possible that neither exists nor ever will, then this possible neither exists nor ever will') there is a sound conditional in which an impossible consequent follows logically from a possible antecedent.

3) It still remains to determine just what the connection is between Alexander's argument and Chrysippus' predilection for the conjunctive form. Indeed, Alexander's example, just like Sextus', could be misleading. The conditional is sound, according to Chrysippus, because it is impossible that Dion should be dead and this man not be dead. So what is expressed by a sound conditional is the impossibility of the conjunction of the putative antecedent with the contradictory of the putative consequent. Yet that properly Chrysippean form can be given an equivalent expression in terms of Philonian implication. The conditional will be sound, from Chrysippus' point of view, when prefixed by the modal operator 'it is impossible that not'. On the other hand, prefixing the operator of positive necessity, which has a greater logical force, is forbidden. Accordingly, Chrysippus' predilection for the conjunctive form is due less to the form itself than to the negative and weakened nature of the modal operator it supplants. The diviners are guilty not of transforming the *and* into *if, then* but of going surreptitiously from the words 'it is impossible that not' to the words 'it is necessary that'¹⁸ in doing so.

¹⁸Abstracting still from the temporal indices of the modalities, Chrysippus admits $\vdash L \sim p \supset \sim Mp$, $\vdash M \sim p \supset \sim Lp$, $\vdash Lp \supset M \sim p$ and $\vdash Mp \supset \sim L \sim p$ but rejects their converses.

It is the rejection of

$$* \sim M \sim p \supset Lp$$

that invalidates (B'), namely:

$$* \sim M \sim (p \supset q) \supset (\sim Mq \supset \sim Mp).$$

Were it legitimate, with the interdefinability of L and M, it would be possible to obtain

It is easily seen how the form chosen by Chrysippus is imposed by both Alexander's example and the Master Argument. In both cases it is only the impossibility of having the conjunction of a proposition p with the negation of a proposition q that can be shown. It cannot be shown, on the other hand, that it is necessary that if p , then q . For, on the supposition of the truth of p , it can be established that the negation of q is not true; but it cannot be concluded, on the same supposition, that q is true, since q is destroyed.

5.3 The non-standard modal system according to Chrysippus.

The Stoic, and probably Chrysippean, definitions of the modalities transmitted by Diogenes Laertius¹⁹ and Boethius²⁰ are complex. The commentators have all taken it for granted that interdefinability was the rule, and their interpretations have accommodated that supposition. Independently of that supposition the texts are ambiguous.²¹

Yet no matter what interpretation one should settle on for the word *possible*, interdefinability will be seen to be invalid. Consider the interpretation the most favorable to its validity:²² 'The possible is that which can be true and which is not prevented by external circumstances from being true'. It is immediately apparent that the necessary entails the not-possibly-not, since the true entails the not-false, but that the converse is not legitimate if the not-false fails to automatically entail the true, as happens in the case where the proposition is 'destroyed'.²³

Let us return now to the conjunctive form that Chrysippus would give to the diviners' predictions instead of the usual conditional form. Most often that conjunctive form consists in the impossibility of the negation of a conjunction having to do with fate. That particular expression evidently suggests a relationship between the rejection of

$L(p \supset q)$, and then, by $(B)''$, $[L(p \supset q) \supset (\sim Mq \supset \sim Mp)]$, the consequent $\sim Mq \supset \sim Mp$.

It is remarkable that Chrysippus uses the sorites form in defense of Stoic theses. He does not use them though in Carneades' hypothetical form, but rather as negated conjunctions (Sedley, *op.cit.*, p. 91, who rightly adds that "this formulation ... is used to show that the relationship of the two propositions is something less than one of strict entailment"; still, the non-necessary is not for all that identical with the simply possible).

¹⁹VII, p. 75.

²⁰*In de int.* II, p. 234.

²¹Frede, 1974, pp. 107-114. On denial of the validity of the interdefinability of the modalities see Vuillemin, 1983.

²²Frede's second interpretation, 1974, p. 108, p. 112.

²³Mignucci, 1978, pp. 327-330.

modal interdefinability, on the one hand, and the substitution of the conjunctive for the conditional form of divination, on the other. What is the nature of this relationship?

Cicero, setting the scene with Chrysippus at grips with the 'Idle Argument', has him distinguish two sorts of assertion. "There are, in reality, isolated assertions and assertions bound together. Here is an isolated assertion: 'Socrates will die on a certain day'. Should he have done some particular thing or have refrained from doing it, the day of his death is determined. But if fate has it that Oedipus will be born of Laius, one cannot say: 'should Laius have had intercourse with a woman or should he not have', because the event is bound and 'confatal', as he says, for fate has it both that Laius will have intercourse with his wife and that he will beget Oedipus who will kill him".²⁴ Chaste, Laius would not have been killed by his son; and that chastity was dependent on his assent, that is to say, his nature. It is illusory to think then that Laius is a plaything of external fates that dragged him along in spite of himself, since the act from which his trouble stemmed was his own. The Idle Argument posits right off that it is impossible that Oedipus not be born of Laius and therefore not kill him, since the impossibility is there whether Laius have intercourse with a woman or not. All that we have the right to assert is that it is not possible that Laius have intercourse with a specific woman *and* that Oedipus not be born of him. For the edict of fate to be legitimate, that is, for the conjunction of these two propositions to be impossible, it must be that, if it is capable of being true, external circumstances prevent its being true. To grant the prophecy its dramatic rather than its apotropaic sense now, suppose that the first of the conjuncts is true (and even necessary) and that Laius has had intercourse with a woman, and that the second conjunct, namely that it will not be the case that both Oedipus will be born of Laius and not kill him, is about the future. For the conjunction to be simply impossible it has, if it is capable of being true, to be prevented from being true by external circumstances. But it is indeed capable of being true, since, Laius' intercourse with Jocaste does not, if considered intrinsically or logically, contain in its analysis the future occurrence of Oedipus' birth and his assassination of Laius. However, there are external circumstances—foreign in themselves to the essence of the past—precisely those precipitating the tragedy, that prevent the conjunct from being true. What fate disallows is that, in the fatal conjunction of two events, the anterior one should have occurred without automatically being followed by the yet future one. But

²⁴Bréhier, 1962, p. 484.

fate does not mean fatalism no more than physical connection means moral constraint. The future event, that Oedipus will be born of Laius and assassinate him, is not necessary. Its necessity would only be derived from the necessity of the intercourse of Laius and Jocaste (which is granted since it has already occurred) if it were positively necessary that it be not the case that this intercourse has taken place and that it will not happen that Oedipus be born of Laius and not assassinate him. But this positive necessity requires not only that, were that conjunction capable of being true, it would be prevented from being so by external circumstances, but also that it be actually false. Then at least one of the conjuncts must be false. It cannot be the past event of Laius' intercourse with Jocaste, which is true and even necessary. Therefore it is the future event of the non occurrence either of Oedipus' birth or of his assassination of Laius. The negative impossibility differs from the positive necessity and does not imply it, because the truth of the future is not already given within the truth of the past. Formulated as an astrological conditional, fate entails not only the impossibility that Oedipus will not be born of Laius, but the necessity that he will be. The Idle Argument becomes legitimate then, since the necessity of the consequent can be detached.²⁵

The prediction concerning Oedipus ought therefore not to have been of the form 'It is necessary that Laius have intercourse with a woman and beget Oedipus' but only 'it cannot possibly not happen both that Laius have intercourse with a woman and that he not beget Oedipus'. It is probable that if the Stoics insisted on the clauses of non-prevention in their definitions of the modalities it is because they expressed the laws of nature in the form: it is not possible not to have such and such a conjunction.

Fate has to do with *condestinates*. The illusion of necessity is due to the fact that one of the assertions—the birth of Oedipus—was isolated and made to support the entire causal chain. This is how the celebrated example of the cylinder can be explained. "Just as in pushing a cylinder one has given it a beginning of motion, but has not given it the capacity to roll, so a sense-presentation will surely impress and

²⁵Taking $Pp =$ 'Laius had intercourse with a woman'
and

$Fq =$ 'Oedipus will be born of Laius',

the law of condestinates will be written: $\sim M(Pp \sim Fq)$. Given Pp , and thus LPp , all that can be concluded without appeal to the interdefinability of the modalities is:

$\sim M(Pp \sim Fq) \sim M \sim Pp$. The conditional law $L(Pp \supset Fq)$, on the other hand, together with the given Pp , allows the detachment of LFq as the Idle Argument would have it.

mark its form in the soul, but the act of assent will be in our power; pushed from without, as we said of the cylinder, it will move by its own force and nature. If something were produced without antecedent cause, it would be false to say that everything happens by fate; but if it is likely that everything that happens has an antecedent cause, what reason could be adduced for not recognizing that everything happens by fate provided that the distinction and difference between the causes is well understood?"²⁶

It cannot be that a certain cylinder should not have a certain motion, if an external impulsion is impressed upon it. It is not for all that that this motion bespeaks a brute necessity. For from the point of view of providence it is simply a question of not possibly not bringing about a certain conjunction, given the maximum perfection of the work in conformity with the principle of the best. And from man's point of view it is simply a question of not being able to avoid the consequences bound to his acts, without its being for all that that the assent he gives to moving mental representations is a brute and inevitable consequence of the impression they make.

Leibniz made a famous commentary on this passage and its continuation by Aulus Gellius²⁷. Chrysippus' cylinder is similar to Leibniz' boat swept along at a greater or lesser speed in a river current. On the one hand such a possible has its own nature and spontaneity which are formal and not material, as is misleadingly suggested by the metaphor of the cylinder and of the boat too; and they constitute the perfection of the individual in which is encompassed its assent²⁸. And that is what accounts for freedom. From the point of view of theodicy on the other hand, if it is objected that the sheer fact that the cylinder is by nature rough, the boat heavy or ungainly, are so many arguments against providence, the reply must be that the partial evil is in view of the general good, like the vulgar epigrams and inscriptions in ancient comedy.²⁹ Cicero's development of these two themes in the *De Natura Deorum* is proof enough of the connection between Leibniz and Chrysippus.³⁰ Neither for the one nor the other do providential

²⁶ *De Fato* XIX, 43; Bréhier, 1962, pp. 489-490; assent is not assigned by the specific chain of mental images (Cherniss, 1976, II, p. 591, note e).

²⁷ *Noct. Att. L. VI, c. 2*, quoted by Leibniz, Gerhardt, VI, p. 321; Jalabert, p. 326.

²⁸ Leibniz, Gerhardt, VI, p. 314; Jalabert, p. 328.

²⁹ Leibniz, Gerhardt, VI, p. 313; Jalabert, p. 327. Plutarch takes up the Stoic view of comedy (*Moralia*, XIII, Part II, 1065; Cherniss, 1976, II, pp. 709-713) and turns it against Chrysippus as impious. "But that refutation doesn't amount to much," says Leibniz (Gerhardt, VI, p. 313; Jalabert, p. 326).

³⁰ Especially *L. II, ch. XIV and XXXIII*; Hamelin, 1978, pp. 94-95.

foreknowledge and preordination entail necessitarianism.³¹ Both admit that mental representations incline one but do not compel. To the condestinates correspond the compossibles, and the origins of Leibnizian preformation are to be found in the order of natures according to Zeno and Cleanthes followed by Chrysippus. It even happens often enough that Leibniz expresses the conditional in negative form: "They say that what is foreknown cannot fail to exist, and they are right; but it doesn't follow that it is necessary".³² Leibniz, like Chrysippus, explains evil by concomitance and repeats the adage of jurisprudence: *incivile est nisi tota lege inspecta judicare*. There is a difference between them however. Leibniz retains the interdefinability of the modalities and escapes necessitarianism in distinguishing two kinds of necessity. The first or metaphysical and brute necessity goes back to the principle of non contradiction, while the second which may, and in the final analysis must, be moral is based on the principle of the best. Chrysippus, on the other hand, does not appear to make that distinction. But consider just what he does do. Suppose he distinguishes two sorts of laws. The first, the mathematical ones, include the affirmative form of the second premise, according to which the necessary follows logically from the necessary, along with the interdefinability of the modalities. Here the second premise is therefore valid in its negative form: the impossible does not follow logically from the possible. For physical reality, by contrast, other laws obtain. Here the second premise is still valid in its affirmative form, but the interdefinability of the modalities and therefore the negative form of the second premise no longer hold. The exclusion of interdefinability then would serve to separate the purely logical brand of incompatibility, which falls under it, from physical incompatibility or causal law, which are not subject to it.

There is a passage in Diogenes Laertius (75) that bears this out. "The necessary is a proposition which being true is not capable of being false, or which is capable of being false but is such that it is prevented by external circumstances from being false". Of this complex alternative characterization it is clearly the second term that concerns us here. Just what is a proposition which is capable of being false but is prevented by external circumstances from being false? It is a proposition such that it is simply impossible that it be not the case. Without ever being able to be false it can happen that this last proposition is not true, if it comes to be destroyed.

³¹Leibniz, Gerhardt, VI, p. 330; Jalabert, p. 345, who quotes Cicero: "*Sequitur porro nihil deos ignorare, quod omnia ab iis sint constituta*".

³²Leibniz, Gerhardt, VI, p. 123; Jalabert, p. 130.

What difference there is then between Chrysippus and Leibniz on this point can only be attributed to the following principle (which is nevertheless a fundamental one). True to the pantheism of the Stoa's founders, Chrysippus cannot conceive of a distinction between divine will and divine understanding. There is then but one necessity of fate. To limit its harshness for freedom as well as for theodicy, it will be necessary to refuse the right of passing from the impossibility of not being to the necessity of being, from conjunctions to conditionals. By contrast, the trans-worldly God of Leibniz allows of such a distinction. To Chrysippus 'sound' conditionals correspond Leibniz' brute necessities, to his simple 'astrological' conjunctions correspond conditional necessities. But these last give rise to yet a further distinction which is missing in Stoic philosophy. For with Leibniz there is a science of simple intelligence having to do with the conditional necessity of compossibles, and there is a science of vision having to do with the decree of creation considered as being in view of the best.

5.4 A system related to Prior's system Q; the double logical square of Chrysippean modalities and the double temporal index in the Master Argument's second premise.

It was Prior who constructed a system of modal or temporal logic, called system Q, to account for situations in which, the propositions not existing, it can be said neither that they are true nor that they are false.³³

³³Prior, 1957, p. 49, shows how the system Q enables us to avoid the paradox: 'If it is not possible that I don't exist (in the sense that it is not possible that there are no facts about me), then it is necessary that I exist'. The paradox is a bit forced though, for who would admit the antecedent? It is in 1967 that Prior makes specific reference to Chrysippus and the argument about Dion (p. 152)

"... 'If nothing exists, this man doesn't' is never false [$\sim M \sim (\sim (\exists x)E!x \supset \sim E!a)$] where 'E!x' stands for 'x exists', for it is true whenever there is such a proposition. And it is possible that nothing should exist [$M \sim (\exists x)E!x$]. So here we have a [$\sim M \sim (\alpha \supset \beta)$] and a $M\alpha$ which are true, though the corresponding $M\beta$, namely [$M \sim E!a$] is false, i.e. [B'] does not universally hold. And the example is *almost* Chrysippus' own, except that I have replaced his 'Dion doesn't exist' by 'Nothing exists', the entailment by which of 'This man doesn't' is perhaps clearer. It is perhaps a little contentious to say that it could be that nothing exists, but if one held that being of the basic *sort* one is, e.g. being a man, is 'essential' or 'necessary' in anything that is of that sort, one could say that it could not be false that if no man exists, then this man doesn't, that it could be that no man exists, and that it couldn't be (*isn't* the case in any possible state of affairs) that precisely *this* man doesn't exist". (Prior, 1967 p. 152). One might think that the idea of the destruction of propositions in the Chrysippean theory would allow us by the same

System Q as conceived of by Prior, however, diverges from the Chrysippean model in that 1) despite its temporal character, it supposes that the second premise of the Master Argument is a thesis of pure modal logic, 2) it implies the discontinuity of time, 3) it entails an assimilation of Chrysippus and Diodorus on the fundamental question of defining the possible as that which is sometimes realized, even if it distinguishes them by means of the destroyed propositions. Alexander has formally rejected the Diodorean interpretation of Stoicism, which the addition of destroyed propositions would not essentially change. He writes: "But, it will be said, by affirming that it is fate that produces everything, neither the possible nor the eventual are denied. This indeed can be that nothing prevents from being though it not be".³⁴ Cicero and Plutarch confirm Alexander on this point.³⁵ System Q and the system of Chrysippus can be said to be related, but not identical.

System Q has been axiomatized³⁶ and studied from the point of view of semantic models.³⁷ If modally and temporally non-standard in its refusal of interdefinability, it does conserve the logical laws both of propositions and quantification. If the modalities are interpreted as having a causal sense—and this is close to the Stoic intuitions about fate—it is seen that the 'standard' logic of causality leads³⁸ to 1/ a collapse of the modal distinctions if substitutivity of identity is maintained, as would be normal given a univocal sense of identity, and 2/ causal necessitarianism. To remedy the second fault the interdefinabil-

token to deny the validity of the retrogradation of futures. Indeed, the quantified form of $p \supset PFp$ will read $(\exists x)\varphi x \supset P(\exists x)F\varphi x$.

The antecedent can be true and the consequent false if the consequent is 'destroyed', that is to say, where for want of facts the proposition doesn't exist. But that way out is blocked by divination: the ineluctability of the consequent will be expressed simply in negative ($\sim M \sim P(\exists x)F\varphi x$) and not affirmative ($LP(\exists x)F\varphi x$) form. One does not have the interdefinability of G ('it will always be the case that') and F nor of H ('it has always been the case that') and P and $\sim P \sim$; $\sim P \sim (p \supset q) \supset (Pp \supset Pq)$ is not a theorem. (Prior, 1967, p. 156).

One might be tempted here to draw a parallel between Chrysippus and Aristotle, comparing the 'destroyed' propositions of the first with the truth-valueless propositions of the second. But the parallel would be misleading.

1/ All Stoic propositions have a truth-value. A destroyed proposition that is, one such that the conditions necessary for its use have disappeared (M. Frede, p. 49), is altogether different from a proposition without truth-value.

2/ Destroyed propositions have no assigned relation to futures.

3/ The impossibility of a conjunction is in no way equivalent to a contingency; although it cannot be assimilated to a positive necessity

³⁴ Arnim, *S.V.F.*, II, n° 959.

³⁵ Hamelin, 1978, p. 87.

³⁶ By Bull (Prior, 1967, p. 154; Hughes and Cresswell, 1972, p. 304).

³⁷ Føllesdal, 1965 and 1966.

³⁸ Proposed by Burks; see Føllesdal, 1966, p. 12.

ity of the causal modalities will be given up. To remedy the first, the only singular terms admitted will be descriptions that keep the same reference in all physically possible worlds. But this comes down to saying that if it is causally necessary that a , and further that $a = b$, then it is causally necessary that b as well, where causal necessity has nothing to do with our subjective manner of describing individuals, but has to do rather with their proper essence.³⁹ This 'causal essentialism' is quite in keeping with the Chrysippean notion of fate.⁴⁰

What is still missing in such systems is the explicit reference of the Chrysippean modalities to a double temporal index and the corresponding interpretation of the Master Argument's second premise.

The definitions,⁴¹ or rather, elucidations (of the modal terms figuring in the *definiens*) of Chrysippus are complex. Following M. Mignucci we shall lay down the following conventions:

A means: ' p ' is true

B means: ' p ' can be false

C means: external circumstances prevent p 's being false.

Replacing the word 'false' by 'true' and vice versa in these stipulations we get:

A' means: ' p ' is false

B' means: ' p ' can be true

C' means: external circumstances prevent p 's being true.

The definitions of necessity and of not-possibly-not (which will be distinguished from that of necessity), for example, will read as follows:

p is necessary (Lp) if and only if p is true and if p can be false it is prevented by external circumstances from being so, that is

$$Lp \equiv A.(B \supset C).$$

By contrast, the negative form 'it is not possible that not p ' will be written simply:

$$\sim M \sim p \equiv B \supset C,$$

where the assertion that A has disappeared. It is asserted only that if ' p ' can be false, external circumstances prevent its being so. But there

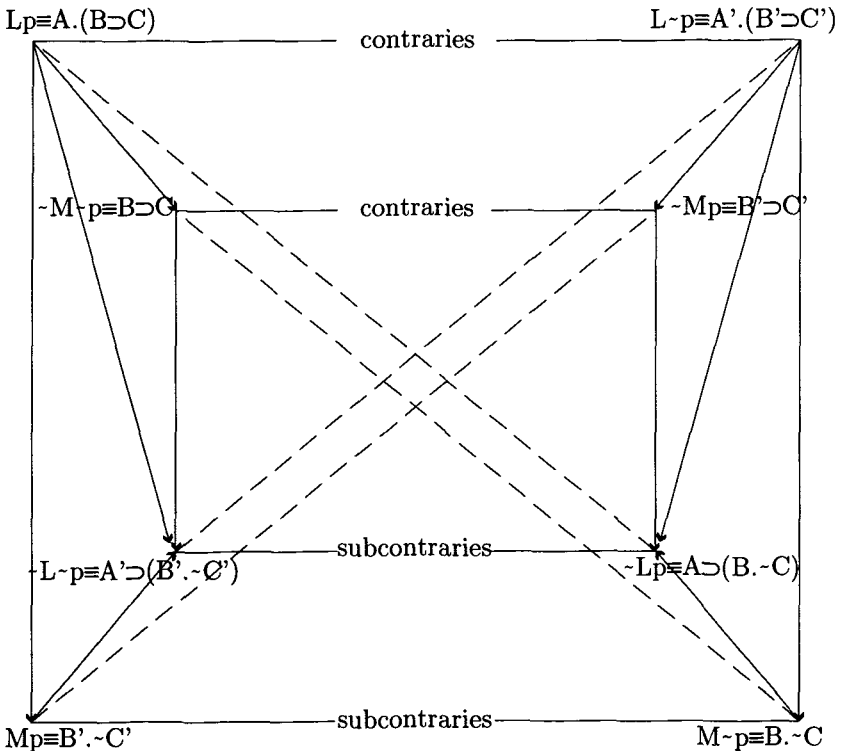
³⁹Føllesdal, 1965, pp. 272-273, who thus finds Prior's example again.

⁴⁰In such a system past and future are symmetric. But adjunction of the Master Argument's first premise, admitted by Chrysippus, re-establishes the asymmetry of time (that tempers the notions of absolute numerical identity in the eternal return and thus opposes Chrysippus to Cleanthes). As for the system Q, it would seem to have a good deal of affinity with the Aristotelian logic of possibles *ad unum* (the virtues).

⁴¹Mignucci, 1978; Vuillemin, 1983.

is nothing to oblige supposing that the proposition p exists. It may be 'destroyed' as happens for propositions containing a deictic, when the object designated by the deictic no longer is. In the case of affirmative necessity Lp this circumstance is ruled out by the occurrence of A . According to Diodorus the necessary was that which, being true, never became false. Chrysippus modifies that definition in two ways. He dissociates the positively necessary from that of which the negation is impossible, the second term being deprived of the truth-clause. He introduces the notion of external circumstances as constituents of compossibles and fate.

Because it rejects interdefinability the Chrysippean system will have eight modalities and differentiated relations of implication and contradiction.



It is readily seen from this square of opposition why an impossible, $\sim Mp$, is compatible with the negation of a negative necessity, $\sim L\sim p$, for there is opposition of contradiction between 'it is impossible that

p ' and 'it is possible that p ', but not between 'it is impossible that p ' and 'it is not necessary that not p '.

The question that arises at this point is that of the relation between the double *logical* square of Chrysippean modalities and the double temporal index figuring in the Master Argument's second premise. The preceding arguments about the interdefinability of the modalities are not sufficient to determine their meanings. But the occurrence of the letters B and C in these definitions are enough to show that these meanings do not belong to pure modal logic.

When Chrysippus says that from the strong Chrysippean antecedent

'If Dion is dead, then this man is dead'
the conditional consequent

'If it is possible that Dion is dead, then it is possible that this man is dead',

logically follows, the strong antecedent would mean, according to the definition of positive necessity, that 'If Dion is dead, then this man is dead' is true and that if it is capable of being false external circumstances will prevent it from being false. But these strong antecedents make the consequent false, in the case where 'Dion is dead' is true and 'this man is dead' is destroyed. We must therefore accept only the weaker antecedent that it is not possible that Dion be dead and this man not be dead, a conjunction which is indeed always true. Unfortunately this conjunction does not authorize the consequent.

In the strong rejected antecedent as well as in the weak authorized conjunction, we find two synchronic sentences, while the consequent is a conditional whose antecedent states a diachronic modality and the consequent a synchronic modality. This suggests that there must be a connection between the Chrysippean non standard square and the second premise.

Let us first examine what would mean for Chrysippus himself as well as for the whole Stoic-Megaric school the second premise of the Master Argument as put under its complete Aristotelian form, if no distinction were made between weak and strong modalities:⁴²

$$(B)_s^c(t)(M_N p_t \supset \{(\exists t_1)[M_{t_1} p_{t_1} \cdot (t \leq t_1 \leq N \vee N \leq t_1 \leq t)] \cdot (M_t p_t \supset p_t)\}).$$

⁴²The complete second premise says:

$$(t)\{M_N p_t \supset (\exists t_1)M_{t_1} p_{t_1} \cdot (t \leq t_1 \leq N \vee N \leq t_1 \leq t)\} \cdot (t)(M_t p_t \supset p_t)$$

This formula is equivalent to:

$$(t)\{(M_N p_t \supset (\exists t_1)[M_{t_1} p_{t_1} \cdot (t \leq t_1 \leq N \vee N \leq t_1 \leq t)]) \cdot (M_t p_t \supset p_t)\}$$

and entails the consequence by the law: $[(P \supset Q) \cdot R] \supset [P \supset (Q \cdot R)]$.

In conformity with the Chrysippean square, the strong diachronic possible ($M_N p_t$) entails the following conjunction (1) that 'this man is dead' can be true (B'), and (2) that external circumstances do not prevent it from being true ($\sim C'$). The first conjunct is true if and only if there is an intermediary instant t_1 such that $M_{t_1} p_{t_1}$, since then p_t 'can be true'. The second conjunct is true if and only if no circumstance prevents p_{t_1} from being true, which would unmistakably be the case would $\sim p_{t_1}$ obtain. The second premise of the *De Caelo* text had been analyzed into two parts: the proper second premise of Epictetus—the synchronic contraction of the diachronic possible—and the principle of conditional necessity. Both parts are again found in the Chrysippean definition of the strong possible.

What singles out Chrysippus in the school is his distinction between a strong possible ($B' \cdot \sim C'$) and a weak one [$A' \supset (B' \cdot \sim C')$]. A possible simply contradicts its corresponding impossible in a standard square of modalities. A weak possible still contradicts its corresponding strong impossible in the Chrysippean square. But the conjunction of a weak possible with the weak corresponding impossible does not express here a contradiction. For example, the conjunction ($\sim L \sim p \cdot \sim Mp$), or, expressed in the terms of the square:

$$[A' \supset (B' \cdot C')] \cdot (B' \supset C')$$

may be compatible. For it to be compatible, A' must be false and the conditional ($B' \supset C'$) be true. And A' is false when it is not the case that 'this man is dead' is false, a situation that obtains when the proposition 'this man is dead' is destroyed.

Therefore, if a weak possible is substituted for the strong one in the antecedent of the complete strong second premise of the Master Argument:

$$(B)_W^C(t)(\sim L_N \sim p_t \supset \{(\exists t_1)[M_{t_1} p_{t_1} \cdot (t \leq t_1 \leq N \vee N \leq t_1 \leq t)] \cdot (M_t p_t \supset p_t)\}),$$

the conditional becomes false for Chrysippus, since the consequent may be denied without contradiction.⁴³

Though Chrysippus' model may seem artificial, it expresses an im-

⁴³For Chrysippus, the formula:

$$(\exists t)\{\sim L_N \sim p_t \cdot [(t_1)(t \leq t_1 \leq N \vee N \leq t_1 \leq t) \supset \sim M_{t_1} p_{t_1}] \vee (M_t p_t \cdot \sim p_t)\}$$

is compatible if one of the disjuncts or the consequent is true and if ' p_t ' is destroyed. It is to be observed that for the first disjunct of the consequent being true p_{t_1} must be destroyed at all t_1 past or future comprised in the intervals $[tN]$ or $[Nt]$. The fitness of Chrysippus' example shows itself: Dion's death is either future and the proposition "this man is dead" is "beforehand" destroyed (since it has never been existing) or past and the same proposition becomes then "normally" destroyed.

portant representation of natural law as prediction. Every prediction connects two different instants of time through a necessary relation. This necessity, however, cannot be positive, since, in that case, it should suppose the actual truth of the connection. It must therefore be negative, stating only the impossibility of the conjunction between the capacity for truth of some proposition and its prevention from being true by the whole of the *confatalia*. This weak necessity is apt for connecting two terms to which their respective temporal indices assign different roles. The first predictive term has, by definition, two distinct temporal indexes. It therefore belongs to the class of theoretical terms, which no finite class of observational terms implies. The second term consists in a connection between two simultaneous observations; one observation is about the general state of affairs at that moment, the other observation is about the actualization of the capacity at that same moment for a singular deictic proposition to be true at that moment. Now it is the fate of these propositions of observation to be destroyed. Chrysippus' counterexample was well chosen, because only a natural law thus expressed may generate a theoretical or diachronic prediction with destroyable synchronic verifiers.

Human perception, except when liberated by dream⁴⁴ or divinatory⁴⁵ vision, is confined within the present.⁴⁶ The chain of fate must therefore contract time into the conjunction of the sound 'conditional', as is shown by the theorem or argument of the sign. In the theorem: "If somebody is wounded at heart, he will die", the wound at heart is a sign not that he will have to die, but that he is having to die "the present sign being the sign of a present thing".⁴⁷ The theorem delimits the share of fatalism, a fatalism which is broken as soon as the temporal identity of a sign is destroyed.

5.5 A Philonian doubt about the second premise?

Two simple objections are made against the Master Argument's second premise.

Suppose, first, that it is now possible for me to catch a train at the station tomorrow at 8 a.m. and that going to the station needs half an hour. If I neglect to leave my house tomorrow before 7.30 a.m., it will be true that what had been possible until 7.30 has ceased to be so, though no instant before 8 has offered the occasion of a synchronic possible. The objection, however, forgets that the conditions to be filled for doing

⁴⁴Goldschmitt, 1977, p. 85.

⁴⁵Goldschmidt, 1977, p. 81.

⁴⁶Goldschmidt, 1977, p. 97.

⁴⁷Sextus, *Adv. Math.*, VIII, 254-256; Goldschmidt, 1977, p. 44.

an action are explicitly mentioned, when the Ancients require that external circumstances do not hinder the realization of the possible, failing which the verbal modal judgment should not be given its genuine sense. Aristotle agrees with this general understanding. Speaking of the rational potencies, he specifies: "To add the qualification 'if nothing external prevents it' is not further necessary; for it has a potency of acting, and it is this not in all circumstances but on certain conditions, among which will be the exclusion of external hindrances; for these are barred by some of the qualifications of our definition".⁴⁸

More pertinent reservations seem to arise from the reflection that it is possible now that a certain bit of radium emit radiation at t , though no emission occurs between now and t . But what we mean then is that there is a very high probability P that, during the space of time $|N - t| \leq e$, the given bit of radium will emit a radiation. We imagine a very large number N of such identical states, *i.e.* of similar bits of radium. The more the number N grows up, the more next to P will be the relative frequency of emissions during the time $|N - t| \leq e$. At the same time, we shall verify that no emission has been made by this particular bit, while the next one has emitted three times. The probability is a property belonging to a class of individuals and it must not be confused with individual potency. It is therefore not contradictory to maintain that this bit of radium which has emitted no radiation during the time $|N - t| \leq e$ had no potency of doing it, in conformity with the second premise, although the probability P of its emitting was estimated very high.

Could then the second premise still be doubted, while maintaining the individual character of the possible? If such a hypothesis was formulated it could only have been by a philosopher who would have stripped away a maximum of their reality from the real possibilities imbedded in time that the Master Argument is all about. In other words, that philosopher would have had to abstract from the external circumstances of the temporal realization of the modalities to concentrate solely on their logical aspect. He would not have challenged conditional necessity, as Plato has done, throwing suspicion on the reality of sensible existence. He would simply have purged the modal concepts of their temporal adherences, separating them from what involves their realization in time.

⁴⁸ *Métaphysique*, Θ 5 1048^a 16–21; see 2.4, p. 22; on this objection and the following one see below, 10.1.

Philo, the Megarian, was surely a philosopher of the like. It is only a conjecture, but not a groundless one, to attribute doubts about the second aspect of the second premise to him.

Philonian or material implication reduces natural laws to formal implications and therefore to simple universals lacking in any modal surplus. So it diverges from the conception Diodorus as well as Chrysippus and Aristotle had of laws. One can presume that after having dissociated assertoric logic from all modal engagement Philo should have continued his effort in trying to dissociate the temporal modalities as much as possible from their temporal engagement. He says that "the possible (which is moreover none other than the non-necessary) is that which is susceptible of being by virtue of its own essence, so that a thing that isn't and will not be, but is prevented from being by external circumstances only, is none the less possible than that which is or will be. Straw and a shell at the bottom of a well have the capacity, the one of being burned, the other of being seen, wood in the middle of the ocean remains combustible. In a word, possibility consists in mere aptitude or virtuality".⁴⁹ The clause identifying possible with non-necessary blocks Chrysippus' escape route. There is nothing to indicate, on the other hand, that Philo contested either the first premise or conditional necessity. This last principle has to do only with the relation of present existence to present necessary existence, and it is doubtful that a Megarian would abandon their connection.

We must therefore examine the second aspect of the second premise. The unique index of the synchronically possible takes its place between the two indices of the diachronically possible. As truistic as it seems, that operation supposes that the second diachronic index is not shifted to infinity. Removing that supposition, the Diodorean definition 'the possible is that which is or will be true' gives way to the Philonian 'the possible is that which is or will be true or of which the truth will be deferred to infinity by external circumstances'. Never assignable, the ever prevented possible of the third premise would not give rise to a synchronic possible that would contradict the simultaneous conditional necessity given rise to by the hypothesis that the possible will not be realized. This shift to infinity will simply push to the extreme Diodorus' solution, of which it can be considered the limiting case. It would resolve the *aporia* economically without for all that giving up conditional necessity which seems indispensable once one dogmatically identifies being and event. It would still fit in too with the general

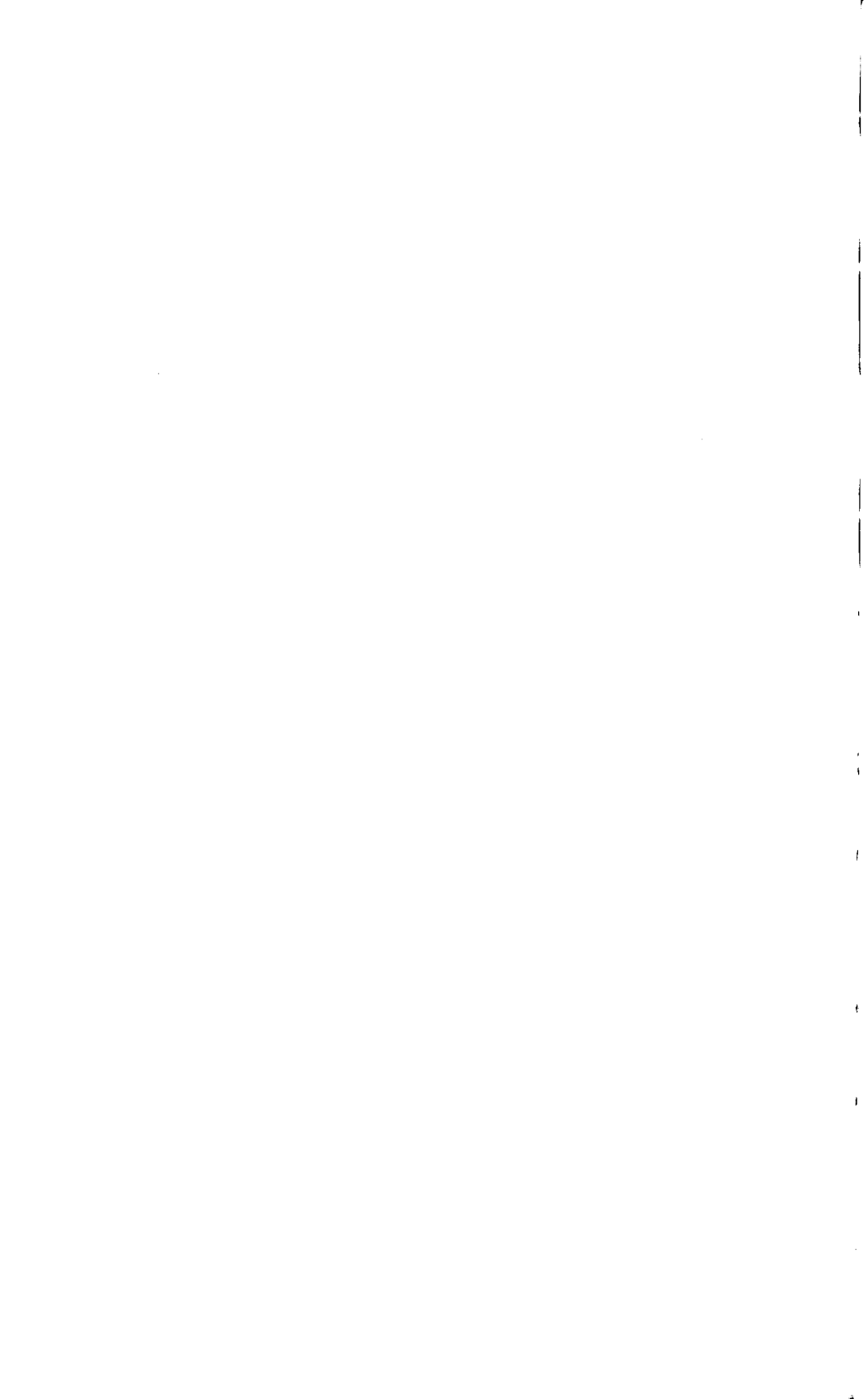
⁴⁹Hamelin, 1978, p. 84; Döring, 1972, fr. 135-137, pp. 41-42.

necessitarianism to which Philo, like his Megarian masters, seems to have remained faithful.⁵⁰

That elegant and economic solution from the logical point of view has the drawback of obliterating at the limit the distinction between real and logical possibility just as Philo's extensional definition of sound conditionals had the particularity of eliminating any consideration of necessity from the concept of law. It is understandable that the Stoics, attentive as they were to the relation of the modalities to physical reality, should have paid little attention to a solution that suppressed rather than resolved the question raised.⁵¹

⁵⁰Hamelin, 1978, p. 84.

⁵¹"Although that definition [the one Philo gives of the possible] does retain something Megarian, not only ... in that it leaves no room for genuine contingency, but also in that it makes no appeal to external circumstances for weighing their possibility, it is seen nevertheless to invoke these circumstances for determining the real. This is meeting Stoicism midway" (Hamelin, 1978, p. 84). But Philo mentions external circumstances only to eliminate them in his definition of the possible, τῆ ἐπιτηδεϊότητι μόνῃ ('by virtuality alone'), so goes the Philonian definition according to Simplicius (*In cat.*, 195, 31–196, 24 Kalbf.; Döring, 1972, p. 41, fr. 137). There we have the point of view of a logician concerned with separating his science from physics.



Part III

Systems of Contingency: The Lyceum, The Garden, The Academy

Towards Rehabilitating Opinion as Probable Knowledge of Contingent Things. Aristotle.

The Epictetus passage reporting the Master Argument makes mention only of the Megarian School and the Porch.

Yet the other schools of philosophy met Diodorus' challenge as well. Cicero's *De Fato* attests to this as far as the Academy and the Epicureans are concerned, and in all likelihood Chapter IX of the *De Interpretatione* contains Aristotle's own response.

Cleanthes and Chrysippus, in contesting the argument's first and second premises respectively, were not acting solely as logicians. Both the thesis of numerically identical eternal recurrence and that of *confatalia* presuppose a physical image of the world and consequently a certain representation of causality. In the one case this was the sympathy of connection where there was room left for spontaneity, in the other it was the constantly renewed order of palingenesis.

The solutions proposed by the other schools have that same characteristic. When an Aristotle, an Epicurus, a Carneades or a Plato challenge one or another of the Master Argument's implicit logical premises, they do so in the name of a certain conception of physics and a determinate principle of causality. Chance and fortune, the *clinamen*, the specificity of deliberate cause and the specificity of spiritual movement are all brought in for freedom's sake.

6.1 *De Interpretatione*, Chapter IX.

Chapter IX of the *De Interpretatione* is one of Aristotle's most difficult and most disputed texts. A translation will be given and the introduction analyzed to specifically determine the problem raised: that of

future contingents. According to Aristotle, the solution must conform to the two logical principles of non-contradiction and the excluded middle. On the other hand, it requires abandoning the Megarian theory and in particular the Diodorean definition of the possible. Aristotle's explicit solution is grounded in two principles: a distinction must be made between absolute and conditional necessity and the validity of the principle of bivalence is subject to limitation. The Stagirite's overall conception confirms the lesson of *De Interpretatione*, Chapter IX. He did not attempt to reform logic, whose principles he had just established, but to rehabilitate the notion of opinion as constituting knowledge of the contingent. Such knowledge would have a probabilistic value.

Translation of the text

18^a28. Applying to what is or has been, it is necessary for the affirmation or the negation to be true or false. Applying to universal things taken universally it is always so that one is true, the other false, and applying to singular things it is the same, as we have said. But applying to universal things not taken universally it is not necessary; this we have also spoken of. Applying to future singular things, however, it is not the same.

18^a34. For if every affirmation or negation is either true or false it is necessary too for everything to be the case or not be the case. For if one person says that something will be while another says that that same thing will not be, it is clearly necessary that only one of them is saying what is true, since every affirmation is either true or false. For applying to this sort of thing it will not be the case that both are simultaneously saying what is true.

18^a39. For if it is true to say that something is white or not white, then it is necessary that it be white or not white (18^b); and if it is white or is not white, it was true to affirm it or to deny it. And if it is not the case it is false, and if it is false it is not the case. It follows that it is necessary that either the affirmation or the negation be true.

18^b5. Nothing then either is or happens either by chance or indeterminately, nothing that will or will not be, but everything happens necessarily and without any indetermination. For either he who affirms says what is true, or he who denies. Otherwise it is indifferently that an event would occur or would not occur. For that which is indeterminate does not or will not happen rather in one way than another.

18^b9. Again, if something is white now it was true before to say that it will be white, so that it was always true to say of anything that has happened that it will occur. But if it was always true to say that it is or that it will be, it is not possible that it not be or that it will not

be. But it is impossible that that which cannot not occur should not occur. And that of which it is impossible that it not happen happens necessarily. Therefore all futures come about necessarily.

18^b15. Consequently nothing will be the case indeterminately or as a result of chance, for that which depends on chance is not of necessity.

18^b17. Nor is it possible to say that neither the affirmation nor the negation is true: that an event for instance neither will take place nor will not take place. Firstly, if the affirmation is false the negation is not true and if the negation is false it happens that the affirmation is not true. Secondly, if it is true to say that a thing is white and black, both qualities must belong to it. Should they belong to it until tomorrow, they will belong to it until tomorrow. Suppose, by contrast, that tomorrow the event neither will nor will not occur: nothing indeterminate, such as a naval-battle for instance, would then take place. For it would be necessary that the naval-battle neither take place nor not take place.

18^b26. Such, among others, are the absurdities that arise if for every affirmation and negation, either applying to universals as universals or applying to singular things, it is necessary that one of the opposites be true, the other false, and if there is nothing indeterminate in events but that everything is and happens as a result of necessity. Consequently there would no longer be any point in deliberation nor in taking any pains, with the idea that if we accomplish a certain action a particular result will follow, whereas if we do not accomplish it the result will not follow.

18^b34. For there is nothing to prevent one man's saying ten thousand years beforehand that something will be the case, another that it will not, so that it necessarily will be that of one of the two cases it was true to predict it then. Nor does it really matter whether people did make the affirmation or not. For it is clear that reality is what it is even if there was not this one to formulate the affirmation, this other one the negation. For it is not because of the affirming or the denying that the event will or will not occur, even (19^a) had the announcement been ten thousand years beforehand rather than at any other moment. Hence, if it has been from all time that one of the contradictory propositions said the truth, it was necessary that that should happen and every event has always come about then in such a way as to happen necessarily. For it is not possible that what anyone has truly said will be the case should not happen; and as to what has happened, it was always true to say that it will be.

19^a6. But what if these consequences are impossible? For we see that there is also an origin of what will be both in deliberation and in action

and, that the potentiality of being and of not being resides entirely in things that do not always exist in act, things that, since they may be or not be, the one as well as the other, may therefore also come to be and not come to be. There are many manifest cases of this kind. This garment, for example, may be cut in two and yet will not be cut in two, but will wear out first. In the same way, it may not be cut, for it could not wear out first were it not possible for it not to be cut in two. This holds for all other events as well which are mentioned as having the same kind of potentiality. Clearly then not everything is or happens of necessity but some things come about indeterminately, and for them neither the affirmation nor the negation is more true, the one than the other, whereas for some other things one of the two is true more frequently, though it does come about that the other should happen and not it.

19²³. It is necessary that what is be, when it is, and that what is not not be, when it is not. Yet it is not of necessity that everything that is is or that everything that is not is not. For it is one matter for everything that is to be necessarily, when it is, and another for it to be necessarily simply. The same holds for everything that is not.

19²⁷. The same argument applies to contradiction as well. Everything necessarily is or is not, will or will not be, without saying for all that, if we divide, that one of the two is necessary.

19³⁰. Let me take an example. It is of necessity that tomorrow there will or will not be a naval-battle. Yet it is neither that the naval-battle takes place necessarily tomorrow nor that it does not. What is necessary is for either to take place or not take place.

19³². Consequently, since propositions are true in so far as they conform to the things themselves, it is clear that whenever the things behave indeterminately, and have a potentiality for contraries, the same necessarily goes for the contradiction as well. This is the case for things that are not always existent or that are not always non-existent. For it is necessary then that one of the two contradictory propositions be true or false, but it is not this one or that one but either, and when one is more true than the other it is nonetheless not already true or false. It is clear then that it is not necessary for every affirmation or negation taken from among opposite propositions that the one be true, the other false. For what is non-existent but has the potentiality of being or not being does not behave after the fashion of what is existent, but in the manner just explained.

6.2 Outline of the passage: Introduction (18^a28-34): The problem raised.

The passage quite naturally falls into two parts: the first (18^a34-19^a22) in which the Megarian theory is set forth and refuted, together with an interlude on the validity of the excluded middle (18^b17-25), and the second in which Aristotle sets forth his own solution (19^a23-19^b4). A remark of Pacius¹ who classifies all the possible theories concerning contradictory pairs of singular statements about the future into four groups, suggests a more finely tuned division of the text and of its logical articulation. (The different possibilities according to Pacius are that both are true, that both are false, that one is true and the other false now, and finally, that one is true and the other false, but only potentially). The passage is then reduced to a chain of implications. If one admits the principle of non-contradiction—the two statements are not both true—then, if one admits the law of the excluded middle—the two statements are not both false—then if one further admits the universal validity of the principle of bivalence—one of the statements is true, the other false now—one cannot avoid holding that all statements about the future are necessary. By contraposition, the existence of future contingents will require that if non-contradiction and excluded middle are to be maintained the principle of bivalence will be called into question and, consequently, one of the statements will be true and the other false, but only potentially.

At first Aristotle singles out three sorts of contradictory statements that are not to figure in the subject of his inquiry. The first two sorts, singular statements about the present or the past and universal statements taken universally, are such that the one is presently true, the other presently false. The third class consists of universal statements not taken universally. These are indeterminate or indefinite opposites (man is white/man is not white). When treated as two particular opposites they are subcontraries and can both be true, but not both false, at the same time. No difficulty arises with respect to them for the one is not really the negation of the other.²

While universal statements, taken universally or not, follow the same laws whether they are about the future or not, singular statements are different with respect to the future. The quantitative distinction is a necessary criterion for determining the fourth class of statements. To obtain a sufficient criterion the matter involved must also be taken into

¹Quoted by Edghill in *The Works of Aristotle*, 1928, p. 18^a, footnote 5.

²Aristotle, *De Interpretatione*, VII, 17^b28-33.

account, as St. Thomas points out.³ The word designating the future here is usually opposed to the future participial of the verb *to be* like what is in contingent matter is opposed to that which is in necessary or impossible matter.⁴ Assertions resulting from an essential predication, such as 'Socrates will be a man' or 'Socrates will not be an ass', affirm or deny a property of a subject not in so far as it is a particular singular subject but after the manner of universals,⁵ as is required by science. The opposition between these singular future assertions is therefore treated as an opposition between a universal and its contradictory, where the opposite truth-values obtain presently. The inquiry will be limited then to deciding whether it is necessary in the case of singular statements about the future in contingent matter that one of the opposites be true and the other false, presently.⁶ These statements may be of two different grammatical forms. There are singular predicative statements such as 'this will be white', and pseudo-dated existential statements, e.g. 'there will be a naval-battle tomorrow'. Not every future singular predication or future singular existence statement is accidental. Socrates will be essentially reasonable and the Heavens will necessarily have a given motion. But a particular man will be standing or sitting by accident, and he will exist by accident. It is to this twofold domain that the inquiry is limited

³St. Thomas, *De Interpretatione*, Liber Primus, Lectio XIII, 1-3 (Oesterle, *De Interpretatione*, 1962, 6, pp. 102-103).

⁴The text says "ἐπὶ δὲ τῶν καθ' ἕκαστα καὶ μελλόντων" in *singularibus et futuris*. Ammonius (quoted from William of Moerbeke's Latin translation, *In De Int.*, 138-139, *Corpus Latinum Commentariorum in Aristotelem Graecum*, t. I, pp. 264-265) specifies that what is to be understood by 'future' here is what is thought of as of contingent matter. He reminds us that in the *De Generatione et Corruptione* (II, 11, 337^b3) Aristotle opposes 'mellon' (or 'future') and 'esomenon' (future participle of the verb εἶμι). The latter signifies what will happen in any case, as when we say 'winter or summer will come' or 'an eclipse will take place', whereas the former involves a future that can happen or not, as 'I'll go for a walk' or 'I'll sail'.

⁵St. Thomas, *Peri Hermeneias*. p. 43: "*secundum universalium rationes*".

⁶As St. Thomas points out: "Aristotle has not mentioned contingent matter until now because those things that take place contingently pertain exclusively to singulars, whereas those that per se are inherent or are repugnant are attributed to singulars according to the notions of their universals". "Aristotle is therefore wholly concerned here with this question: whether in singular enunciations about the future in contingent matter it is necessary that one of the opposites be determinately true and the other determinately false" (Oesterle, *op. cit.*, p. 104).

6.3 Validity of the principle of non-contradiction and the law of excluded middle (18^a38 and 18^b17-25).

A simple remark suffices to exclude the possibility of making an exception to the principle of non-contradiction for statements in contingent matter about the future. Its application to the future adds no further plausibility to the negation of this principle—a negation some attribute to Heraclitus and that corresponds to the thesis of universal motion in physics.⁷

On the other hand, Aristotle spells out his reasons for maintaining the validity of the excluded middle for statements about the future. He does this after having set forth the Megarian theory, where he argues that not only would rejection of the law fail to enable us to escape the difficulty proper to that theory as regards the modalities, but would further open the door to the specific aporias regarding truth, analyzed in the *Metaphysics* in connection with the doctrine of Anaxagoras.⁸

As to truth, since, if the true and the false do not form a disjunction, once the futures will have to come to be so as to verify one of the two opposite statements, the other will still not be falsified.⁹

As to modality, suppose in conformity with the doctrine of universal mixture, which is the physical counterpart of the negation of the excluded middle, that two present contraries are true; it is true that something is simultaneously black and white¹⁰ (principle of homoeomerics). The conjunction of these two states is thus necessary ($\delta\epsilon\iota$). But with nothing to distinguish the logical behavior of present from future singular statements, it must be concluded from the fact that it is true that a certain thing will be white tomorrow and that that same thing will be black that necessarily that thing will be simultaneously black and white tomorrow. The only thing specific to the doctrine of homoeomerics is that it expresses the necessitarianism of

⁷ *Metaphysics*, Γ , 3, 1005^b24; 7, 1012^a25.

⁸ *Metaphysics*, Γ , 7, 1012^a26.

⁹ This is a particular case of the general refutation given at *Metaphysics*, Γ , 7, 1011^b25-29.

¹⁰ The principal manuscripts read $\mu\acute{\epsilon}\gamma\alpha\nu$ (large) instead of $\mu\acute{\epsilon}\lambda\alpha\nu$ (black); and Ross, following Boethius and Moerbeke, adopts this reading which renders the text unintelligible. The correction of $\mu\acute{\epsilon}\gamma\alpha\nu$ adopted by Minio Paluella, 1949, is necessary. What is more, it is in agreement with the Aristotelian passages treating of Anaxagoras' theory of homoeomerics. According to him all was mixed in the beginning (*Metaphysics*, A, 8, 989^a30-989^b20). As an example of a real intermediate—therefore essential when treating, as is the case here, of the logical conditions of the thought of change—Aristotle specifically give grey between black and white (Γ , 7, 1011^b29-30 and 36).

the future in terms of conjunction instead of expressing it in terms of disjunction.

Let us substitute 'non-white' for 'black' in the above example in order to make it deal with just a single predicate. It follows that if it is true that a certain thing will be white tomorrow and that that same thing will be non-white tomorrow then it is necessary that it will be white and non-white tomorrow. Doesn't such an expression come down to denying the principle of non-contradiction? As shown at the end of Book Γ of the *Metaphysics*, the specific absurdity produced by the negation of the excluded middle becomes apparent only when one goes from the affirmation that all is true to the affirmation that all is false. The example above then, with negations substituted for affirmations, will read 'If it is true that a certain thing will not be white tomorrow and that that same thing will not be non-white tomorrow, then it is necessary that it will be neither white nor non-white tomorrow'. Aristotle then introduces the example of the naval-battle. If the excluded middle is wanting it will have to be that the battle neither will take place nor will not take place tomorrow. Intuitionism,¹¹ in abandoning one of the fundamental principles of logic, hoped to make room for indeterminacy. All it issues in is a conjunction of impossibles.

With indeterminacy giving way to necessity, by way of impossibility, the difficulties of necessitarianism are added to those of intuitionism.

It is not known whether there were philosophers in Aristotle's day who rejected the excluded middle to escape necessitarianism, as Epicurus was later to do; in any case the Stagirite rejects such a solution as illusory.

¹¹ εἰ δὲ μήτ' ἔσται μήτε μὴ ἔσται αὐριον οὐκ ἂν εἶη τὸ ὁπίωερ' ἔτυχεν (18^b22-23). Those Aristotle has in mind here are the partisans of the intuitionistic logic peculiar to Anaxagoras, where one hasn't the right to substitute an affirmation (white) for a double negation (not-not white). The conjunction of a simply negated statement and the same statement doubly negated

$$(\sim p \cdot \sim\sim p)$$

does not violate the principle of non-contradiction. For that to happen it would have to be possible to substitute 'p' for ' $\sim\sim p$ ', which is precisely what intuitionistic logic disallows. In *Metaphysics*, Γ , 8, Aristotle remarks "Again, there are obviously contradictories which cannot be at the same time true. Nor on the other hand can all statements be false, yet this would seem more possible in view of what has been said". (1012^b2-4) (Ross translation). On Anaxagoras' intuitionism and theory of continuum, see H. Weyl, *Principles of mathematics and natural science*, New York, Atheneum, 1963, p. 41.

6.4 Critical examination of the Megarian theory (18^a34-18^b17 and 18^b25-19^a22).

Aristotle records two demonstrations of the Megarian thesis. The first is of a rather logical nature, the second refers more to reality.¹²

The first demonstration (18^a34-18^b9) consists of two stages. The first of these is brief and general (18^a34-39). It expresses the principle of correspondence, foundation of Aristotle's semantics, though retaining only the passage from truth to being. But it gives this passage an apodictic force and a modal aspect heralding the principle of conditional necessity.

(C) If every affirmation or negation is true or false, it is necessary or it is impossible for the corresponding state of affairs to exist.

The overall sense here requires us to take necessity in the divided sense.¹³ Since the thesis is universal we are free to apply it to an affirmation or a negation about the future, as long as its truth-value is present:

(C_F) If every affirmation or negation about a future is true or false it is necessary or it will be impossible for the corresponding state of affairs to have to exist.¹⁴

¹²Ammonius, *op.cit.*, 1961, pp. 251-252.

¹³εἰ γὰρ ἀληθὲς εἶπεν ὅτι λευκὸν ἢ ὅτι οὐ λευκὸν ἔστιν, ἀνάγκη εἶναι λευκὸν ἢ οὐ λευκὸν (18^a39-18^b1).

From the grammatical point of view there is no decisive argument for choosing between the division ($Lp \vee L \sim p$) and the composition $L(p \vee \sim p)$ of necessity. (On this point see Mrs. Frede, 1968, pp. 16-17). One might then reject our decision as unfounded and follow Edghill (*op. cit.*, 19^a30-32) in taking the consequent of (C) in the composite sense. The formula (F)

$$*L(p \vee \sim p) \supset (Lp \vee L \sim p)$$

is not a thesis of modal logic.

(Notice Hughes and Cresswell, 1972, p. 38, who are mistaken in saying that the antecedent is false and the consequent true). On the composed-sense interpretation, the denial of (F) would suffice for avoiding Megarian necessitarianism. But that would mean supposing that Aristotle had attributed to the Megarians the thesis (F) whose modal invalidity is patent.

¹⁴Ammonius presents this passage as follows. "Suppose two individuals feign divining with respect to a particular event, attempting to predict of an invalid for example, one that he will recover, the other that he won't. Clearly it is necessary to hold one of the two cases true, the other false. If the one predicting recovery therefore says what is true, it is necessary that the invalid recover (for it has already been postulated that the truth of discourse is followed by the occurrence of the thing in any case); but if the one denying it says what is true, clearly it is impossible that the invalid recover. That is why either it is necessary that the thing happen or it is impossible that the event take place. Therefore contingency is excluded..." (1961, pp. 267-268).

The second stage of the first demonstration (18^a39-18^b9) has to do with an example in contingent matter, the white. One would have expected that, from the explicit position of bivalence (curiously deduced at line 18^b4 as necessary) and by detachment, the conclusion of Megarian necessity would have been drawn, as the rest of the text which describes necessitarianism seems to demand. But these lines, perhaps corrupted, simply apply (C) to the example, rounding off the analysis of the true by that of the false. In addition, they evoke the passage from being to truth, and more precisely from the present being of a state of affairs to its past truth.¹⁵ This is probably an encroachment on the second demonstration.

The second demonstration, from what Ammonius says, proceeds from "more evident and more generally accepted theses".¹⁶ In stating the principle of the retrogradation of truth, it shows why the Megarians accord themselves the right to affirm the antecedent of (C) and of (C_F), i.e. the principle of bivalence. Aristotle gives successively three different forms to the statement of the principle of retrogradation: the first indefinite (R_I, 1.10), the second quantified (R_Q, 1.11), the third Diodorean (R_D, 1.12).

(R_I): If something is the case now it was true at any moment of the past whatsoever that it will be the case.

(R_Q): If something is the case now then it always was true that that thing will be the case.

(R_D): If something is the case now then it always was true that that thing is or will be the case.

Given that Diodorus defines the possible as that which is or will be the case, (R_D) says that the Diodorean possible is necessary.¹⁷ Consequently, if it is granted that that which was true and foreseeable from all eternity cannot not happen, then everything that is in act is necessary. But just as (C_F) could be concluded from (C) in the first demonstration, if it is legitimate to posit (R_D), retrograding from

¹⁵The transfer of time is expressed by the imperfect $\eta\nu$. If there is white or if there is not white it was true that there will be white now or it was true that there will not be white now. Ammonius insists on the imperfect (1961, p. 269): "it is not only according to the time in which the things are and subsist that it is true to say they are as they are, but the prediction with respect to them is true also before the event".

¹⁶1961, pp. 274-275.

¹⁷D. Sedley, 1977, p. 80, gives a span of twelve years (334-322) for the possible influence of Diodorus on Aristotle. This would mean assigning a rather late date to Chapter IX of the *De Interpretatione*. The question of relative chronology here remains at least an open one. (cf. Celluprica, 1977, p. 16).

present to past, it is also legitimate to posit (R_F), retrograding from future to past.

(R_F): Supposing that something will be the case, it always was true that that thing is or will be the case.

For, since (R_D) is supposed valid for anything whatsoever, regardless of the modality of its matter, a refusal to apply retrogradation to the future would amount to the postulation that only futures can be contingent. The future world would then be subject to different laws from the past world. This supposition is all the more ridiculous in that, with the scission between past and future in constant displacement, one and the same event which would not always have been true in the past as long as still future would suddenly come to have always been true in the past at the moment of its actualization. If there can be contingency in the future it must have been that there could be contingency in the past.

The retrogradation thesis, which according to Aristotle is characteristic of Megarian philosophy, implies and expresses the second aspect of the correspondence principle, though without respecting its limits. From the existence of a state of affairs we have the right to conclude the truth of the statement of that existence. But the retrogradation principle extends that truth to time in its entirety, consequently entailing necessitarianism on the sole supposition that what was always true is necessary. It allows positing the principle of bivalence on the basis of the law of excluded middle. In virtue of this latter, it can be said of a future event that it will be the case or it won't. In virtue of retrogradation, it was therefore always true or it was always false that it is or will be the case: hence bivalence. It is this detachment of bivalence, once given retrogradation, that justifies the second demonstration. It is this detachment that allowed the steps from (C_F) to the Megarian conclusion:

(M) All futures are necessary or impossible.

Necessity is communicated then to all becoming (18^b25-19^a6). It is apparent for events foretold by divination. It remains valid, all prophecy aside, for it is truth that depends on existence, not the other way around. The necessitation of being by truth is therefore independent of divination.¹⁸ These consequences are in contradiction with the

¹⁸It is the truth of the thing—in the future as in other times—that allows the statement of the thing, not the other way around. (*Metaphysics*, I-1, 1053^a33). St. Thomas, who accepts the principle for human science, rejects it for Divine science (*Summa Theologiae*, I q14, a 8: "*scientia Dei est causa rerum*"). This position is understandable in terms of creation. St. Thomas avoids necessitarianism in qualifying the science of God.

experience of becoming, with contingency and with the capacity for contraries both as regards human volition and as regards events due to chance or to non-necessitating causes. They lead to inaction and to the fatalistic acceptance of what happens (Idle Argument). Without the contingency of things not always existing in act (19^a6-22) the sublunary world would not be what it is.

6.5 Aristotle's solution (19^a22-19^b4); conditional necessity and exceptions to the principle of bivalence.

Before leaving the Megarians Aristotle concluded that since the consequent of (C_F) is inadmissible the antecedent will have to be rejected. From the denial of necessitarianism the invalidity of bivalence will be concluded (19^a16-22). It is thus natural for Aristotle to set forth his own solution in the form of a contraposition. Since not all futures are necessary, there will not be an already given truth-value corresponding to every proposition about the future. Corresponding to the antecedent of this contrapositive is the distinction between two sorts of necessity (19^a23-27), corresponding to its consequent are the exceptions to the principle of bivalence (19^a27-19^b4).

In distinguishing two sorts of necessity Aristotle abstracts from the difference between past and future.¹⁹ He in no way abstracts from the temporal condition in general.²⁰

"The science of God is the cause of things insofar as the things are in the science. But it was not in the science of God that the things would be from all eternity. Therefore, although the science of God is eternal, it nevertheless does not follow that the creatures are from all eternity".

Divination was not simply a matter of vulgar superstition in antiquity. It was also an axiom of Stoicism, as has been seen. Some Peripatetics like Dicaearchus and Cratippus admitted it (Pauly, 1842, Bd II, p. 1118). Aristotle himself counts melancholy as one of the predispositions for divination by extasy (*Probl.* XXX, 1 and 14, and *ibid.*, p. 1121 and 1123 on dreams). But the Peripatetics rejoice the Cynics and Epicureans in opposing the oracles. Overall, the scaling back of the importance of divination in the *De Interpretatione* fits in with the general rationalistic tendency of Aristotelianism.

¹⁹As Ammonius remarks, 1961, p. 289.

²⁰William of Moerbeke translates Ammonius commenting on Aristotle (p. 290) ὅταν ἢ by *quando est* *quando non est*. The text has τὸ μὲν οὖν εἶναι τὸ δὲ ὅταν ἢ καὶ τὸ μὴ ὅταν μὴ εἶναι ὅταν μὴ ἢ ἀνάγκη. For Aristotle, ὅταν ἢ signifies either an iteration involving indeterminate futures or a general cause. Boethius translates by *quando est* (*quando non est*), Moerbeke by *cum fuerit* and St. Thomas by *dum est*. (According to Hintikka this phrase would mean that the statements are dated, 1964, pp. 472-473, reprinted as Chapter VIII of *Time and Necessity*, 1973; cf. also Boudot, 1973, 4, p. 467). Looking at *Metaphysics*, *ϑ* 3, 1046^b29, which treats of the reality of possibilities and the refutation of the Megarians through the analysis

Three interpretations of conditional necessity are possible. The first two express respectively the necessity of the consequent:

If p takes place then it is necessary that p take place,
and the necessity of the consequence:

It is necessary that if p take place, then p take place.

The second interpretation is a tautology.²¹ Necessitarianism cannot be derived from it, but it fails to translate Aristotle's text. The first, in spite of its hypothetical form, asserts the simple or absolute necessity of the event. According to Aristotle it corresponds to the Megarian theses. The necessity has to do in the consequent with an already temporally determinate proposition. It is therefore *de dicto* and corresponds to the proposition's being always true.

We must therefore come to a third interpretation, positing a conditional connection between an event and its necessity, as in the first case, but subordinating the necessity to the duration of the act of the event. Necessity, having itself become the object of a temporal condition, governs an open statement containing a temporal variable and is itself subject to that same variable: it is *de re*. The corresponding principle can be expressed in the following quantified form:

For any time, t , if p takes place during time t , it is necessary during time t that p take place during time t .

The quantification can be over the future as well as the present or past, but the time of the necessity cannot be different from that of the event described by the statement.

Ammonius' examples²² of simple necessity, 'the sum of the angles of a triangle is equal to two right angles', 'the Heavens are always in motion', 'fire is hot', 'Socrates is mortal', express cases of essential predication in which the predicate's necessarily belonging to the subject is either eternal or is co-extensive with the subject's duration. The statements 'the sun is eclipsed by the moon' and 'Socrates is walking' afford examples of conditional necessity, in which the predicate's be-

of motion, we find the expression $\delta\tau\alpha\upsilon\ \acute{\epsilon}\nu\epsilon\rho\gamma\eta\ \mu\acute{o}\nu\omicron\nu\ \delta\acute{\upsilon}\nu\alpha\sigma\theta\alpha\iota$ ("there is potency only whenever there is act"), where the repetitive applies to any indeterminate time whatsoever. This is the sense I shall retain, thus identifying $\delta\tau\alpha\upsilon\ \eta$ and $\delta\tau\alpha\upsilon\ \acute{\epsilon}\nu\epsilon\rho\gamma\eta$. The term 'being simpliciter' has to do with what is in general, therefore in potency or in act (which is eminently the case for future contingents), whereas the phrase 'whenever it is' indicates that the subject has passed to act.

²¹According to Lukasiewicz, 1957, p. 131, it is in this way that Alexander interprets the *De Interpretatione* and many a Scholastic is in agreement with him (Prior, 1962, p. 211).

²²153(289).

longing to the subject is necessary solely during the actuality of the event: the eclipse or the walk.

As the eclipse example shows (the eclipse was the model of foreseeable and necessary events), conditional necessity alone is no guarantee of contingency. It is a necessary but not a sufficient condition of it. In the world of Diodorus the modalities depend on the logical status of the propositions, that is to say, on the behavior of their truth-values over the course of time. It is not the nature of reality and the temporal relation of predicate to subject then that make for the necessity of a state of affairs. It is the stability of the truth-values of a *dictum*. The retrogradation of truth on the one hand, the eternity and simplicity of necessity on the other, go together. By contrast, the Aristotelian modalities depend on the kinds of predication and on the real relation of predicate to subject. When this relation is accidental it imposes on necessity, conceived of *de re*, a merely temporal validity. Taken by itself, such a conditional necessity, inseparable as it is from its temporal context, would not be able to retrograde. There are however extrinsic considerations having to do with causality that may, as in the case of the eclipse, provide a ground for retrogradation.

We must now turn to the consequence conditional necessity has on the contrapositive of (C_F): abandonment of the principle of bivalence (19^a27-32). For as far as future accidents are concerned, all that can be said about the corresponding state of affairs is that it will be necessary for it to be while it will be or that it will be impossible for it not to be while it will not be. On the other hand—except where an extrinsic retrogradation is invoked—one cannot say of the corresponding state of affairs that it is necessary *simpliciter* or that it is impossible *simpliciter* that it must be. Consequently, it will no longer be possible to maintain either that every statement about the future is true or false.

The conditional necessity of the eclipse degenerates, for the laws of astronomy guarantee the retrogradation of truth, since the respective positions and motions of the sun and moon are given. It is therefore already true that there will be an eclipse at a certain moment. Simple necessity applies to this kind of accident. But if there is nothing actually in the present causes to render the naval-battle ineluctable, that battle will be necessary only if it takes place and while it will be taking place. It is therefore not already true that it will take place. It can be said in the composed sense that it is true that tomorrow a battle will take place or not, for time occurs only vacuously in the disjunction. On the other hand, there is no more ground for distributing truth, than there was for distributing necessity, in saying that it is already

true that there will be a naval-battle tomorrow or that it is already false.²³

²³The example of the naval-battle is taken up in three different propositions. In the first the necessity is composed, in the second distributed and in the third composed again. Aristotle rejects the second while accepting the two others.

Bekker's text at 19^a30 contains the word *ἔσσεσθαι* that Minio Paluello corrects to *γενέσθαι*.

The first choice was that of Boethius who translated both terms by *futurum esse*. But then the third proposition (line 32) would be but a repetition of the first (line 30). If we want to avoid such a shocking redundancy within just a two-line interval we must take Minio Paluello's lesson to heart, recalling the passage at 18^b13. It is thus that William of Moerbeke, in his Latin translation, opposed *fore* and *fieri*. In *Metaphysics*, K, 8, 1065^a14-17, Aristotle discusses the thesis of universal determinism based on the arguments of physics borrowed from the notion of causal regression. All chance, all contingency will be impossible if one admits in the case of an accidental being the regression of causes all the way back to a cause which is itself necessary. But there are two ways of conceiving of that regression, accordingly as it is applied to what is or to what is becoming. "Even if the cause were supposed no longer what is (*ὄν*) but what is becoming (*γιγνόμενον*), the consequences would be the same: everything would happen necessarily; for the eclipse will take place tomorrow if such-and-such happened, and such-and-such happens if some other thing happens in turn, and that other thing if a third thing happens". We could in no way agree with Bonitz, vol. II, 1849, p. 464, and Tricot, II, p. 611, in taking Aristotle in the *Metaphysics* passage to be opposing present or past being to future being. That was St. Thomas' interpretation who commented (p. 540, number 2282): "Someone might object to the argument saying that the cause of future contingents is not already posited as the present and the past but is up to now contingent like the future. For the result is that then everything happens necessarily as was the case before [where we reasoned about what is, not about what is becoming]. For if a certain cause is future it must be future in some determinate time and determinately distinct from the actual present. Suppose it to be tomorrow. If therefore the eclipse, which is itself the cause of certain accidental futures, is a future that will take place tomorrow, and if all that happens happens through some cause, it must be that that eclipse itself, as a future that will take place tomorrow, happen if this will have happened, that is to say, as a result of something pre-existing it..." St. Thomas thinks then that after having reasoned about present or past causes Aristotle goes on to reason about causes to come, thus extending to the future a causality that that extension does not preserve from determinism. But it is clear that Aristotle considers the eclipse phenomenon to be determinate. He is not concerned with the effects that will result from this phenomenon posited as future. He is inquiring whether for this phenomenon, itself posited as a future effect, there exists a causal chain, an already given regression, which is precisely the case. Moreover, the word *γιγνόμενον* refers to the present just as the word *ὄν* does. The opposition of the two cases distinguished by Aristotle has nothing to do with the temporal index of causality, but with the question of whether causality applies to beings or to events (as Ross well notes in his *Commentary*, t. II, p. 322). And if one wonders why Aristotle introduces that opposition when he defends the possibility of contingency both in the context of causality and in that of logic, the example of the deterministic eclipse will have to be taken into account for finding the answer. He was clearly aware that any solution of the difficulties that would limit itself to reserving contingency to *events*, even future ones, abandoning *beings*

When a property belongs essentially to a subject it belongs to it of simple necessity. The statement expressing that belonging is necessarily true, its contradictory impossible. Division of the modality is therefore legitimate. When a property belongs to a subject accidentally it belongs to it according to conditional necessity. Division of the modality in the disjunction of contradictories is then illegitimate, since the attribution of a truth-value to a determinate one of the parts cannot be made in the absence of a condition that, by the very nature of things, can be specified only at the moment of the event. This is precisely what contingency in the universe means.

There is a correspondence between the truth of statements and the reality of states of affairs (19^a32-19^b4). Contingency lies in what is not always existent but not always non-existent either. As for the corresponding contradictory statements, although each of them is necessarily true or false, it must be said that it is not determinately either, whether it be that the chances are equal for both events or again that one of them has a greater probability of occurring than the other. Even in this last case one could not already ($\xi\delta\eta$)²⁴ attribute a definite truth-value to the stronger statement. Ontologically speaking, contingency is proper to beings in potency ($\delta\upsilon\nu\alpha\tau\tilde{\omega}\nu$), susceptible of only a conditional necessity, whereas simple necessity characterizes beings properly speaking ($\delta\upsilon\tau\omega\nu$) which are always in act. Logically speaking, given that all disjunctions of contradictories are true, it is only those disjunctions dealing with essence, that is to say, with beings, that allow the distribution of the true and the false that can then retrograde. Disjunctions dealing with beings in potency admit neither of retrogradation nor of the division of yet indeterminate truth-values.²⁵

to determinism, would fly in the face of astronomical observations. In *Metaphysics*, K, he shows that the regression of causes enchains both beings and events in a same determinism. In the *De Interpretatione* he shows that what allows for a future event's being contingent, such as the naval-battle tomorrow, is the non-distributivity of the modal operator. And it is this same non-distributivity that allows for events in general that happen presently to happen contingently. If one of the factual proofs of contingency is voluntary deliberation, since we deliberate in the present, contingency must not be reserved to futures but it must issue from the very present taken as potency of the future, that is, becoming. Here we come again to the lesson of line 18: "it is evident that not everything either *is* or *happens* as a result of necessity".

²⁴G. H. von Wright, 1979^b, pp. 237-250. The principle contested by Aristotle is therefore that of the retrogradation of truth under the form:

$$(t')t' < t \supset (p_t \equiv T_{t'}p_t).$$

For this principle applied to that of the excluded middle would legitimate the universality of the principle of bivalence.

²⁵By his substantialist conception of duration, Bergson is in opposition to all Greek

6.6 Aristotle's general conception confirms the *De Interpretatione* solution; the difference between Aristotle and Diodorus.

In *De Interpretatione*, Chapter IX, Aristotle distinguishes two sorts of logical necessity. By the same token, he must distinguish two corresponding sorts of causality from the ontological point of view. It is *Metaphysics*, K, Chapter 8, that furnishes this complement of proof and shows, in the limitation of the principle of causality, the reason for rejecting the universality of retrogradation and bivalence.

"Evidently there are no causes and principles of the accidental, of the same kind as there are of the essential; for if there were, everything would be of necessity. If A is when B is, and B is when C is, and if C exists not by chance but of necessity, that also of which C was cause will exist of necessity, down to the last *causatum* as it is called (but this was supposed to be accidental). Therefore all things will be of necessity, and chance and the possibility of a thing's occurring or not occurring are removed entirely from the range of events. And if the cause be supposed not to exist but to be coming to be, the same results will follow, everything will occur of necessity. For tomorrow's eclipse will occur if A occurs, and A if B occurs, and B if C occurs; and in this way if we subtract time from the limited time between now and tomorrow we shall come sometime to the already existing condition. Therefore since this exists, everything after this will occur of necessity, so that all things occur of necessity".²⁶

thinkers and in particular to Aristotle, who identified freedom with choice among many possibles. Nevertheless, Bergson bases his conception on the rejection of the principle of the retrogradation of truth, which for Aristotle is none other than the refusal to attribute a determinate truth-value to future contingents. "Yet the conviction always persists that, even if [a given state of the universe] was not conceived of before coming to be it could have been, and that in this sense it figures as a possible from all eternity in some real or virtual intelligence. Looking deeper into this illusion, it will be seen to depend on the very essence of our understanding. Things and events come about at determinate moments: our judgement recognizing the apparition of the thing or of the event can only come after it; it has therefore a date. But that date is immediately effaced in virtue of that principle, anchored in our intelligence, that all truth is eternal. If the judgement is true at present it must, so it seems to us, always have been true. Little does it matter that it should not yet have been formulated: it posited itself by right before being posited in fact. In this way we attribute a retroactive effect to every true affirmation; or rather, we impress upon it a retrograde movement." (1934. p. 21).

²⁶1065^a6-21 (Ross, Works of Aristotle, vol. VIII). St. Thomas, quoted by Tricot (II, p. 611), winds up his commentary on the eclipse example as follows. "If the eclipse must occur tomorrow, and if everything that occurs results from some cause, it must be that the eclipse itself occurs because of something that pre-exists it, and that thing in turn because of another; and in this way, always because of the

Aristotle responds to this argument by making a distinction. Taking over for the eternal beings the Platonic expression of being *per se*, Aristotle declares them to be determinate and necessary, identifying them with being *qua* true (ὡς ἀληθῆς ὄν). Accidental beings are not necessary but indeterminate (ἀόριστον), and their causes are unordered and infinite.²⁷ But the principle of correspondence transmits the properties of the things and their causes to the statements about them. It is because of the causal indetermination of accidents that it is not already true, from all eternity, to say they are so; and since once *p* is true it is always true, a statement about an accident cannot be true before it happens.

anticipation or removal of causes, we shall take something away from the time there is between the present moment and the future eclipse. Therefore, since this is a finite time, and since everything finite is exhausted when something is taken away from it, everything that will occur will be due to a cause existing now". This is the reasoning used by Lukasiewicz (*op.cit.*, 1967, p. 29). Its principle is the following: "...every fact G occurring at instant *t* has its cause in some fact F occurring at instant *s* earlier than *t*, and at every instant later than *s* and earlier than *t* there occur facts which are both effects of fact F and causes of fact G" (*ibid.*, p.28).

²⁷1065^a21-26. To the argument of necessity Aristotle replies that for contingent beings there is a limit to the search for causes (Ross, 1924, vol. I, 363). This is precisely the point of view defended by Lukasiewicz, which he does moreover by recourse to the analysis of infinity and continuity. Causal chains have inferior limits beyond which it is pure imagination to want to go back (*op.cit.*, pp. 30-33). For Aristotle the limit to the regression of causes is due to the fact that "some condition will arise, if it does arise, not by a process but instantaneously" (Ross, *ibid.*). The Stoics in particular blamed Aristotle for thus having introduced an ἀναίτιος κίνησις that would liken him to Epicurus. Mrs. Frede (pp. 115-117) attributes this objection to a misunderstanding which was quite natural given the obscurity of the terms Aristotle uses. The only difference between 'essential' and 'accidental' causality is that the former involves a unique and immanent teleology whereas the latter involves several distinct teleologies. The accident is therefore necessary once reinserted in its proper teleology (someone hid a treasure in the field) and is properly accidental only when referred to an extraneous teleology (a man plowing the field finds the treasure). Aristotle's doctrine is therefore not unlike that of Cournot when he defines chance as the meeting of two independent causal series. That is a classic and plausible interpretation. The Stoics in fact will insist on the universal connection of the causal chain, the *fatum*. In this sense, the limit in the regression of causes as conceived by Lukasiewicz would be Epicurean, not Aristotelian, in spite of the mention made of instantaneous and ungenerated conditions. The instantaneous character of the accidental event would be due to the instantaneity of the meeting of the two independent series, each of which by itself would be indefinitely generated. There is still, besides, a fundamental difference between Aristotle and Cournot. For Cournot chance results from the meeting of two series determined only by efficient causality, whereas for Aristotle it results from the meeting of two teleological series.

In declaring in *De Interpretatione*, IX that the truth-value of propositions about future contingents is not already assigned,²⁸ Aristotle is not then introducing a thesis that is to be without echo in the rest of his work. It remains to be shown that, as a consequence of his adoption of conditional necessity, this thesis is indispensable if we hope to preserve what is specifically Aristotelian as over and against Diodorean necessitarianism.

Considered by itself, the principle of conditional necessity brings the two doctrines dangerously close to one another. Although Aristotle explicitly mentions possibles that will not be realized,²⁹ the principle of conditional necessity would seem to exclude them *a priori*. Since it is supposed that the act contrary to a given potentiality, the inexistence of something supposed contingent for example, lasts indefi-

²⁸Mrs. Kneale, 1962, p. 51, thinks that it is the right we assume to speak of a dated truth that vitiates the whole of *De Interpretatione*, IX. She says that Aristotle's definition of truth "gives the most important fact about the predicate 'true', namely that, if 'P' is any propositional sign, the proposition that $\neg P$ and the proposition that it is true that $\neg P$ mutually entail one another. This holds also when the propositional sign is a sentence in the future tense. For it is true that there will be a naval battle tomorrow if, and only if, there will be a naval battle tomorrow. By introducing the phrase 'it is true that' we make no assumption about determinism which is not made by use of the simple sentence in the future tense. We mislead ourselves, however, when we speak, as Aristotle does, of its being true *now* that there will be a naval battle tomorrow, for we thereby induce ourselves to suppose that *this* will not be true tomorrow evening, when the battle is over, but something else will, i.e. 'There has been a naval battle today'. Two different sentences are plainly involved here, but they both express the same proposition in the sense that to convict any person who uttered either of error would also be to convict of error any person who uttered the other at the appropriate time." Aristotle's error here would come down, therefore, to attributing the predicates 'true' and 'false' to sentences, whose tenses vary, instead of reserving them for *propositions*, whose tense does not vary since the propositional content bears no relation to the moment of utterance. But the question remains whether a *proposition* about the future has a truth-value before the event, even when one has decided with Mrs Kneale to eliminate sentences in favor of propositions.

²⁹As pointed out by Chevalier, 1915, p. 273, the third proposition of the Master Argument, *δυνατὸν εἶναι ὁ οὐτ' ἔστιν ἀληθὲς οὐτ' ἔσται*, takes over the letter of *Metaphysics*, 1047^b8-9, οὐδὲν κωλύει δυνατόν τι ὄν εἶναι ἢ γενέσθαι μὴ εἶναι μὴ ἔσεσθαι (Cf. Hamelin, 1978, p. 60). St. Thomas comments on the *De Caelo* in the following way. "It is therefore clear that everything destructible will be destroyed one day. And the same holds if something is generable in its nature, it is necessary that it should have been generated. *This is not to be understood, however, as if everything generable was generated one day. For many things can come about which never do. But this cannot be the case: namely, that something already existing in its nature (jam in sua natura existens) be generable, and nevertheless should not have been generated but should have pre-existed eternally*". (St. Thomas, 1952, ad 283^b8, p. 138).

nately, throughout all this time it is necessary for that act to be,³⁰ and consequently the potentiality in question is reduced to an impossibility. Mustn't we conclude then with Diodorus that the possible is what is or will be?³¹ The repeated oppositions concerning modality, accident, non-being, motion and even language,³² between the ontology

³⁰*De Caelo*, I, 281^b1, 281^b15-25, 282^a22-25, 283^a24-29. See Cherniss, 1962, p. 416, on these passages.

³¹At *De Caelo*, I, 283^a27, Aristotle writes that "the destructible is at some time destroyed".

J. Hintikka, 1973, pp. 93-113, systematically assimilates the Aristotelian and the Diodorean conceptions of modality. The *Metaphysics* passage (Θ, 4, 1047^b3-14) that he alleges in favor of his thesis enjoins us only not to consider as semantically possible that which is in principle prevented from passing to actuality, whether by an already given material or efficient cause or by a logical principle. It in no way says that a possible will have to be, but simply that a possible that certainly will not be is not an authentic possible.

³²According to Aristotle "potency and actuality extend beyond the cases that involve a reference only to motion" (*Metaphysics*, Θ, 1, 1046^a1-2), and they therefore go beyond the range of the corresponding notions according to Diodorus. Time can fall within the scope of an Aristotelian modality. Moreover, infinity, the void, and matter are eternally in potency and never pass to actuality. When Diodorus criticizes the possibility of motion regarded as a passage (Fr. 128 in Döring, 1972, p. 37: "...so something living dies neither in the time in which it lives nor in the time in which it does not live, therefore it never dies") he is in fact denying generation. He is to be counted therefore among those who, through awkwardness, imitate the Ancients and according to whom "no being is generated or destroyed, because whatever is generated must necessarily be so either from being or from non-being, two equally impossible solutions; indeed being cannot be generated for it existed already, and nothing can be generated from non-being for there must be something to underlie it" (*Physics*, I, 8, 191^a27-32). But how does Aristotle reply to those who simply deny motion, like the ancient Megarians, or to those like Diodorus who reduce it to a cinematographic succession of discontinuous states? He invokes the distinction between essence and accident (*Physics*, I, 8, 191^b12-17; *Metaphysics*, Λ, 1069^b14-34). Accidental non-being, that is to say, privation, gives rise to generation. But all material beings, as such, suffer privation in virtue of their contrariety, and the missing contrary has a sort of ghostly existence that Aristotle calls potentiality and that, because of its incompleteness, produces an uneasiness calling for change. As regards quality for example, potentiality is the ghostly presence of the contrary (white) in something having a given quality (black). Of course when the potentiality passes to actuality, if ever it does, it will be a development in present or future time just as much for Aristotle as for Diodorus. But for Aristotle it is there at the very core of the thing before becoming actuality. In the same way, motion for Aristotle is the fulfillment of what is in potentiality in so far as it is in potentiality. The act of motion, which would be expressed grammatically by the present progressive tense (this is moving), could in no way therefore be confused with a succession of immobilities, that is to say, with the positions occupied successively by the mobile in the course of discontinuous time. The 'now', says Aristotle, is a limit, not a part of time (*Physics*, IV, 10, 218^a6) and motion cannot be reduced to a correlation between the points occupied by the mobile and the 'nows' dividing time.

of Aristotle and that of Diodorus, would be reduced then to so many delusions, completely in Diodorus' favor.

It is precisely this closeness due to the principle of conditional necessity, that suggested the Master Argument to Diodorus on the basis of a passage of the *De Caelo*. To extricate Aristotle from such a compromising companionship it will suffice to show that the exceptions to the principle of bivalence allow him to answer the Master Argument without invalidating the *De Caelo* passage for all that.

The *De Caelo* demonstrates that nothing created, that is to say nothing having the possibility of being destroyed, can avoid destruction. For suppose a world of which it would be true that it would forever be preserved. By the principle of conditional necessity it would be necessary that it should be preserved forever. It would therefore forever be impossible that it should be destroyed, contrary to the consequence of the premise by which it was posited that it had been created. Why the Master Argument, which is nevertheless modeled on the *De Caelo*, is not valid according to Aristotle can readily be seen through a comparison of the premises. The *De Caelo* reasons about the eternal or destructible existence of the world and therefore about an essential property that it is either true or false to attribute to the subject. The Master Argument's third premise, on the other hand, says that what

For the Megarians, Diodorus included, there is no distinction to be made between non-being as such and accidental non-being. There is consequently no distinction that can mark off an essential disposition predicate that, once supposed the thing exists, will inevitably pass to actuality within a given limit of time, from an accidental disposition predicate that could, once the thing is supposed to exist, not pass to actuality. The only difference between essence and accident or between possibles *ad unum* and possibles *ad utraque* that Diodorus' philosophy allows us to retain is that regarding the irrevocability of a unique event that takes place in time (growing old, dying) and the succession of contrary states (to be sitting, standing), but which also inevitably pass to actuality in time.

Then again, either motion as succession is an illusion or in any case, understood as the fulfillment of what is in potentiality in so far as it is in potentiality, understood that is as the passage of becoming, it melts away into chimera: it is only to the trace that it leaves in space and time that any reality can be accorded.

This opposition between Aristotle and Diodorus is borne out further by their theories of language. Euclid, followed by Diodorus on this point (Fr. 30 in Döring, p. 10) rejected all reasoning by analogy, and Diodorus himself admitted of no ambiguity as regards words (Fr. 112 in Döring, p. 31). For Aristotle, by contrast, it is one of the essential mistakes of the Eleatics, so like the Megarians, to take 'being' in an absolute sense when its uses are numerous (*Physics*, I, 3, 186^a24-25) and it is only by analogy that matter can be known, since it is potentiality that cannot become actualized (*Physics*, I, 7, 191^a7, just as for Plato it was knowable only by a bastard reasoning). Thus Diodorus maintains the synonymy of 'being' and 'potentiality', Aristotle their ambiguity. Diodorus reduces potentiality to occurrence in present or future time, Aristotle sees it as the simultaneous coexistence of contraries.

neither is true now nor will be so is possible. But such a potentiality, though contradictory for Diodorus, would not be so for Aristotle on the supposition (the only interesting one) that we are reasoning about an accident, unless there were already existent the causal chain allowing us to say that the statement about the future is already false now and will remain so. But it is precisely this present assignation of truth-value that is missing when the futures are in contingent matter, and it is this absence that invalidates the Master Argument.

6.7 First interpretative hypothesis: More than two truth-values.

It is therefore in limiting the validity of the principle of bivalence that Aristotle resolves the aporia of the Master Argument. We must try now to determine whether his solution is logically satisfying and whether the intuition it is based on can be legitimately systematized. With this inquiry we leave the realm of history and fact for that of interpretation and hypothesis.

There are three hypotheses that come rather naturally to mind. The first is that Aristotle's system demands the construction of a logic of more than two truth-values. The second requires modifying the standard definition of truth. The third suggests the introduction of probabilities.

Leibniz, in spite of his overall sympathy for Aristotle and his conciliatory approach in interpreting the texts, often criticized calling the principle of bivalence into question. Such a challenge fails to recognize "the very nature of truth which is determinate in enunciations we can form about future events as it is in all other enunciations, since the enunciation must always be true or false in itself, although we don't always know which it is".³³ Taking a tip from this final concessive clause, we can specify the nature of the confusion that would have crept into Aristotle's conception: he would have taken the uncertain for the indeterminate³⁴ and fallaciously derived an objective property

³³Leibniz, Gerhardt, 1875-1890, VI, p. 30; Jalabert, 1962, p. 30.

³⁴"Philosophers agree today that the truth of future contingents is determinate, that is to say that future contingents are future, or that they will be, they will come to pass, for it is just as sure that the future will be as it is sure that the past has been. It was already true a hundred years ago that I would write today, as it will be true after a hundred years that I have written. So the contingent is no less contingent for being future; and the *determination*, which would be called *certainty* if it were known, is not incompatible with contingency. The *certain* and the *determinate* are often taken for one and the same thing since a determinate truth is one that is susceptible of being known, so that determination might be said to be an objective certainty" (pp. 129-130). Like the Megarians, Leibniz accepts

regarding the order of things from a subjective property regarding our knowledge. Cavalier in questions of freedom, Leibniz takes aim here at Epicurus just as much as at Aristotle.³⁵ His explanation applies better in fact to the former, who contested the excluded middle, than to the latter, who simply rejected retrogradation.

What about the logical question? Wouldn't it be only out of extreme desperation that one would come to imagine that the truth of 'p or q' is not a sufficient condition for the truth of 'p' or the truth of 'q'?³⁶ But the difficulty is perhaps that the reconciliation of truth and freedom requires more than two truth-values.³⁷ It is this consideration that prompted Lukasiewicz to propose first a system of three truth-values, then another of four. Neither of them however can be regarded as Aristotelian. The first makes the possibility of p equivalent to the assertion that if not-p then p. This corresponds to none of the senses of the word 'possible' according to the Stagirite and, above all, it invalidates the principle of the excluded middle which is contrary to the letter of both the *De Interpretatione* and the *Metaphysics*.³⁸ The

the retrogradation of the truth of future statements, which is nothing other than the determination of their truth.

³⁵Leibniz, Gerhardt, VI, p. 211; Jalabert, p. 222; the text is quoted pp. 167-168.

³⁶Quine, 1966, p. 21; Hamelin, 1978, p. 35.

³⁷Von Fritz, 1972, I, pp. 241-250.

"Quine's criticism seems right—and very strongly so at first—only because the decisive factor, namely the relation of the statements to the states of affairs stated, is left out of the picture. The proposition 'it is true that p or q' holds without limitation for all events, past and future, when 'p' and 'q' stand for the contradictory opposition of 'be' and 'not-be'. At each instant a determinate event can but occur or not occur, or a (transitory) state can but obtain or not obtain: *Tertium non datur*. The principle of the excluded middle (*ἀξίωμα ἀντιφάσεως*) has in that measure an absolute validity, even though the corresponding state of affairs is realized only when the event in question or the temporary state takes place or does not take place. The proposition 'it is true that p or it is true that q' follows from the proposition 'it is true that p or q' in the following sense: with the occurrence or the non-occurrence of the event in question, necessarily either the proposition 'it is true that p' or the proposition 'it is true that q' is verified (*wahr wird*). In this sense the first proposition really is, as Quine supposes, the 'sufficient condition' for the second, but not in the sense in which the second must be valid already before the realization of the state of affairs corresponding to p or to q, since it is just this that determines the appropriateness of p or q."

Though justified in showing the possible inadequacy of one or another given logical formalization, this argument is not justified in abolishing all symbolism, as the author himself points out in adopting a logic of more than two truth-values.

³⁸Lukasiewicz, 1967, pp. 40-65.

Three axioms are formulated:

1. $\sim Mp \supset \sim p$ (axiom of necessity)
2. $\sim p \supset \sim Mp$ (axiom of conditional necessity)
3. $(\exists p)(Mp \cdot M \sim p)$ (axiom of contingency)

second system preserves the excluded middle, but there is no echo in Aristotle's work³⁹ of the twin definitions of the possible in which it

Lukasiewicz gave an altogether different interpretation of the axiom of conditional necessity in 1950 and 1955 (*Aristotle's Syllogistic*, Oxford, Clarendon Press) as signifying either $L(p \supset p)$, in which case no difficulty is raised, or the rule $(x)\alpha \supset L\alpha$, which, without further restriction, leads to paradoxes (1955, pp. 153-154). If it is posited then that 1) the modal operators are truth-functores, 2) there are two and only two truth-values, the true and the false, it can be shown that a) the axiom of conditional necessity leads to a collapse of the modalities, b) the axiom of contingency leads to the admission that everything is possible, c) there is incompatibility between the last two axioms whose simultaneous assertion is inconsistent. Still taking the modal operators to be truth-functores, if a third truth-value is admitted so that a proposition can be neither true nor false, it can be shown with a fairly intuitive assignment of matrices for the propositional connectives that a) the axiom of conditional necessity no longer leads to a collapse of the modalities, b) the axiom of contingency no longer leads to the admission that everything is possible, c) the three 'Aristotelian' axioms then form a coherent system. The only truth-functional definition of the possible that can be given in such a system is one leading to the equivalence of the possibility of p to the assertion that if not- p , then p (Tarski, in Lukasiewicz, 1967, p. 55: $Mp \equiv (\sim p \supset p)$; Prior, 1962, pp. 246-247). In the system in question (p. 59) disjunction is defined in terms of implication: $p \vee q \equiv_{Df} (p \supset q) \supset q$. For the truth-value $1/2$ (neither true nor false) the value of the principle of excluded middle differs from true (i.e. 1).

$$(p \vee \sim p) \equiv (p \supset \sim p) \supset \sim p \equiv \left(\frac{1}{2} \supset \frac{1}{2}\right) \supset \frac{1}{2} = 1 \supset \frac{1}{2} = \frac{1}{2}.$$

As Prior remarks (1962, p. 244), "At this point there is therefore a divergence between Lukasiewicz's system and that suggested in the *De Interpretatione*; for Aristotle held that even when neither 'There will be a sea-battle tomorrow' nor its negation is definitely true, the alternation 'Either there will be a sea-battle or there will not' is definitely true. ... It is not because its components are both neuter, but because they are contradictories, that Aristotle affirms this disjunction".

(On a similar difficulty, but regarding conjunction rather than disjunction, cf. Prior, *op. cit.*, 1962, pp. 246-247).

³⁹In his *Aristotle's Syllogistic* (*op. cit.*, 1951 (1957)) Lukasiewicz surmises that Aristotle rejected the axiom of conditional necessity (1957, p. 136) on the interpretation already proposed but in fact gave it another form through his acceptance of true individual apodictic propositions (pp. 153-154), whereas acceptance of the axiom of contingency, i.e., of true contingent propositions, always leads to the possibility or any proposition whatsoever (pp. 154-157).

Lukasiewicz then constructs a four-valued system in which there are no true apodictic propositions. As a result, the distinction between truths of reason and truths of fact simply vanishes. But within this system it is also possible to formulate twin definitions of possibility. Using the two definitions simultaneously makes it possible to establish, without paradox now, the existence of contingent propositions (pp. 158-180, 205-208). Lukasiewicz likens the two possibilities ' Mp ' and ' Wp ' (p. 173) to a pair of identical twins that cannot be distinguished if met with separately but can be distinguished when seen together. In other words, there are sequences such as

MWp and WMp

results. A more general objection still is that Aristotle is nowhere seen to evoke more than two truth-values.

6.8 Second interpretative hypothesis: propositions without a determinate truth-value.

We must then, without leaving the realm of two-valued logic, construct a system that validates conditional necessity and excluded middle while not validating the principle of bivalence.

This is an impossible task so long as one admits an intuitive criterion of truth maintaining the equivalence of a statement's assertion and the assertion of the statement's truth (the Tarski criterion).⁴⁰ But in the analysis he gives of *The Concept of Truth in Formalized Languages*, Tarski cites the example of Aristotle as one of those to have given an intuitively acceptable semantic definition of truth for the statements of everyday language. It will be seen that this allegation is not well-founded.⁴¹

According to Aristotle, "to say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true; so that he who says of anything that it is, or that it is not, will say either what is true or what is false, whereas [the partisan of an intermediate between contradictories] says neither of what is or of what is not either that it is not or that it is".⁴² So goes the first

which are valid without our being able to substitute the twins, for the formulas

$$MMp \quad \text{and} \quad WWp$$

are invalid. Such a system has its proper paradoxes though (Prior, 1957, pp. 127-130; Hughes and Cresswell, *op.cit.*, 1972, pp. 307-310).

If we keep the definition of disjunction: $p \vee q \equiv (p \supset q) \supset q$ and we choose the value $p = 2, 3$, in table M9 (Lukasiewicz, 1957, p. 168), the following facts will be noticed:

$$p \vee \sim p = (2 \supset 3) \supset 3 = 3 \supset 3 = 1$$

$$p \vee \sim p = (3 \supset 2) \supset 2 = 2 \supset 2 = 1.$$

The principle of the excluded middle therefore, verified for the values $p = 0, 1$, is verified for the non-classical values as well.

⁴⁰ "... assume that 'A or not A' is logically true, but that 'A' is truth-valueless. If 'A or not A' is logically true, it is true. By the Tarski truth-criterion, therefore, we have A or not A. Further, the Tarski truth-criterion, and the convention that "A is false" means "(not -A) is true" entails the conditional: if A or not-A, then 'A' is true or 'A' is false. But we already have A or not-A. Hence, the consequence that 'A' is true or 'A' is false follows immediately by *Modus Ponens*, a consequence which contradicts the assumption that 'A' is truth-valueless". (Lambert, 1969, p. 96; also Haack, 1974, p. 68).

⁴¹Tarski, A., 1956, p. 155.

⁴²*Metaphysics*, Γ, 7, 1011^b26-29; I follow Tricot, especially for the bracketed addition, I, p. 235.

of the arguments Aristotle brings to bear against those who challenge the universal validity of the excluded middle and postulate an intermediate between contradictory statements. This argument is "evident for anyone who defines what the true and the false are" (1011^b25). Since this same appeal to evidence is also made in Aristotle's third argument (1012^a2) we shall join it to the first: "Again, discursive thought affirms or denies every object of discursive or intuitive thought; this much is evident, by definition, whenever it says what is true or is false. When it connects in a certain way, either through affirmation or negation, it says what is true; and when it connects in some other way, what it says is false".⁴³

The entire set of these arguments aims at reducing the adversaries of the principle of excluded middle to the absurd.⁴⁴ It proceeds as follows:

- (1) Every proposition is the connection or union of a subject (*S*) and a predicate (*P*) (*De Interpretatione*, 5, 17^a12).
- (2) Such a connection, therefore, can have any of four possible forms: *S* is *P*, non-*S* is *P*, *S* is non-*P*, non-*S* is non-*P*.
- (3) A proposition is true if it connects *S* and *P* as they are connected in reality; it is false if it connects them otherwise (*Metaphysics*, Θ, 10, 1051^b25).
- (4) The definition of true and false is most easily applied when subject and predicate designate the same thing. The four forms enumerated in (2) then become: that which is is, that which is

⁴³ *Metaphysics*, Γ, 7, 1012^a2-5.

⁴⁴ That this is a question-begging reduction can be seen by referring to the truth-value matrix for negation by Lukasiewicz in 1930 and to the three-valued logic of Bochvar, in which the excluded middle is invalidated for the *internal* connectives ($\sim i = i; i \vee \sim i = i \vee i = i$) and the assertion of the principle of the excluded middle ($a(i) = f$) is false. (S. Haack, 1974, p. 170).

The intuitionist says that "τὸ μεταξὺ ἀντιφάσεως" is. Hence he says neither of that which is nor of that which is not that it is or that it is not, which is the condition for saying truly or falsely. His statement is therefore neither true nor false, which is absurd. Ross (1924, I, pp. 284-285) justly remarks "(1) that Aristotle does not assume merely that to say of what is that it is not or of what is not that it is false, and that to say of what is that it is or of what is not that it is not is true, but these are the definitions of falsity and truth, i.e. are convertible propositions. It is only on this assumption that it follows that the opponent, who maintains the existence of what neither is nor is not, is saying what is neither true nor false. (2) That the opponent is assumed to admit (a) the correctness of the definition of truth and falsity, and (b) that every judgement must be either true or false. Thus Aristotle is inferring the metaphysical form of the law of excluded middle—that there is no objective intermediate between contradictories—from the logical form. The argument thus has value only *ad hominem*. But of this Aristotle is well aware; he knows that first principles cannot be demonstrated".

not is, that which is is not, that which is not is not. Applying (3) it is evident that the two extreme forms are true, whereas the two middle ones are false (1011^b25-27).

- (5) Therefore, to say that what is is, or that what is is not, is to say what is true or what is false and there is no third (1011^b27-28). To say that *S* is *P* is true if *S* is *P* and false if non-*S* is *P*, and to say that non-*S* is *P* is true if non-*S* is *P* and false if *S* is *P*, and there is no third possibility.
- (6) On the other hand, to say that the intermediate between *S* and non-*S* either is or is not *P*, is not to say what is true or what is false (1011^b28-29).
- (7) Therefore, the negation of the excluded middle, taken as disjunction of the subject (intermediate between what is and what is not), is neither true nor false.
- (8) To say that *S* is *P* is true if *S* is *P*, and false if *S* is not-*P*; and to say that *S* is not-*P* is true if *S* is not-*P*, and false if *S* is *P*, in virtue of (3)(1012^a4-5).
- (9) Therefore, to say that *S* is *P* or that *S* is not-*P* is to say what is true or what is false, and there is no third possibility (1012^a2-4).
- (10) On the other hand, to say that *S* is an intermediate between *P* and not-*P* is not to say what is true or what is false.
- (11) Therefore, the negation of the excluded middle taken as disjunction of the predicate (to be intermediate between what is and what is not) is neither true nor false.
- (12) The negation of the excluded middle, therefore, whether taken as disjunction of the subject (first argument) or as disjunction of the predicate (third argument), is neither true nor false.

This argument uses the definition of truth through its most evident instance (4) or through its consequences (5) and (8). It does not contain it explicitly and we had to import it from another passage of the *Metaphysics* (3). The text where Tarski thinks he sees the Aristotelian definition of truth is in fact not one. Just what relation then does the Aristotelian definition have to the Tarski criterion? According to Aristotle, "truth and falsity, insofar as things are concerned, depend on their combination or separation, so that he who thinks that what is separated in fact is separated, or that what is combined in fact is combined, thinks truly, while he who thinks what is-contrary to the nature of the things is in error."⁴⁵ But that definition differs from Tarski's criterion, not only by the subject to which true and false are

⁴⁵ *Metaphysics*, Θ, 10, 1051^b2-5.

attributed—the statement for Tarski, thought and its affections for Aristotle⁴⁶—but above all, by the range of that attribution and also by the relation posited between the thing and that to which true and false are attributed.

Aristotle attributes truth and falsity to composites, either contingent (the wood is white) or necessary (the diagonal is incommensurable), but also to simple natures as well. Whereas the composites are objects of discursive knowledge, the simples are known through intuition. Aristotelian correspondence, which is between two structures for composites, reduces to the simple presence of the thing for truth, to its absence for falsity, in the case of simple natures. In the last case error is impossible: here you either see or you don't see.⁴⁷

Tarski's criterion leaves no room for this difference which Aristotle holds to be fundamental once the truth of logical principles is at issue. But the shortcoming of Tarski's criterion can already be shown by insisting on the priority of the thing with respect to the very existence of an intuitive thought. Where there is a question of simple natures, Aristotle specifies, "if the object is existent it exists in a particular way, and if it does not exist in this way it does not exist at all; and truth means thinking these objects, and falsity does not exist, nor error, but only ignorance".⁴⁸ Where the object is inexistent, there is no thought, therefore no falsity. So, contrary to Tarski's criterion, there is a truth for simple natures that no falsity can come to deny.

In *De Interpretatione*, Chapter nine, the same priority of the object with respect to thought and truth has the consequence that, as future contingents are still indeterminate as to the object, it is in no wise plausible that the corresponding affirmations and negations have a determinate truth-value before the occurrence of the event.

The formal expression of such a conception requires a system that 1) preserves all the logical tautologies, 2) does not, however, make an already-determinate truth assignment to every statement, 3) does not introduce a third truth-value. Van Fraassen's languages of supervaluations meet these three conditions. To a molecular proposition of which some elements lack a determinate truth-value, a supervaluation will assign what would be the ordinary valuation in cases where that determination is unique (as with tautologies and contradictions) and no determinate truth-value where that is not the case. For example, a molecular proposition ' $p \vee q$ ', which is sometimes true and some-

⁴⁶ *Metaphysics*, E, 4, 1027^b25-1028^a 2.

⁴⁷ *Metaphysics*, Θ, 10, 1051^b30-1052^a 11.

⁴⁸ *Metaphysics*, Θ, 10, 1051^b35-1052^a 2.

times false on a classical evaluation, will not receive a determinate truth-value on a supervaluation. On the other hand, the proposition ' $p \vee \sim p$ ' and ' $p \cdot \sim p$ ' will receive respectively the supervaluations 'true' and 'false', which they are assigned universally on the ordinary evaluations.⁴⁹ Some inferences considered valid in classical logic cease to be so in this system. Such is the case for the 'disjunctive dilemma' (if $A \vdash C$ and $B \vdash C$, then $A \vee B \vdash C$). It is right to contest such inferences when they involve formulas lacking determinate truth-values, but once that indetermination is removed the classical rules are restored.⁵⁰ Van Fraassen's system fits into the program of free logics, *i.e.* logics without existential presuppositions, as formulated by K. Lambert. Adopting it to Aristotle would mean effectively fixing the status of fictive objects, so important in the poetics and the rhetoric, and treating singular statements with non-referential subjects as truth-valueless.⁵¹ In that respect, they would be assimilated to future contingents. On the other hand, in freeing the whole of logic from existential presuppositions we would surely come into conflict with the assumption of the non-empty universe of discourse which is at the very base of the logic of terms and therefore of the entire Aristotelian syllogistic.

6.9 Third interpretative hypothesis: probability.

Chapter IX of the *De Interpretatione* and Chapters 4 and 6 of book II of the *Physics* enable us to separate the sets of contingent events—those that are neither necessary nor impossible—into three classes: 1/ those with a low frequency (*ut in paucioribus*) and involving chance or fortune, chance and fortune resulting from the meeting of two independent final causal series, the first not accompanied by the representation of the end, the second accompanied by it; 2/ those that have as much chance of coming about as not (*ad utrumlibet*) and which depend on deliberate choice; 3/ those with a high frequency (*ut in pluribus*) which represent the unimpeded effect of natural causality, though nevertheless in the sublunary world it doesn't always follow its course.⁵²

⁴⁹Bas C. van Fraassen, in K. Lambert, 1969, pp. 67-91. Mr. Heinzmann points out that it would be possible to imagine another model that would preserve the syntactic validity of the excluded middle, but only on a non-standard interpretation identifying truth with demonstrability. While such a model would satisfy Aristotle's formal requisites it would be in flagrant violation of the objective conception of truth that he proposed.

⁵⁰Haack, *op.cit.*, 1968, p. 216.

⁵¹Lambert, *op.cit.*, 1969

⁵²"We say that everything either is always and of necessity, or is what happens for the most part, or is neither what happens for the most part nor what is always and of necessity, but what happens merely as it chances. That it might be cold in the

A representation such as the above is completely at odds with the idea of probability, since it prevents bringing together under a single heading the set of events whose foreseeability is neither null nor absolute. It holds every law to be necessary, since deploying the finality of nature. It is by principle then that the exception escapes the law. Failing in finality, it could in no way be counted among the cases falling under a law.⁵³ That, according to Aristotle, is the reason why natural selection can in no way simulate finality.⁵⁴

In spite of all this, there is one circumstance that should have, and in fact did, provoke an inquiry on the part of Aristotle and his school into just what there is in common between chance, fortune, freedom, and the approximative laws of the sublunary world. The privative

day-days, for example, is something that happens neither always and of necessity, nor for the most part; it can happen only sometimes. The accidental, then, is what occurs, but not always nor of necessity, nor for the most part" (1064^b32-1065^a3). Chance and fortune are defined in the *Physics*, II, 6, 197^b18-21. "In the realm of things that take place in the absolute sense for the sake of some end, when they come about without having in view the result and having their final cause outside of it, then we speak of the effects of chance; and for the effects of chance of the kind susceptible of being chosen and touching beings capable of choice, we speak of the effects of fortune". The examples given are that of a horse coming to a certain place and finding safety there (197^b14-15), and the unexpected recovery of a debt through meeting the debtor (196^b34-197^a4).

⁵³Science cannot then content itself with assigning the probability of an event in a series: it must explain its occurrence in a determinate way. "For all science deals either with what is always or with what is for the most part. How else is one to learn or to teach another? The thing must be determined as happening always or for the most part. It can thus be said that honey-water is good for one having a fever, for the most part; but one cannot give an account of exceptional cases, say at what moment honey-water will not produce that effect, at the new moon for example: for even what happens at the new moon happens either always or for the most part, while the accidental is outside the always and the most part" (*Metaphysics*, E, 2, 1027^a20-26; Δ, 30, 1025^a14-30). It can be seen here why Cournot writes: "It follows from all this that Aristotle has an inkling, though in a most confused way, of the applications of the doctrine of chances and of probabilities, and of the future science of statistics, though not knowing whether to place it in 'science' or 'opinion'" (1975, p. 450). It is evident then that neither Aristotle nor Aristotelianism developed a clear and distinct concept of probability and that they weren't even aware of the questions that arise concerning its measure. It is nevertheless so, as Cournot says, that the attention they paid to the imperfect determinations of the sensible world and to accidents must—as in Chapter IX of the *De Interpretatione*—have given rise to themes that enter into conflict with logic if there is no room made in the system for a primitive theory of probabilities.

⁵⁴"And of course it is the beings in which everything was produced as if it had been for a purpose that have been preserved, being organized in a fitting way; the others, like the man-faced oxen in the theory of Empedocles, have perished and continue to perish. This, among other ways, is how those who raise this difficulty argue, but it is impossible that this should be the case" (*Physics*, II, 8, 198^b29-32).

concepts that the determinist doctrines of Diodorus and the Stoics denied or assimilated to fate⁵⁵ took on, at the hands of the adversaries and then consequently of the defenders of Aristotelianism, a unity that was at first awkward but quickly assumed some consistency. Polemical pressure made Aristotle and his disciples more and more attentive to the individual, to opinion, to contingency: in a word, to probability.

It is not an unfounded fiction then to suppose a disciple of the Lyceum to be wondering about the probable in itself. With a view to harmonizing his principles, it will not have taken him long to posit the probable in nature, and not in our imperfect knowledge.⁵⁶ His attitude can be found in taking the opposite view of the Stoics. In speaking of them, Alexander says "whatever happens, and no matter how diversified the causes, our adversaries still think it equally true of all that it is impossible that under the same circumstances, either with respect to the cause or with respect to what follows from it, that things should sometimes not happen in a certain way and sometimes happen in that same way".⁵⁷ Our philosopher will be committed therefore to recognizing that, under the same circumstances with respect to the causes, what follows from the causes will happen at one time in one way, at another in another, where the set of these 'ways' or eventualities constitutes an exhaustive set of contraries and with the probable resulting from the contrariety among these eventualities.⁵⁸ There will be 'identical circumstances' when the total combination of eventualities is the same. Without denying the univocal and entirely determinate action of a 'causal disposition' he attributes to essence, the Peripatetic will also admit that the probable, an accidental cause as regards facts stemming from an irrational causality, an essential cause as regards those stemming from free choice, ends the supposedly infinite⁵⁹ causal chain in the fabric of a supposedly universal determinism. In this latter case, that of free causality, he will bring to bear possible causes that will not all become real, as the effect is reduced to the choice (even only aleatory at that) of a single case.

The Ancients do not seem to have distinguished the concept of event from that of eventuality, an event being defined as a determinate subset of the set of eventualities. Having defined a set of eventualities X, they reason about only a part of that set, a part corresponding to

⁵⁵ Alexander of Aphrodisias, in Nourrisson, 1870, X, p. 211.

⁵⁶ Alexander of Aphrodisias, *op. cit.*, X, p. 211.

⁵⁷ *Ibid.*, XXII, pp. 262-263.

⁵⁸ *Ibid.*, IX, pp. 208-209; XII, p. 223.

⁵⁹ *Ibid.*, XXII, pp. 260-261.

a natural event, and the complement of that part.⁶⁰ It would suffice to generalize this process to meet the first requisite of the calculus of probabilities. Instead of a natural part of X , extracted from the set of its subsets, let us consider the set of subsets of X in its entirety. Let us construct on the set X of eventualities what is called a field of sets F : *i.e.* a non empty family of sets (in this case events) closed under complementarity and logical sum.

A field of sets F has the following two properties. a/ Since when it contains event A it also contains its complement non- A and their logical sum, it contains the entire set X . By definition this latter set contained all that will occur no matter what—it corresponds, so to speak, to the total cause. b/ Since F contains the sum of A and non- A it contains their logical product as well: *i.e.* the complement of that sum.⁶¹ This product is the empty set and corresponds, so to speak, to the absolute absence of cause. In other words, if the contingent proposition ‘The two processions will meet’ corresponds to event A , while its contradictory ‘The two processions will not meet’ corresponds to event non- A , it will be possible to form both the disjunction and the conjunction of these two contradictory propositions, namely ‘The two processions will meet or they will not meet’ and ‘The two processions will meet and they will not meet’.⁶²

The Ancients’ concept of the probable was a qualitative one. But as is witnessed by the opposition of the words “rare, equal, frequent”, they conceived of an order of greater and lesser probabilities; and in the case of two events having an equal probability, as well as in the frequent situation of drawings, they conceived of a numerical assignment of equal probabilities. Imagine our Peripatetic then thinking to himself

⁶⁰Consider the following elementary eventualities: there will be a procession \underline{a} at time t in one of the two places x or y and there will be a procession \underline{b} at the same time t in one of the same two places x or y . The set X of eventualities is formed by combining these elementary eventualities. It will contain four elements, namely: 1/ \underline{a} will take place in x and \underline{b} will take place in x , 2/ \underline{a} will take place in x and \underline{b} will take place in y , 3/ \underline{a} will take place in y and \underline{b} will take place in x , 4/ \underline{a} will take place in y and \underline{b} will take place in y . The set of subsets of X , say Y , has $2^4 = 16$ elements, among which are to be counted the empty set and X itself. One might for example conceive of the event $\{\underline{a}$ takes place at x and \underline{b} takes place at x or \underline{a} takes place at x and \underline{b} takes place at $y\}$ which would be the set $\{1, 2\}$, that is to say, the event in which \underline{a} always takes place at x . The Ancients would tend to raise the question as to whether the two processions would meet or not. They would focus then on the event $A = \{1, 4\}$, the processions meet, and the complementary event $\sim A = \{2, 3\}$, they do not meet.

⁶¹ $\sim(A \cup \sim A) = A \cap \sim A = \emptyset$.

⁶² $\pi(A \cap \sim A) = \pi(\sim X)$. But $\pi(X) = \pi(A \cup \sim A) = 1$;
 $\pi(X) + \pi(\sim X) = 1$. Therefore $\pi(\sim X) = 1 - 1 = 0$.

of a field of sets constructed on the set of eventualities X . If he defined in this field an event A comprised of half the eventualities of X , he will spontaneously assign the probability $1/2$ to A and to non- A , since the eventualities of X are symmetrically disposed, and the probability 1 to their sum and the probability 0 to their product. If he extends his considerations to general cases he will quite naturally define an additive finite probability space with every probability being a number between 0 and 1 and such that every event has a probability equal to or greater than 0, the probability of the set X of all eventualities is equal to 1 and if two events are without a common part, as is always the case in the examples of the Ancients, the probability of their sum is equal to the sum of their probabilities. And this is the second requisite of the calculus of probabilities.

Let us come back now to event A . To say with Alexander that from a same set of causes contrary future contingents may occur is to say that on the same set of eventualities X one can assign a probability to A , $0 \leq \pi(A) \leq 1$, such that the probability of the complementary event $\sim A$ will be $1 - \pi(A)$ and therefore itself ≥ 0 . Thus A and non- A can both occur, contrary to what the Stoics held, and one can say that it is possible that A at the same time that it is possible that non- A . A proposition such as this affirming the contingency of A will be analyzed simply in terms of the conjunction of two propositions assigning complementary probabilities to these two events. But the sum of these probabilities, equal to the probability of their sum, is none other than 1. As Aristotle said, it is not determinately certain either that A will occur or that non- A will occur, if it was supposed that neither $\pi(A)$ nor $\pi(\sim A)$ was equal to 1 or to 0. On the other hand, it is determinately certain that $\pi(A \cup \sim A) = 1$, so that the excluded middle is always verified. Nor is the principle of non-contradiction ever violated since $\pi(A \cap \sim A) = 0$.⁶³

It is therefore possible to make sense of *De Interpretatione*, IX and to interpret it speculatively without historical claim by translating it into the language of probabilities with A being the event of which p asserts the existence.

⁶³If we were to interpret conditional necessity as Leibniz does and as Lukasiewicz tended to do in 1955 (*viz.* as meaning, it is necessary that if p , then p) its translation in terms of probability would be simply $\pi(A \cup \sim A) = 1$, since 'if p , then p ', is true if and only if 'not- p or p ' is true. Conditional necessity would be indistinguishable from excluded middle, another form of the principle of propositional identity.

1. p is possible	$\pi(A) > 0$
p is necessary	$\pi(A) = 1$
p is impossible	$\pi(A) = 0$
2. It is necessary that p or not- p	$\pi(A \cup \sim A) = 1$
It is impossible that p and not- p	$\pi(A \cap \sim A) = 0$
Even if it is necessary that p or not- p	$\pi(A \cup \sim A) = 1$
it is not for all that that it is	does not entail
necessary that p or that it is necessary	either $\pi(A) = 1$
that not- p	or $\pi(\sim A) = 1$
3. p is absolutely necessary	$\pi(A) = 1$
p is conditionally necessary	$\pi_B(A) = 1$
(an incomplete expression which means	
that p is necessary on condition that q ,	
B being the event attested by q)	

This table must not be read as a translation since the probability sentences do not preserve the truth-value of the corresponding modal sentences. However modal sentences entail *a fortiori* the corresponding probability sentences; while the probability sentences do not fix sufficient conditions for their modal counterparts.

To go further it would be necessary to recast the modal expressions with two temporal indices in terms of probabilities. If it does not seem advisable to affect the probabilities themselves with temporal indices, there is nevertheless an interpretation of probability theory, advanced by certain physicists, suitable for producing the desired result. These physicists define probability as a prediction or mathematical expectation of relative frequency.⁶⁴ Such a definition, which for that matter seems to be required by a reflection on the modalities of double temporal index⁶⁵ and, more especially, on Aristotle's⁶⁶ future statements in contingent matter, makes it immediately possible to translate the Master Argument's premise (A) and the principle of conditional necessity.⁶⁷ The second premise (B) would say that if it is possible now that p at t we predict for a large number of systems similar to p at t_1 ($N \leq t_1 \leq t$) a relative frequency > 0 . To say, finally, that there are possibles that will not be realized is to assert that, in such a set of systems, there are

⁶⁴C.F. von Weizsäcker, *Aufbau der Physik*, Carl Hanver, München-Wien, 1985, p. 110; M. Drieschner, *Voraussetzung-Wahrscheinlichkeit-Objekt*, Lecture Notes in Physics 99, Springer, Berlin, 1979, p. 73.

⁶⁵von Weizsäcker, 1985, p. 81.

⁶⁶*Ibid.*, pp. 52-53.

⁶⁷*Ibid.*, p. 93. If probability is predictive (A) and (NH) are verified.

some that will not verify p at t_1 , although the relative frequency of p at t_1 is different from 0.

Looked at in this way, the intuition developed in the *De Interpretatione* is not logically contradictory. It does nevertheless require, that, within the domain of the possible as regards reality and which is therefore already different from the pure logically or mathematically possible, a distinction be made between essence and accident. Only the accident is subject to probability. To be already true or false, for an accident, is to have already fallen within the domain of reality, that is to say, to have a probability equal to 1 or to 0. Not to have a determinate truth-value is to have a probability falling between those two extremes. Then all the modal expressions used in Chapter IX translate into probabilist expressions and the paradoxes pointed out by the logicians vanish.

A theoretical difficulty, however, dismisses this interpretation, at least if we use it in its direct and literal sense. When Aristotle doubts that bivalence is universally valid, his doubt is cast on individual contingent factors. The premises (A), (B) and (NH) of the Master argument are considered as true. What is questionable in (C) is not the notion of a possible deprived of realization but the assertion of its actual truth before the non-realization has occurred. On the other hand, the probabilistic interpretation requires that probabilities be interpreted as frequencies, since any hint of subjective probabilities is barred in Aristotle's dogmatic system. Then there are not individual events, but frequencies that are able to verify or to falsify probabilistic sentences.

However, introducing frequencies into the Master Argument's interpretation seems to recommend Diodorus' rather than Aristotle's solution.⁶⁸ Moreover, when physicists today make reference to Aristotelian potentialities, they indeed allude to second order probabilities depending on probability amplitudes, all concepts that lead us far away from the letter of the Master Argument.⁶⁹

⁶⁸See later, p. 249.

⁶⁹See later, 10.5, pp. 259–263.

Epicurus and Intuitionism.

Epicurus escaped the Master Argument by rejecting Dialectic.¹ He insured freedom, for the atoms and the soul, in admitting that the atoms move² with an uncaused swerve.³ Antiquity mocked him for these decisions, nor were Bayle or Leibniz more favorable to him on this count. The latter writes: "It seems that in order to preserve freedom and avoid an absolute necessity, Epicurus, after Aristotle, held that future contingents were in no way capable of a determinate truth. For if it was true yesterday that I would write today, it could not therefore fail to happen: it was already necessary; and, by the same reason, it was so from all eternity. Thus everything that happens is necessary and it is impossible that it should happen otherwise. But since that is by no means the case, it would follow, according to him, that future contingents have no determinate truth. To uphold this sentiment Epicurus went so far as to deny the first and greatest principle of the truths of reason; he denied that every statement was either true or false. For here is how he was driven to that extremity: You deny that it was true yesterday that I would write today, therefore it was false. The poor

¹Cicero, *Academica*, II.XXX.97; Bréhier, 1962, pp. 231-232. "For if they don't get Epicurus, who scorned and ridiculed dialectic, to admit the truth of the following statement to them: 'Either Hermarchus will be alive tomorrow or he will not', while the dialecticians establish that every disjunctive statement such as 'yes or no' is not only true but even necessary (see now how clever he is, that Epicurus you take to be slow-minded: for they say that if he admits that the one or the other is necessary it will be necessary that, tomorrow, Hermarchus be alive or that he not be alive; but there is no such necessity as that in nature), then while the dialecticians, that is to say, Antiochus and the Stoics combat Epicurus, Epicurus turns the whole of dialectic upside down. For if a disjunctive proposition made up of contraries (I call contraries statements of which the one affirms what the other denies), if then such a disjunction can be false, then no other is true."

²Cicero, *De Fato*, X(23); Bréhier, 1969, p. 482.

³Cicero, *De Fato*, IX(19), X(22); Bréhier, pp. 480-481; *De Fato*, XX(46-48), Bréhier, p. 491.

man, unable to admit that conclusion, was forced to say that it was neither true nor false. After that, there is no need to refute him".⁴

Leibniz' judgment leaves the difference between Aristotle and Epicurus in the dark. Aristotle, along with the dialecticians, explicitly admits the truth of "either Hermarchus will be alive tomorrow or he will not"; he even establishes its necessity. What he denies is the distribution of truth—and therefore of necessity too—over each of the disjunction's components. Epicurus rejects the truth, and therefore the necessity, of the disjunction itself.⁵ What Epicurus denies is therefore not the determinate character of the truth-values of the disjunction's components, but the truth of that very disjunction. His criticism is directed not to the principle of bivalence but to the principle of the excluded middle. Naturally, a challenge to this latter is a challenge to the former, for then the excluded middle is neither true nor false.⁶

Historians of philosophy have paid little attention to Epicurus' denial of the principle of the excluded middle, either for lack of interest in logical questions or for the suspicion they bore towards Ciceronian testimony. Logicians have given two different interpretations of that denial. The only one that can be retained since it is compatible with the texts will lead to considering Epicurus' doctrine as a form of intuitionism. We shall have therefore to examine the nature of the Epicurean criteria. From this examination we shall draw consequences regarding certain hypotheses as well as the principle of the excluded middle, which will permit us to assign the probable attitude of Epicureanism towards the Master Argument. Thereafter we shall look at other intuitionist representations of modality in Descartes and Kant.

⁴Leibniz, Gerhardt, VI, p. 211; Jalabert, p. 222. Leibniz based his remark on Cicero's *Academica* reporting something Carneades said, perhaps ironically, in praise of Epicurus (Bk. 2, XXX(97); Bréhier, 1969, pp. 231-232).

⁵See note 1.

⁶On the contrary, denying the validity of the bivalence does not entail denying the validity of the principle of the excluded middle, since the first denial only bears on future contingents and does not question logical truths. Cicero makes mention of Epicureans who, ashamed to have to declare that there are propositions that are neither true nor false, "declare more impudently still that alternations of contradictories are true, but that neither the one nor the other of their terms is true" (*De Fato*, XVI. 37; Bréhier, 1962, p. 487). This Ciceronian passage evokes that of Quine on the fantastic character of Aristotle's position. The difficulty there is in distinguishing logically between the principle of bivalence ($'p' \vee \sim p'$) and the principle of the excluded middle ($'p \vee \sim p'$) explains how the Epicureans, natively contemptuous of logic, could have taken over the Aristotelian position which perhaps recommended itself to their eyes by virtue of the Stagirite's reputation as a logician.

7.1 First logical interpretation of the Epicurean denial of the excluded middle: the three-valued logic of Lukasiewicz; reasons for rejecting this solution.

At the same time as he proposed a three-valued logic to explain *De Interpretatione* IX,⁷ Lukasiewicz emphasized Aristotle's reticencies and obscurity. He added that in according a truth-value, albeit indefinite, to propositions about future contingents, the Aristotelians had retreated in face of the consequence of the principle gingerly advanced by their master. To them he contrasted the Epicureans who had espoused Aristotle's indeterministic intuition.⁸

Let us look then at this three-valued system where characteristic matrices have been defined for the propositional connectives taking as arguments these three truth-values and having as values these same truth-values, whereas the characteristic matrices of the modal operators take the three truth-values as arguments but have as values only the true or the false. On this first interpretation, 1) the principle of the excluded middle is invalid,⁹ 2) the interdefinability of the connectives 'and', 'or' (laws of De Morgan) is respected, 3) the disjunction ($p \vee q$) is no longer definable as ($\sim p \supset q$), 4) the modal operators are truth-functional¹⁰ and 5) the possible is defined as that which is not false.¹¹

⁷See above, 6.7, pp. 153-155.

⁸Appendix to "Many-valued Systems of Propositional Logic" in *Polish Logic*, edited by Storrs McCall, 1967, p. 64.

⁹See above, p. 154, Note.

¹⁰Lukasiewicz in Storrs McCall, 1967, pp. 40-66. With the three-values chosen as 1, $\frac{1}{2}$, 0, the following must be verified: 1) $\sim p = 1 - p$, 2) $p \supset q = 1$ if $p \leq q$ and $p \supset q = 1 - p + q$ if $p > q$; see also Prior, 1962, pp. 240-250. Disjunction is defined: $p \vee q \equiv (p \supset q) \supset q$.

¹¹Lukasiewicz, *op. cit.*, p. 55. ($Mp =_{Df} \sim p \supset q$).

The matrix is:

p	$\sim p$	$\sim p \supset q$
1	0	1
$\frac{1}{2}$	$\frac{1}{2}$	1
0	1	0

whence $M0 = 0$, $M\frac{1}{2} = 1$ and $M1 = 1$.

whence

$$\begin{aligned} L0 &\equiv M \sim 0 \equiv \sim M1 \equiv \sim 1 = 0, \\ L\frac{1}{2} &= 0, L1 = 1; M \sim 0 = 1, M \sim \frac{1}{2} = 1, M \sim 1 = 0; \\ &\sim M0 = 1, \sim M\frac{1}{2} = 0, \sim M1 = 0. \end{aligned}$$

The theorem of two-valued propositional calculus ($\sim p \supset p$) $\supset p$ is falsified here for $\frac{1}{2}$: $(\sim \frac{1}{2} \supset \frac{1}{2}) \supset \frac{1}{2} \equiv (\frac{1}{2} \supset \frac{1}{2}) \supset \frac{1}{2} \equiv 1 \supset \frac{1}{2} \equiv \frac{1}{2}$

But does this three-valued system of Lukasiewicz enable Epicurus to refute the Master Argument? The refutation can evidently bear only on the conclusion since the excluded middle figures in the rest of the argument only to say that every moment of time belongs either to the past or to the union of present and future, and that is hardly contestable. Epicurus would have to have held then that the Master Argument proved the incompatibility of its three premises but that, even admitting this incompatibility, one was not forced for all that to relinquish one or another of them, following the consequence drawn by Diodorus, Cleanthes and Chrysippus. Formally, that comes down to saying that the negation of the conjunction of several propositions does not entail the disjunction of their negations. To escape Diodorus, Epicurus had then to abandon the law of De Morgan.¹² But that law is specifically verified by the matrices of the first calculus Lukasiewicz proposed.¹³ And this is so precisely because those matrices still verified the interdefinability of 'and' and 'or'. If Epicurus' parry was to suffice in saving him from necessitarianism, it had to be that in denying the excluded middle he was assured of denying De Morgan's law as well, and that he had thus already denied the interdefinability of 'and' and 'or' in adopting a new definition of negation. The choice Epicurus was faced with was comparable to that of Chrysippus; and both of them fell under the same reprobation on the part of Cicero and Bayle.¹⁴ Chrysippus gave up the interdefinability of the modalities.¹⁵ More radical yet, it was at the level of the propositional calculus itself that Epicurus asserted the independence of the connectives. For that, it

It will be noted that the same goes too, for the same value, when a stronger negation-matrix, $\sim \frac{1}{2} = 0$, is adopted.

There $(\sim \frac{1}{2} \supset \frac{1}{2}) \supset \frac{1}{2} \equiv (0 \supset \frac{1}{2}) \supset \frac{1}{2} \equiv 1 \supset \frac{1}{2} \equiv \frac{1}{2}$.

¹²Here, for example, with p, q and r designating the three Diodorean premises:

$\sim (p \cdot q \cdot r) \supset \sim p \vee \sim q \vee \sim r$.

¹³Let us show this for the case of two variables:

p	q	$p \cdot q$	$\sim (p \cdot q)$	$\sim p \vee \sim q$
1	1	1	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
1	0	0	1	1
$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	0	0	1	1
0	1	0	1	1
0	$\frac{1}{2}$	0	1	1
0	0	0	1	1

It is seen that there is reciprocal implication between the last two expressions.

¹⁴Leibniz, Gerhardt, VI, pp. 211-212; Jalabert, pp. 222-223.

¹⁵See above, 5.3, p. 113.

suffices to change the matrix for negation. It will take its arguments from the three truth-values but, as is the case with the modal operators, it will have as values only the true and the false: namely, the true for the false and the false for the other two argument-values.¹⁶ Negation then has the same matrix as did the impossible in the previous system.¹⁷

7.2 Second logical interpretation of the Epicurean negation of the excluded middle: The Intuitionist System.

With this truly ‘modal’ definition of negation and the old characteristic matrices of the propositional connectives, some of De Morgan’s laws¹⁸ are falsified but not the one that interests us. It can be shown that the system of axioms verifying these new matrices, on the condition that one of the axioms is omitted, corresponds to Heyting’s intuitionist propositional calculus.¹⁹ Should we try to construct a model of this last mentioned calculus, no set of matrices with a finite number of elements renders it valid; but validation may be obtained from an infinite number of elements²⁰ and therefore with an infinite number of

¹⁶Prior, 1962, pp. 250-259.

p	$\sim p$
1	0
$\frac{1}{2}$	0
0	1

¹⁷See note 11.

¹⁸

p	q	$p \cdot q$	$\sim (p \cdot q)$	$\sim p \vee \sim q$	$\sim (\sim p \vee \sim q)$
1	1	1	0	0	1
1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1
1	0	0	1	1	0
$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	1
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1
$\frac{1}{2}$	0	0	1	1	0
0	1	0	1	1	0
0	$\frac{1}{2}$	0	1	1	0
0	0	0	1	1	0

Consequently:

$$\vdash \sim (p \cdot q) \equiv \sim p \vee \sim q$$

$$\vdash \sim (p \cdot q) \supset \sim (\sim p \vee \sim q)$$

but not

$$* \sim (\sim p \vee \sim q) \supset (p \cdot q).$$

¹⁹The axiom in question is $\vdash (\sim p \supset q) \supset \{[(q \supset p) \supset q] \supset q\}$ which is verified by the three-truth-valued matrices.

²⁰Prior, 1969, p. 253.

irreducible degrees of possibility.²¹ But then it becomes impossible to interdefine any of the propositional connectives *no*, *and*, *or*, *if then*. In particular, the conditional of which the antecedent is the negation of a conjunction of propositions and the consequent the disjunction of the negations of those propositions is invalid.²² Then, and only then, can Epicurus escape the Master Argument's conclusions.

7.3 Are the Epicurean 'criteria' compatible with intuitionism?

But is it reasonable to foist off such a conjecture on Epicurus, who held dialectic in contempt? Anaxagoras is the only Greek philosopher mentioned by Aristotle as having challenged the excluded middle. But then his theory of *homoiomereia* and of infinite divisibility gives additional good reason for considering him an ancestor of intuitionism. Epicurus, by contrast, adheres to the finite divisibility of matter and the existence of atoms. Both these dogmatic themes inherited from Democritus are contrary to the spirit of intuitionism, at least if we give the word 'intuitionism' its full sense: a constructive methodology applied to the mathematical continuum and to the whole building of our theoretical and practical ideas. As we do not know what was Epicurus' conception of mathematics, it will then be safer to regard *a priori* as doubtful the logical interpretation that has been advanced to explain the Epicurean denial of the excluded middle.

Let us consider however just what it was that distinguished Epicurus from Democritus. It is what made him formerly the object of universal derision: the *swerve* in physics that prompted Cicero²³ to ask whether the atoms "draw straws for which one will deviate and which one will not deviate", and his forsaking of the excluded middle in logic. It is easy to imagine the unexpected new vigor modern atomism has

²¹Lukasiewicz in Storrs McCall, p. 61. That infinity of possibles will be defined by the unique formula:

$$Mp = \sim p \supset p$$

in giving p first two, then three, then four, etc. distinct truth-values, thus with the matrices

$$M_1(0) = 0, M_1(\frac{1}{2}) = 1, M_1(1) = 1$$

$$M_2(0) = 0, M_2(1/3) = 1, M_2(2/3) = 1, M_2(1) = 1, \text{ etc.}$$

²²This is thesis 55b in Kleene, 1971, p. 24 which is not valid in intuitionist logic: $(\sim(p \cdot q) \supset (\sim p \vee \sim q))$.

²³Cicero, *De Fato*, XX(46), Bréhier, p. 491. Sedley (*op.cit.*, note 137, pp. 115-116) thinks that in his *Letter to Herodotus* (307/6) Epicurus "still seems quite unaware of the determinist threat". His *clinamen* theory and his wariness of necessity are explicit in the *Letter to Menoeceus* and the fragments of his book on free will written after 296/5.

given to Epicurean physics and to its discontinuity theses.²⁴ Then again, his Canonic that accords a fundamental importance, though in a quite different sense from Anaxagoras, to the activity of the subject in the formation of the criterion of truth, becomes clearer in the light of intuitionism.

Intuitionism has often been considered to admit a definition of truth reduced to verification or, in mathematics, to construction. From the logical point of view that would mean, consequently, that in any formal system expressive enough, since not all that is true in the classical sense is demonstrable, this truth that goes beyond demonstration is only chimerical and it would be wise not to linger on it. More generally, it would emphasize this theme of reasonableness: given that not all that is true in the classical sense is verifiable, it is important to restrict oneself to the verifiable only, at least to the extent that only it produces direct or indirect consequences in our present experience. But what is it that the *Letter to Herodotus* (51) says, for example, when it comes to distinguishing between illusion and error? There is illusion when our disposition receives mental images (phantasms) resembling external objects from sensible material. But we can do nothing about an illusion; it arises therefore in connection with an 'immobile' apprehension. Error, on the contrary, requires a sort of action on our part, a 'mobile' apprehension. Epicurus says, "Error would not exist if we did not receive also in ourselves another associated but different motion. It is from this motion associated with the imaginative apprehension, but different from it, that, if there is no confirmatory evidence or there is counterevidence, error arises, and truth if there is confirmatory evidence or no counterevidence".

Even though the commentators on Epicurus seem to be at one, for once, on the criteria of error (lack of confirmatory evidence or counterevidence) and truth (confirmatory evidence or no counterevidence), this part of the doctrine is not without problem. It is repeated in other texts²⁵ that throw no light upon it. What they do seem to agree on is that "the control of opinion by experience, positive or negative, splits in two here, leaving open a margin of uncertainty: the non-confirmation may be only temporary, and the absence of falsification is not equiva-

²⁴Mugler, 1953; Vlastos, 1965.

²⁵Cf. the translation of the *Letter to Herodotus* (50-51) in Bailey, p. 29 and in Furley, p. 207. There are other similar texts on the criteria in Epicurus: *Vita Epicuri* (34). "Opinion they also call supposition (ὀπλόγησιν), and say that it may be true or false: if it is confirmed or not contradicted, it is true; if it is not confirmed or is contradicted, it is false". (Bailey, p. 163).

lent to a direct confirmation".²⁶ But that margin of uncertainty may, on the one hand, lead to a resuscitation of fears and myths,²⁷ and on the other hand, it squares poorly with Epicurean empiricism in general as well as with other explicit texts. For Epicurus denounces as the principal source of error the temptation to put on an equal footing that which awaits confirmation (*i.e.* opinion about sensation) and that which does not await confirmation (*i.e.* sensation itself).²⁸ The *Life of Epicurus* passage setting forth the definitions of true and false that seem "to leave open a margin of uncertainty" is immediately followed by this empiricist declaration: "For this reason was introduced the notion of the problem awaiting confirmation: for example, waiting to come near the tower and seeing how it looks to the near view".²⁹ Such 'awaiting' would be senseless if every non-falsified opinion were true.

If the empiricist principle is fundamental, if "all investigations must be controlled by sensations",³⁰ we must first inquire just to what the given definitions of truth and falsity apply. They cannot apply to opinions about sensible things which are liable to confirmation or falsification, since such opinions are only problems;³¹ 'clear vision' confirms or falsifies: it leaves no room for a margin of uncertainty. If this is so, we must look for another level of knowledge upon which to apply the definitions. There is a passage in Sextus Empiricus that gives us a lead in explaining what is to be understood by non-falsification. "Non-falsification is conformity of the invisible thing, posited by hypothesis and conjecture, to the phenomenon. For example, Epicurus asserts the existence of the void, that which specifically is invisible (*adelon*). But

²⁶Rodis-Lewis, 1975, p. 111.

²⁷Principal Doctrines, XII, in Bailey, p. 97.

²⁸Principal Doctrines, XXIV, in Bailey, p. 101 and commentary, p. 363. "XXIV. If you reject any single sensation and fail to distinguish between the conclusion of opinion as to the appearance awaiting confirmation and that which is actually given by the sensation or feeling, or each intuitive apprehension of the mind, you will confound all other sensations as well with the same groundless opinion, so that you will reject every standard of judgement. And if among the mental images created by your opinion you affirm both that which awaits confirmation and that which does not, you will not escape error, since you will have preserved the whole cause of doubt in every judgement between what is right and what is wrong." (Bailey Translation).

Cf. also Goldschmidt, 1977, pp. 270 and 272 as well as his commentary on confirmation, falsification and prenotation.

²⁹*Life of Epicurus* (34) in Bailey, p. 163.

³⁰*Letter to Herodotus* (50). Commentary in Bailey, p. 195.

³¹Bailey, p. 197, writes "The vision of an object at a distance should always be regarded as a problem awaiting the confirmation of the nearer view, by which, if it is either confirmed or not contradicted, it is true." But *if* the nearer vision is possible there is necessarily confirmation or falsification!

one believes in it because of the evident fact of motion; for if there is no void, motion should not have existed either, if there is no place towards which a body in motion can direct itself, since the whole would be full and compact. So the (evident) phenomenon of motion does not falsify the invisible thing which is conjectured".³² Non-falsification is therefore a relation between an hypothesis, unverifiable in itself because not amenable to clear vision, and the clear vision of a phenomenon. Such a relation can only be of a logical order, namely, one of implication.

Mrs. Rodis-Lewis goes on with the following commentary: "On the contrary, the existence of motion falsifies the Stoic denial of the void. It is in this way that use of reasoning by the absurd, which has been noted to be at the base of atomism, is justified. Without the void, there is no motion; without the principle 'nothing is generated from nothing', we fall into the absurdity that everything comes from anything at all. If the principle denied has but a single contradictory, the negation of the negation amounts to confirmation".³³ Even if there is a complete parallelism in the definitions given of the true and the false, are we able to say for all that that the existence of motion at once falsifies the Stoic negation of the void while confirming the Epicurean affirmation of it? It doesn't seem so. For if it were admitted that a double negation amounts to an affirmation, the atomist thesis would be susceptible not only of non-falsification in Sextus' sense, but of confirmation. Distinction must be made then, in the definitions given of the true and the false, between what applies to opinions expressed in observation statements and what applies only to theoretical principles and therefore to hypotheses either about invisible things or about things such as heavenly bodies which, though visible, are beyond the reach of a 'clear view'.

The interpreters of Epicurus³⁴ have discussed the obscure notion of ἐπιβολὴ τῆς διανοίας, projection of the mind. It seems to correspond to the notion of scientific hypothesis. But of that there is no direct confirmation if it is true, but only non-falsification through something sensibly evident resulting from it by clearly understood principles. Take, for example, the question of the speed of the atoms and the *eidola* in the void. Epicurus says, "Next, no observation falsifies the theory that the *eidola* (idols) are of maximum fineness: hence they have maximum speed, since they all have a path big enough to ensure that nothing

³² *C. Dogm.*, 1, 213, quoted by Rodis-Lewis, 1975, p. 111.

³³ *Op. cit.*, p. 112. Likewise, Furley (quoted by Goldschmidt) and Goldschmidt speaking of the inverse supposition (P , for if $\sim P$, then Q which is observed to be false; therefore $\sim\sim P$, that is to say, P), 1977, p. 219, note 4.

³⁴ Bailey, pp. 259-274.

collides, or few things collide...".³⁵ Furley rightly draws a connection between this passage and the Aristotelian doctrine. "Aristotle held that the speed with which bodies move through a medium varies with the weight of the bodies and the density of the medium. Void has no density at all, so that the speed of motion through a void can have no ratio to other speeds at all. Since this is impossible, Aristotle argues that there is no void. Epicurus turns this argument upside-down...: there is a void, so atoms move through it at 'inconceivable' speed. They do not, of course, move at *infinite* speed; but their speed is such that it cannot be related at all to the speed of observable motion".³⁶

This is a remarkable argument. Let p be 'there is a void' and q be 'the speed of the motion of atoms through a void has no ratio to the speed of observable motion'. Epicurus and Aristotle both admit the thesis:

If p , then q .

Aristotle then gives a *reductio* argument:

If 'if p , then q ' is a thesis and it is not possible that q ,
then it is not possible that p ,

and from the impossibility of a speed having no ratio to observable speeds, he deduces the impossibility of the void. This is precisely an application of the Master Argument's second premise interpreted as a thesis of pure modal logic. But both the premises, p and q , the condition and its consequence, belong to the realm of invisible things. For Epicurus then, both of them must be 'inferred' from phenomena without its being possible for a reduction to the absurd to eliminate the condition on the ground of some supposed internal impossibility that would characterize the consequent. The consequent cannot be declared impossible, nor possible either, simply on an examination of its nature.

For Epicurus there is something dialectical about Aristotle's reasoning, for the internal possibility of a concept such as that of inconceivable speed is a dubious one. We must therefore come back to the relation of the invisible to phenomena. Just what is it, on Aristotle's account, that is given in sensation? Motion is. It is therefore on the relation of the void to motion, and only on it, that it will be possible to ground a subsequent inference as to the legitimacy or illegitimacy of the concept of inconceivable speed, itself a consequence of the concept of the void.

³⁵ *Letter to Herodotus* (47), Furley Translation, p. 127.

³⁶ Furley, p. 127.

Let there be the following principle then: no void (not- p), no motion (not- r)—change of place supposing the existence of place and hence of the void.³⁷

If 'if not- p , then not- r ' is a thesis and if r , then it is not the case that not- p .

There is motion, therefore by *modus tollens* it is false that there is no void. There is refutation or falsification of the denial of the void by the phenomenon of motion. Therefore the Aristotelian hypothesis of that denial is false. One might think that from there one would be justified, according to Epicurus, to conclude that there is void. But it would be a futile conclusion for, being based only on a refutation by the absurd of a dialectical thesis, it would be subject to the same uncertainty as that thesis is. To give grounding to the existence of the void there must therefore be a positive argument of such a nature that a *sensation* or a phenomenon could give support to the hypothesis. Epicurus contents himself with saying that the hypothesis of the void is not falsified by the phenomena. In other terms,

1) the *modus ponens* if p , then r and p therefore r is not employed because, while the 'if not- p , then not- r ' is a clear principle for the ἐπιβολὴ τῆς διανοίας, it is not at all the same for the affirmative conditional 'if p , then r '. For it is not sufficient that there be a void for there to be motion. We don't know exactly in what sense Epicurus interpreted his conditionals, but in the weakest sense of material implication p might be true and r false (in a possible, contrary to factual, immobile world). All one can say is that the existence of the void renders motion possible. From the existence of motion as given, the necessity of the existence of the void cannot be concluded; but the hypothesis of the void is not falsified by the fact of motion. On the contrary, once the void is supposed, the incommensurable speed of the atoms follows ($p \supset q$), or rather, that speed is the necessary consequence of the conjoint supposition of the void and of the motion of atoms.

2) It is not justified to pass from a double negation to an affirmation, or, what comes down to the same thing, the excluded middle cannot be applied to principles having to do with invisibles. The most that can be attained is a relation of non-falsification. Posit the void, and the phenomena do not falsify it, since motion, an evident phenomenon, would not exist without it. And since the void entails inconceivable

³⁷Letter to Herodotus (40). "If there were not what we call void, space and intangible nature, bodies would have nowhere to be nor anywhere to move, as they do indeed appear to move". This is of course not a reduction to the absurd of Aristotle, since he explicitly rejected this implication.

speed, this last hypothesis, just as its condition, should be regarded as not falsified.

So in the definition of true and false two aspects, or even two independent elements will be distinguished according to whether we have to do with opinions consisting in anticipations of perception or, on the contrary, in theoretical statements. But this distinction requires at the same time a clarification of the relations obtaining among the four terms: confirmation, non-confirmation, falsification, non-falsification, and between the terms of this set and the two terms: true and false. An observation statement is true if it is confirmed. For it to be false it suffices that it be non-confirmed. When I remark that the tower is not square, the opinion that I had had, 'the tower is square', is seen to be false. Here, non-confirmation amounts to falsification. There is no room for a margin of uncertainty. By contrast, when the question is one of a mental projection about sensible but inaccessible things, or invisible things, there will be neither confirmation nor direct confirmability. The true will be reduced to non-falsification. Falsification of the hypothesis remains possible, however, through the phenomenal consequences that can be attached to it. Theoretical non-falsification must be regarded as a kind of verification, as the passage quoted from Sextus proves it. As the concepts and principles they deal with are not amenable to direct comparison with experience, not being properly empirical concepts, the decision about them is made in connection with the entire set of empirical consequences associated with them by virtue of 'projections of thought'.

Consequently, whereas truth and falsity are subject to a homogeneous and symmetric criterion in the case of opinion about the 'near' sensible, and there is conformity with the canons of empiricism with true meaning 'verifiable' and false 'verifiable-that-not', that is, confirmed and falsified, it is a different story for theoretical hypotheses. These may be falsified although they can never be confirmed. Epicurus then had an inkling of the asymmetry between falsification and confirmation in this domain. His Canon announces Popper. If it doesn't reach exactly the same conclusions as Popper, it is that for Popper non-falsification does not amount to truth, and further, every protocol observation statement is subject to being ulteriorly corrected. This last aspect is understandable in the light of Popper's polemics against psychologism and the conventionalist interpretation of 'basic statements' that is rightly and resolutely rejected by the empiricist canon.³⁸ As for the first, Epicurean intuitionism tends to efface the

³⁸Popper, 1973. (There is a note, p. 103(1), to this often obscure text that

difference between the truth it attributes its hypotheses and the absence of positive truth proper to hypotheses according to Popper. On the one hand, any perceptive anticipation is in itself simply a problem awaiting confirmation or falsification by direct and 'near' experience. As such it is therefore without truth-value, since it is only experience that will confer that upon it. An hypothesis, on the other hand, is verified or falsified by the phenomena following from it. There is no truth or falsity apart from these procedures of confirmation. Moreover, the verification of an hypothesis being limited to non-falsification is asymmetric to falsification, and strictly speaking, given the invalidity of the excluded middle, it is not absolutely positive nor does it in any way authorize the conclusion to *truth* in the dogmatic sense of adequation to the object.

The Epicurean theory of law gives a remarkable illustration of the precepts of the Canonic. For in the procedures of verification and falsification of laws, more complex than those operative on the level of sensible opinion since they can deal only with the far-off consequences of the laws, the method of anticipation has to do no longer with *simulacra* passively received but with effects "whose cause we ourselves have posited".³⁹ There will be two distinct cases. Either confirmation will be inherited from the past as with a body of laws left by the ancestors. In this case the law brings its confirmation (hence, non-falsification) with it and can fulfill its essential function of establishing security, just as confirmation and non-falsification establish security in the order of knowledge.⁴⁰ Or again, there may be a proposed law of which the confirmation or non-confirmation is 'in suspense'. Goldschmidt has likened this to the Attic procedure of 'revision of laws' and the action taken against the author of a detrimental law.⁴¹ In this last case the law is like a hypothesis about the future. If there is often confirmation and hence justification of a rule by its present conformity to the common interest, or non-confirmation and rejection of its legality because its consequences do not conform to the interest of the community, the immediate confirmation assimilating the juridical criterion to the criterion of opinion about 'near' sensible things, it also happens that law changes as a result of constitutional upheavals. What was just ceases

seems to lessen this first difference.) In the other direction, as Goldschmidt aptly points out (1977, p. 217), Epicurean sensation differs from Humean impression by the "*fundamentum in re* accorded it in Epicurean realism by the theory of *simulacra*".

³⁹ Goldschmidt, 1977, p. 217, pp. 221-222.

⁴⁰ *Id.*, *ibid.*, pp. 218-219.

⁴¹ *Id.*, *ibid.*, pp. 196-197, 222-223 ; Goldschmidt, 1981, p. 87.

to be useful. In this case it will have to be admitted that the consequences of a law that was accepted and obeyed for some length of time did not really confirm that law, but simply did not falsify it temporarily. When untoward consequences arise and are more and more clearly recognized as such, they falsify the law and oblige its revision. In other words, from the non-falsification that justified the law up to that time, we go to the falsification which takes that justification away.⁴²

A proposed law conceived of as a juridical hypothesis is different from a scientific hypothesis about invisible or out-of-reach things, because the future will assimilate it one day to an opinion about proximate sensible things. That is why the criterion of what is just will change, as required by positive law. Let us note one last analogy between legal theory and the Canonic. Suspension of the positive legal order or anomia inevitably produces a state of insecurity where everyone harms everyone else. Security, which consists simply in not mutually harming one another, therefore supposes a state of law. The non-falsification of an hypothesis would produce its truth not in the strong, dogmatic sense of correspondence, but in the weak sense of a substitute for verification for that which cannot be verified. Likewise, the non-falsification of a rule produces its justification not in the sense of an ideal justice, but in the weak sense of what plays that role in positive law: namely, peace in the city.

7.4 Consequences of the Epicurean criteria: Plurality of hypotheses and rejection of the excluded middle.

Two consequences follow from this canon: the indeterminacy of certain hypotheses and the rejection of the excluded middle.

In the first place, to be sure, Epicurus admits that both for searching out the hidden causes of phenomena, the invisible (*ἄδηλον*), including both the atoms and the things of the heavens and meteors, and to reach the future (*προσμενον*), opinion must inevitably go beyond the bounds of perception. But the truth-criterion of these mechanisms

⁴²Epicurus, *Principal Doctrines*, XXXVII and XXXVIII, Goldschmidt, 1977, pp. 280-282. Epicurean legal theory could be compared to that of today's neo-positivists. If the annullability of a legal norm is not generally the same as its being void *ab initio*, and if specific legal bodies are required to decide of it, it is that a norm may be subject to revision only in the name of another norm (Kelsen, 1945, XI He). But in order that an explicit contradiction does not upset the edifice of law, the older norm must be regarded precisely as a non-invalidated hypothesis that was active as long as it was accepted and whose annulation coincides with the formal act of invalidation.

of projection is still perception itself. Hypotheses, whether about the imperceptible reasons for things or about inaccessible or future things, have their criteria in perception alone in so far as it confirms or at least does not falsify them. In the case of explanatory hypotheses, it can but not falsify them, in the sense we have given to this term. But then we are in no way obliged to choose between incompatible hypotheses meant to explain one and the same natural phenomenon.⁴³ Epicurus condemns those "who for not having clung to the method of the possible have fallen into that (impious) vanity, because they think that the phenomena come about only in one way and reject all the other ways in harmony with the possible, tending towards the unintelligible and incapable of bringing together in one theory the phenomena that must be accepted as signs".⁴⁴ Is that reason enough however to make a skeptic out of Epicurus and to say that "the Epicureans shouldn't have regarded their own atomist metaphysics as any more than an hypothesis, not refuted but not confirmed either by the facts—an hypothesis which only served them moreover to repel other hypotheses that seemed morally dubious to them"?⁴⁵

Are we to completely subordinate physics to ethics and judge that "Epicurus recognizes that an indefinite number of different explanations could be as true as materialism", the 'truth' of this last residing in its pragmatic consequences, that is to say in its capacity to "more completely liberate man from his fears"?⁴⁶ That would be a complete misinterpretation. First of all, it is within the unique, categorical framework of atomism that these multiple explanations arise.⁴⁷ Then too, precisely in the case of celestial phenomena, Epicurus was seen to require unification in a single theory of the diverse explanations, that is, to absolutely reject the dualism some of his adversaries held there to be between supra- and sublunary worlds,⁴⁸ and the astrological consequences they derived from this representation.⁴⁹ One could in no way then invoke the plurality of certain hypotheses to infer that Epicurus had a skeptical attitude. The eminently peremptory thesis of the non-divinity of celestial things is fundamental in his system. Skepticism attracted some of Democritus' disciples, like Nausiphanes,

⁴³ *Letter to Herodotus* (79-80) in Bailey, p. 51; Sextus Empiricus, VII, 211.

⁴⁴ *Letter to Pythocles* (97), translated by Rodis-Lewis (pp. 59-60); Lucretius, *De Rerum Natura*, V, 526-533.

⁴⁵ Windelband-Heimsoeth, 1935, p. 171.

⁴⁶ Sartre, 1949, p. 191.

⁴⁷ Boyancé, 1953, pp. 428-429.

⁴⁸ *Letter to Herodotus* (77), in Bailey, p. 49.

⁴⁹ *Principal Doctrines* (XI), in Bailey, p. 97.

because Democritus depreciated sense knowledge. Epicurus, by contrast, reestablishes the credentials of that knowledge and, through it, of phenomena,⁵⁰ the only things proper to providing a criterion. What is more, when Epicurus speaks of the plurality of hypotheses, he often seems to be defending simply the plurality of causes. In particular, that is the case in the *Letter to Pythocles* (98), when he looks for the explanation of the inequality of days and nights.⁵¹ The only conclusion that can be drawn then from the plurality of certain hypotheses has to do with logic. It is experience, never the logical organization of the premises, that lets us decide between hypotheses. Logical reasoning is empty when it assigns things essences such that immateriality, eternity or necessity should derive logically from them.

This negative conclusion entails in the second place, another concerning the very principles of logic. It is its non-falsification, that is to say, the confirmation of its experimental consequences that assures us of the truth of atomism. It is not simply the fact that the contrary hypothesis is false. That is why the excluded middle and the law of double negation are themselves powerless to determine the truth of an hypothesis, whatever its nature. But now take the case of the future. How could it be said that a prophecy about it is true or false, if divination is ridiculous, as Democritus had already shown? Since there is no confirmation or falsification to be had before the awaited event itself, the excluded middle could have no application to the future.

It has been said that Democritus' system suffered in having been transmitted through the system of Epicurus which subordinated theory to practice and introduced the metaphysical concept of freedom into philosophy.⁵² And indeed this concept of freedom of indifference or balance of will, that was the admiration of a Marcus Aurelius,⁵³ is the corner-stone of the philosophy of Epicurus. But that freedom is in the first place the freedom to refuse the solicitations of opinion, the representation of future evils for example, in order to accept only the present, i.e. sensation, cut off from the active movement of error.

⁵⁰Rodis-Lewis, 1975, p. 65; Bailey, p. 404.

⁵¹Bailey, p. 67, p. 277 and p. 251.

⁵²Windelband-Heimsoeth, *ibid.*, p. 162.

⁵³*Thoughts*, Bk. IX(41); Bréhier, 1962, p. 1220. "Whoever uses the Democritean argumentation, saying that there is no free motion of the atoms, because of their respective impacts, and that it seems from that that everything moves according to necessity, we shall say to him: Don't you know, whoever you are (Diogenes addressing the passers-by of Oenoanda), that the atoms too have a free motion, not discovered by Democritus but brought to light by Epicurus; it is the existence of a swerve, as he shows from phenomena." (fr. 32, in Rodis-Lewis, p. 79); Lucretius, *De Rerum Natura*, II, 252-293.

The connection with the denial of the principle of the excluded middle becomes apparent once we consider the weight of representations or motives in voluntary decision making. Both adversaries and partisans of the freedom of indifference agree in denying that a motive can be at once sufficient and insufficient for entailing decision. But it is not for all that that the partisans of will admit that a motive is either sufficient or insufficient for entailing decision. For it is the indifference of freedom that bestows upon the motive the power of determination in which the representation in itself, no matter what its weight, is lacking; and that is why it isn't true that such a motive in itself either is or is not sufficient for entailing decision.⁵⁴

7.5 Epicureanism and the Master Argument.

What then is Epicurus' possible relation to the Master Argument? And why, as Cicero has it, did he invoke the invalidity of the excluded middle in order to avoid the necessity he thought resulted from that argument? Diodorus' influence on Epicurus has been played down as far as the theory of elementary motion is concerned.⁵⁵ The influence of the Master Argument on the other hand is great: the Ciceronian testimony is unimpeachable. We know too that Epicurus wrote a treatise *Against the Megarians* and another *On Fate*.⁵⁶ In his *Letter to Menoecus*⁵⁷ (133-134) he protests against the fate of the philosophers, judging it to be worse than what the common people believe in. Then too, he explicitly admits Diodorus' first premise.⁵⁸ In so far as the third premise is concerned, it has been noted in connection with Aristotle's potential infinity that Epicurus "would be impatient with the idea of a potentiality which never can be actualized"⁵⁹

At the same time, we should expect the same impatience with a Diodorean potentiality that is actualized at a time as far removed in

⁵⁴It is this indifference of equilibrium that Leibniz refuses above all in Epicurus (Gerhardt, IV, p. 297; Jalabert, pp. 310-311). In short, if there is an uncaused physical or psychological movement, there are exceptions to the principle of the excluded middle. According to Carneades, both Epicurus and his adversary Chrysippus agree in accepting that proposition (Cicero, *De Fato*, X (20-21); see below, 8.1, p. 206). Let A be the proposition 'it is possible that the motive *p* should not be accompanied by the decision *q*' (case of insufficient motive) and B the proposition 'it is impossible that the motive *p* should not be accompanied by the decision *q*'. What the partisans of free will refuse is the validity of the conditional ' $\sim (A \cdot B) \supset (\sim AV \sim B)$ '. (See above, note 22).

⁵⁵Furley, p. 134.

⁵⁶*Vita Epicuri* (27-28) in Bailey, p. 159.

⁵⁷Bailey, p. 91.

⁵⁸Fragments (LV) in Bailey, p. 115.

⁵⁹Furley, p. 155.

the future as one might wish. If we take into consideration the infinity of time, we will not be able then, according to the Canonic, either to affirm or deny the third premise. Moreover, the overall accommodation made in Epicurean thought for the free play of what is the aleatory would hardly allow discarding the third premise, whose truth is an experiential given for free will. As for the second premise taken as a thesis of pure modal logic, it is employed by Epicurus in the *Letter to Herodotus* (57), for example. He starts from the thesis (δῆλον) “if there is an infinity of parts, however small they may be, having whatever size they do, the body composed of them will itself be infinite in size”.⁶⁰ The consequence is impossible: “how then could a size like that be limited?” Therefore the antecedent is impossible as well.⁶¹ We have reason to presume then that Epicurus escapes the Master Argument’s necessitarian consequence by indirectly challenging the dogmatic scope of the third premise, and that to this end, he denied the validity of the excluded middle, given the overall intuitionistic style and the exigencies of his canonic.

When serious attention is paid to the fundamental innovations Epicurus made in Democritean atomism, it is difficult to take Epicureanism as a kind of model of dogmatic empiricism as Kant does in the Antithetic of the *Critique of Pure Reason*. The characteristics of the Kantian antitheses: infinity of the world in space and time, inexistence of a simple substance, denial of freedom and total subordination of the world to the laws of nature, denial of the existence of an absolutely necessary being are proper to Democritus, though it is difficult to reconcile atomism with the second.⁶² Epicurus tempers all but the last of these theses. To the eternity of the atoms he adds the theme of the plurality of worlds that come to be and pass away and have therefore beginning and end.⁶³ If the soul is a perishable composite, it has, for a certain time at least, a sort of formal permanence that fits in well with its independence, parallel to that of the atoms. Finally, the swerve and freedom break the chain of natural causes. In short, with the excep-

⁶⁰ “For it is clear that the unlimited number of particles have a determinate size and, whatever the size of these components, the magnitude [of the body] would be unlimited as well.” (Bollack, Bollack, Wisemann, 1971, p. 113).

⁶¹ There is no contradiction between the existence of an infinity of atoms and the fact that an infinite number of atoms cannot enter into the makeup of a limited body. For as the *Letter to Herodotus* (40) says, “the whole is unlimited in extension and in the number of bodies”; and it goes on to add (45b), “the number of worlds, some like and some unlike ours, is equally infinite”.

⁶² See the difference Kant makes between monadism and atomism, *Critique of Pure Reason* (B478/A460).

⁶³ Lucretius, *De Rerum Natura*, II, 1047–1089.

tion of the existence of the necessary being,⁶⁴ one would put Epicurus rather on the side of the Kantian theses than the antitheses, that is to say, precisely on the side of practical interest as over and against the speculative interest of reason.⁶⁵

Nevertheless, while placing Platonism (the thesis) decidedly among the dogmatic systems, Kant hesitates about Epicureanism. In an important note he says: "There is still doubt, however, whether Epicurus ever propounded these principles as objective affirmations. If they were perhaps nothing more than maxims for the speculative exercise of reason, he would show a more genuinely philosophical spirit in that than any other of the sages of antiquity. That, in the explanation of phenomena we must proceed as if the field of inquiry was not bounded by limits nor a beginning of the world; that the material of the world must be accepted by experience, if we are to hope to learn from it; that we must determine no production of events, if not by the immutable laws of nature; and finally, that we should have no recourse to a cause distinct from the world—those are even today very good, though little followed, principles for extending speculative philosophy and for discovering the principles of morals, without an appeal to outside help. Nor for that reason can anyone desirous of *ignoring* these principles, as long as mere speculation is involved, be accused of wanting to deny them".⁶⁶ Had Kant taken into account the Epicurean innovations in the domain of physics with the swerve as well as in that of the canonic with his rejection of the excluded middle and of morals with the assertion of free will, he would no doubt have given more weight to this remark and would not have hesitated to see in Epicureanism a sort of precritical philosophy.

7.6 Other intuitionist conceptions of reality: Descartes and Kant.

If Kant is right in noticing an affinity between Epicureanism and critical philosophy, it is still possible to generalize his remark. For since this affinity results from their common intuitionist tendencies, one might well wonder what philosophical traits in general are delineated by such shared tendencies. To find an answer it would be best to compare several philosophical systems differing both in their particular choices and architectonics, but all complying with these same intuitionist tenden-

⁶⁴Which the Epicureans were the only philosophers of antiquity to deny as the prime mover (B478/A450).

⁶⁵B494/A466.

⁶⁶B499/A471.

cies. We shall thus compare Epicurus, Descartes and Kant, examining their systems successively from the point of view of method, of anthropology and of theodicy.

The dogmatic systems accept the Aristotelian adequation of the affirmation to the thing as their criterion of truth. But this adequation is extraneous to the ways by which it is established. In particular, an entity's existence is independent of the mode of its demonstration. So the irrationality of π could be asserted by apagogical demonstrations reducing the contrary supposition to the absurd, long before Hermite was able to give a constructive demonstration of it. Dogmatism is insensitive to this distinction: π was all the same irrational in itself before any demonstration, direct or indirect. The difference disappears because the logical means employed are posited as being genuinely equivalent, since this equivalence is taken either to be given *a priori*, to be founded on the pragmatic success that collective experience has assured equally to these means, or to be as it were programmed in the laws of the language we all use for communicating our experience. It is postulated, therefore, that the laws of ordinary logic are valid for all possible worlds and that the choice of one or another mode of inference from among them is simply a question of will. The principle of the excluded middle and that of the equivalence of double negation to affirmation are among those laws that the order of things, the success of human action, or linguistic habits have selected as being prerequisites of any possible knowledge.

This is the postulate intuitionism challenges in its refusal to exempt the laws of logic from the inquiry it insists on into the criterion of truth. The adequation it requires, therefore, is no longer that between the thing and its true representation, but that between the representation and the canon proper to guaranteeing its truth. For Epicurus this canon is confirmation through perception of all that goes beyond perception in opinion, and consequently, rejection of all that is not confirmed or at least of all that is not 'non-falsified' in the sense Sextus gives to this term. For Descartes an idea is true if it is clear and distinct, or short of that, as happens in the case of the experience of compound substance, if it does not overstep what can be necessarily concluded from truths demonstrated on the basis of clear and distinct ideas. For Kant a concept has content if it is constructible in sensible intuition, and is without objective validity, if not without regulative utility, once it oversteps the bounds of possible experience. In all these cases then, opposed to the dogmatic notion of adequation is a narrower concept of truth based on a specific test. Granted, the test is very differently conceived of by each of the three; but for each of them

though, it is independent of logic and even a precondition of it. This is why Epicurus confines what must be kept of dialectic to within the narrow limits of the canonic and challenges the excluded middle and the necessity of choosing between exclusive hypotheses forming a complete disjunction. Descartes scorns the logic of the Ancients. True, he replaces it by a geometric algebra, but the theory of proportions governing that algebra excludes expressions requiring an infinite number of algebraic operations. Trigonometry and calculus are thus banished from the method;⁶⁷ the continuum remains beyond the reach of reason, the object of simple mechanical processes, for the only clear and distinct idea we have of the infinite is the intellectual one of order, not the imaginative one of a collection. Kant expressly limits the extension of concepts to finite sets. If he does admit the validity of the excluded middle and of apagogical reasoning in mathematics, it is that he thinks that the empirical character of the intuition in which such reasoning necessarily finds its application guarantees *a priori* the possibility of a constructive demonstration that will sooner or later produce the evidence that was lacking and replace the reduction to the absurd. When dealing, on the contrary, with a pure concept of the understanding to which there corresponds no empirical intuition, the concept of the world for example, apagogical reasoning that would assign or refuse to assign it an object is inadmissible, and simply goes to show that some of reason's pretensions are illegitimate in themselves, even if the concerns behind them are not without ground. Consequently, from the fact that the assumption that the world is infinite or that it is finite leads to a contradiction, we have no right to conclude to the necessity of choosing. This would only arise in the case of an analytic opposition where one and the same existent subject, or one constructible in intuition, is assigned two contradictory properties.⁶⁸ In the cosmological antithetic on the other hand, reason is enveloped in a sophism stemming from the fact that in the major premise (when the conditioned is given the entire series of all its conditions is also given) the word *condition* is taken in another sense than in the minor premise (where the objects of sense are given as conditioned). In the minor premise the word is given an empirical sense, in the major a transcendental one. But the empirical sense requires construction of the synthesis of the conditions regressively in time, so to suppose this achieved is

⁶⁷Vuillemin, 1960, pp. 9-73.

⁶⁸Such is the case for the opposition: π is a rational number, π is not a rational number, before Lambert proved the truth of the second proposition (by the absurd). No one would have concluded that they both were false for want of the existence of the subject π .

contradictory. The transcendental sense, by contrast, requires positing the totality of the conditions as realized and independent of any succession. The confusion results from hypostatization of the subject: the world conceived of as the totality of the conditions of phenomena. There is dialectical opposition precisely when contradictory properties, such as finite and infinite, are attributed to a subject without reality: the world in itself. Kant does not explicitly challenge either the excluded middle or the equivalence of double negation and affirmation. But why does the given totality—in itself—of the conditions of a phenomenon fail to define a possible subject of affirmation, when it denies the conditions of the infinite regressive synthesis it implies, and illegitimately frees itself from the bondage of intuition? Why does Kant confine not only the continuous but even the infinite to the province of sensible intuition, which can and must be given (*gegeben*) but could never be delimited (*abgegrenzt*)? It is that the conceptual synthesis of the 'I think', far from being creative, presupposes a manifold extraneous to the understanding and proper to the sensibility, even when that sensibility is taken as pure. A set theory that would countenance a given infinite number, and *a fortiori* cardinal numbers of higher infinity, would run afoul of the same criticism as that made of the world, since it supposes actualized the totality of conditions given only by the progression of numerical synthesis. But there is more. Not only does the entirely negative Kantian dialectic manifest an affinity with the results of the intuitionist criticism of classical mathematics, but it is seen to be indistinguishable from them, once we examine the nature of the transcendental sense given to the idea of the totality of conditions. For how is it known that this sense is empty and provides only an imaginary object? The answer given is that it is because it corresponds to no intuitive given. But how do we make sure of this? Consider two entire series of all the conditions, one of phenomenon B, today's sunrise, the other of phenomenon A included in the first series, yesterday's sunrise. The second series is a proper part of the first and it can be shown that there is a one-to-one correspondence between the elements of the two series, which shows that the part is as 'great' as the whole. In short, when concepts of proven utility within the province of sensible intuition are applied beyond the limits of that province, concepts such as *part*, *whole*, *equivalence* lose their primitive meaning. It is this *experiment*, already mentioned by Galileo, that makes us leery of the objective validity of the concept of an entire series of conditions. There is however something ambiguous about the experiment. It may indicate that there is no possible object corresponding to the entire series of conditions. But then again, it may suggest that certain prop-

erties of concepts must be modified when the field of their application is switched from conditions taken in the series to the series in its entirety.

How is one to decide between these two interpretations? Kant clearly opts for the first. No positive property can be assigned to the entire series of conditions. The first critical answer to the question of the world's magnitude is thus negative: "the world has no first beginning in time and no extreme limit in space".⁶⁹ The dogmatic antithesis concluded that the world is infinite as regards both time and space. This affirmative answer, which was dialectical, is now to be replaced by an affirmation that is acceptable because it has to do with the constitution of the subjective synthesis of the world and not with the world as a thing in itself. "The regression in the series of worldly phenomena as determining the magnitude of the world continues *in indefinitum*".⁷⁰ What distinguishes the world in itself from the empirical world then, is that the former admits of the illusory attribution of absolute magnitude whereas the latter does not. But how do we get from the empirical world to the world in itself? The empirical world is encompassed within the process of the subjective synthesis whose incompleteness is one of the givens of experience. The thing in itself is posited as the total series of the synthesis. But the only difference that constitutes the act of dogmatic projection lies in the application of the excluded middle. It is recognized that the empirical world does not have the property P (that of having limits). Nevertheless one does not assume the right to conclude that the world has the property non-P, which, in attributing infinity to it, would amount to taking it dogmatically as a thing in itself. The property non-P holds only for the subjective series and is expressed in the experience we have of being able to continue the series indefinitely. The subjective series falls under the excluded middle; it is finite or infinite, and we conclude by the absurd that it is infinite.⁷¹ Application of the excluded middle outside the subjective synthesis results in hypostatizing the thing in itself. To maintain that the empirical world is not a thing in itself is in fact to reject the universal validity of the disjunction of finite and infinite, *i.e.* its validity independent of the conditions of intuition and construction. But this just is the intuitionist principle.⁷²

⁶⁹B548/A520. This answer differs from the first proposition of the antithesis only by the adjectives *erster* Anfang and *äusserste* Grenze, marking that the dogmatic confusion is avoided and that the world is taken, both in the major and minor premise, as a thing in itself.

⁷⁰The same remark would apply in the case of the second antinomy, on the one condition of substituting the words *in infinitum* for *in indefinitum* (B551/A523).

⁷¹*Indefinite* for the first cosmological question, *infinite* for the second.

⁷²In the letter to Chanut of June 6, 1647, Descartes adopts a similar position.

From its required subordination of the logical principles to the direct and positive test of fact results the inevitable introduction of some skeptical traits into the intuitionist method. Epicurean freedom often differs little from suspension of judgment.⁷³ This is the case whenever opinion goes beyond the ken of sense. Cartesian doubt becomes metaphysical and forges fictions going beyond the natural errors and illusions, in order to arrive at an unshakable certainty. Kant praises the skeptical method,⁷⁴ which is in fact one with transcendental idealism. All of these skeptical traits in effect assign limits to the domain of necessity. Epicurus admits that what is true is necessary, but there are only truths of fact; whatever does not fall under sensation imposes no necessity either. Descartes distinguishes the uncreated first truths, all having to do with the divine attributes and the consequences following from the idea of the infinite being, from eternal truths which, though they force their necessity upon us, were nevertheless instituted by divine free will and therefore have a necessity which is not primitive or intrinsic but derives from the constancy of the divine decrees.⁷⁵ What is necessary for our understanding is in no way necessary for God himself.⁷⁶ It is remarkable that Descartes considers as one of the absolute impossibilities that God should be able to make what is or what has been not be.⁷⁷ The Master Argument's second premise taken as a thesis of pure modal logic seems to fall in with the cre-

Only God is infinite, he says; the world is but indefinite. "In which there's a very remarkable difference: for to say that something is infinite one must have some reason that makes it known as such, which one can only have in the case of God; but to say that a thing is indefinite, it suffices to have no reason by which it could be proved that it has limits." The remainder of Descartes' reasoning there is altogether comparable to Kant's (B548/A520).

⁷³See Gassendi, *Disquisitio Metaphysica*, p. 68.

⁷⁴B535/A507.

⁷⁵Gueroult, 1953, II, p. 30.

⁷⁶"As for the difficulty of conceiving how it was free and indifferent for God to make it not have been true that the three angles of triangle should be equal to two right angles or, generally, that contradictories could not be together, it can be easily removed by considering that God's power can have no limits, and then further, by considering that our mind is finite and created in such a way as to be able to conceive possible those things that God willed to be truly possible, but not in such a way as to be able to so conceive what God could have made possible yet nevertheless willed to make impossible. For the first consideration lets us know that God can not have been constrained to make it true that contradictories cannot be together, and that He could therefore have done the contrary, while the other assures us that, even though this be true, we should in no way try to understand it, as our nature is simply incapable of that". (Letter to Mesland, May 1644, IV, p. 118).

⁷⁷Letter to Morus, Feb. 5, 1649, V, p. 273: reference to St. Anselm's *Proslogion* (VII) given by Gueroult, 1953, II, p. 29.

ated eternal truths. As for the third, it seems to result directly from the existence of our freedom. Furthermore, these truths for our understanding are not unfounded once that, given the demonstration of God's existence, His veracity is engaged in the clear and distinct idea we have of necessity.⁷⁸ Two sorts of possibility⁷⁹ and necessity are thus distinguished. Assuredly the distinction is reminiscent of Aristotle and foreshadows Leibniz.⁸⁰ But Leibniz found the creation of good and evil,⁸¹ and of the eternal truths generally, so shocking that he suspected there was some 'philosophical trick'⁸² behind the idea: what was traditionally attributed to the divine understanding was made out to be an object of the divine will, endowed with freedom. The two sorts of modalities distinguished by Descartes are altogether different from the dogmatic opposition between the order of essence and the order of existence or between the principle of contradiction and the principle of the best. Descartes writes to Mersenne⁸³ that since some men "understand the truths of mathematics but not that of the existence of God, it is not surprising if they fail to believe that the former depend on the latter. On the other hand, they should judge that, since God is a cause whose power surpasses the limits of human understanding, and since the necessity of these truths does not outstrip our knowledge, these truths must then be something lesser and subject to that incomprehensible power".

So Descartes' absolute or uncreated necessities are not those of essences, objects of the understanding, precisely because these essences are created. They have to do only with God's truth, which lies beyond our understanding because of His immensity that we cannot encompass. As for created necessities, though on the same scale as our understanding, there is nonetheless something about them that shields

⁷⁸Gueroult, 1953, II, p. 34.

⁷⁹Gueroult, 1953, II, p. 39. "God in His omnipotence can do everything in principle—even what we judge to be positively impossible—as long as it is not repugnant to His very omnipotence, that is to say, as long as it is not a question of absolute impossibility. But having instituted as eternal truth whatever our understanding perceives of as positive impossibilities, being immutable and truthful, God will do nothing in the universe that we judge excluded by them. If He should happen to do so however, He will call it to our attention in an indubitable way. Moreover, He can always do what we fail to understand the possibility of, in the case where we haven't a sufficient reason to judge it necessarily as positively impossible; there is nothing, on the other hand, that could make Him unable to realize whatever our understanding conceives of as possible".

⁸⁰Gueroult, 1953, II, p. 39.

⁸¹Leibniz, Gerhardt, VI, p. 219; Jalabert, p. 230.

⁸²Leibniz, Gerhardt, VI, p. 227; Jalabert, p. 239.

⁸³Letter of May 6; *Correspondence*, A.T., I, p. 150.

their modality from the penetration of reason. And that is precisely the arbitrary and dependent state that is theirs as being created. For to maintain the gulf separating the infinite from us it must be that what is *de jure* for us is in some inscrutable way simply a matter of fact for God. These necessities therefore remain founded on the experience of a given that imposes itself upon us; and the claims of several such givens may tangle themselves in such a knot that we are unable either to unravel or cut it. This is precisely what happens in the case of freedom and providence,⁸⁴ a difficulty that seemed strange to Leibniz for whom the modalities, whichever they may be, are dogmatically bound up with being and are fathomable by reason. "Could he have been unaware," he asked, "that it is not possible for there to be an invincible objection to the truth? since such an objection could only be a necessary logical sequence of other truths whose result would be contrary to the truth one holds, and consequently there would be a contradiction among truths, which is of the utmost absurdity".⁸⁵

Kant explicitly reduces the modalities to the connections of the cognitive faculties with the formal conditions of experience, depriving them of any synthetic force as regards the object.⁸⁶ There is no difference of content between a hundred real thalers and a hundred possible ones. This conception, that rules out the ontological proof, does not fall under the classical disjunction of words and things. Kant takes the modalities neither in a *de dicto* nor in a *de rei* sense, but *de cogitatione*, so to speak. Such is the logical consequence of intuitionism. If the object of knowledge 'revolves' about the knowing subject, modality, divorced from the object, does no more than describe the connection of object to subject. Necessity therefore is not distinct from what constitutes the complete system of the possibility of experience: that is to say, the phenomenon. It makes no sense when supposedly applied to things in themselves. There is no longer the opposition of absolute and created necessity. Only the latter subsists in the form of transcendental idealism. It is one with the act of cognition, tied as that is to phenomena, and gives rise only to illusion when supposed to have bearing on things.

⁸⁴ "However the power and knowledge of God should not keep us from believing that we have a free will, for it would be wrong for us to doubt of what we perceive interiorly and know through experience to be in us because we do not understand something else which we know to be incomprehensible by its nature." (*Principes*, A.T., I, vol. 9, art. 41). In opposition to the absolute necessity of divine preordination is the created necessity, irresistible given the interior feeling, and guaranteed by divine veracity, of my freedom.

⁸⁵ Leibniz, Gerhardt, VI, p. 89; Jalabert, p. 94.

⁸⁶ B296/A233.

The inseparability of the modalities from the experiential givens in general, which is characteristic of the intuitionistic method, makes the importance of psychology or at least of anthropology in such a framework immediately clear. This is manifest in the account of error and in the experience of freedom.

Since the criterion of truth is not adequation, but confirmation as witnessed to by the well-ordered relation among the faculties, error is to be accounted for as an intrinsic derangement of this relation. Excessive expectation above and beyond the sense image for Epicurus, prejudice or precipitation of the free will making judgments that go beyond the passively received ideas of the understanding for Descartes, illusion of reason overstepping the limits imposed by sensibility on the understanding as it constructs its objects for Kant, the mechanism of error is different in each of the three cases. It depends in each of them however on the excessiveness of some faculty's emancipation and disregard for the servitudes imposed upon it by the human condition: heed sensation, admit only of the clear and distinct as true, admit of objects only within the limits of possible experience.

But that faculty's overstepping its bounds is nothing other than an expression—albeit a vicious one—of our freedom. Epicurus is said to have introduced the transcendental concept of freedom into philosophy. The intuitionist philosophers are the only ones to have retained it.

Leibniz made the remark: "It is funny that a man like Epicurus, who had set aside the gods and all incorporeal things, should suppose that the will, which he himself makes out to be composed of atoms, could have empire over the atoms and deflect them from their path without it's being possible to say how".⁸⁷ But he adds that this manifest absurdity is preferable to the subtlety of Carneades that muddles things up in situating the freedom of indifference in the soul⁸⁸ where it seems to be more at home. According to Descartes it is true that the freedom of indifference is but the lowest degree of freedom, still deprived of clarity, yet essential to the possibility of merit and demerit, and human action is "entirely free and indeterminate".⁸⁹ Leibniz said that indetermination spoiled freedom;⁹⁰ but Descartes founded free-

⁸⁷Leibniz, Gerhardt, VI, pp. 307-308; Jalabert, p. 321. This ascendancy of the soul can be compared with the power of the horseman to alter the direction of movement. Leibniz will object that the direction too is preserved (Leibniz, Gerhardt, VI, pp. 135-136; Jalabert, p. 143).

⁸⁸*Ibid.*, see also Gassendi, p. 452, on this point.

⁸⁹*Principes*, Part I, art. 41.

⁹⁰Leibniz, Gerhardt, VI, p. 331; Jalabert, p. 346.

dom on an acute internal feeling (equally contested by Leibniz)⁹¹ that was one with the faculty of doubt, the very principle of his method.⁹²

Such a conception of freedom, whose certainty would be guaranteed by an interior sentiment, leaves much to be desired; and Kant sided with Leibniz' criticisms on this point. "But there are many who believe they can explain this freedom with empirical principles, just as they can explain other natural abilities. They regard it as a psychological property, the explanation of which turns solely upon a more exact investigation of the nature of the soul and of the incentives of the will and not as the transcendental predicate of the causality of a being which belongs to the world of sense; but it is this latter which is what really counts. Thus they deprive us of the great revelation which we experience through pure practical reason by means of the moral law—the revelation of an intelligible world through realization of the otherwise transcendent concept of freedom; they deprive us of the moral law itself, which assumes absolutely no empirical ground of determination. Therefore it will be necessary to add something here as a protection against this delusion and to expose empiricism in its naked superficiality".⁹³

This objection intuitionism makes to some of its expressions is worth attending to. The proof required for the ground of truth cannot consist in a brute fact, since the presence of the fact could always be contested; it consists in the methodical production of that fact. Nor can that methodical production be reduced in turn, without sophistry, to a fact. The activity of control required by intuitionism is enough to distinguish it from pure empiricism. But then what exactly is the connection of that activity to fact? Kant established that what is given as fact on the moral plane is the consciousness of duty. If that consciousness makes us posit freedom it is that there would be a contradiction in our being obliged were we not free. Freedom is the *ratio essendi* of the consciousness of duty, which is itself the *ratio cognoscendi* of freedom. We are therefore led to postulate our freedom without ever being able to grasp it directly in an act of consciousness, because we are faced with the specific fact of duty which refers us beyond the realm of experience. Freedom has the status of an *a priori* synthesis made necessary by the confrontation with a *sui generis* experience requiring the relation of the phenomenon to the thing in itself.

⁹¹Leibniz, Gerhardt, VI, p. 130; Jalabert, p. 137.

⁹²Descartes, *Principes*, Part I, art. 39.

⁹³Kant, *Critique of Practical Reason*, L.W. Beck, 1976, p. 200.

This is a characteristic trait of Kantianism and of Kantian moral philosophy. In examining Kant's own treatment of the question of dogmatic illusion however, we can see its general bearing for intuitionism. The *Critique of Pure Reason* teaches how transcendental idealism allows us to free ourselves from the dogmatic illusion. It does not instruct us why we ought to do so, given that that illusion is natural for our reason. It says simply that the fact of this illusion, which would otherwise be without any way out, is an indirect proof of transcendental idealism.⁹⁴ But supposing we were to remain bound to that illusion, what would the consequences be? What would result would be the most absolute necessitarianism. Kant agrees with Diodorus and admits the consequence he draws from the Master Argument. As long as we are considering a thing under the condition of time and are dogmatically confusing this condition with a property of things in themselves, natural necessity radically excludes any freedom and therefore any possibility that would not be realized in time. For necessity "implies that every event, and consequently every action, which occurs at a certain point of time, is necessary under the condition of what preceded it. Since the past is no longer in my power, every action which I perform is necessary because of determining grounds which are not in my power. That means that at the time I act I am never free".⁹⁵ The Master Argument's first premise, together with the principle of causality and the confusion of the phenomenon with the thing in itself, leads inevitably to necessitarianism. As long as the dogmatic confusion is accepted it is impossible to escape the consequences of necessity. In particular, recourse to a *comparative*⁹⁶ notion of freedom, noting with the Stoics and Leibniz that the determining grounds of our actions are interior ones, resulting from our spontaneity, in no way exempts us from the domination of necessity. This is but useless subterfuge, for "here reference is made only to the necessity of the connection of events in a temporal series as they develop according to natural law, whether the subject in which this evolution occurs be called *automaton materiale* when the machinery is impelled by matter, or, with Leibniz, *automaton spirituale*⁹⁷ when it is impelled by ideas. And if the freedom of our will were nothing else than the latter, i.e., psychological and comparative and not at the same time transcendental or absolute, it would in essence be no better than the freedom of a turnspit, which

⁹⁴ *Ibid.*, p. 203.

⁹⁵ *Ibid.*, p. 200.

⁹⁶ *Ibid.*, p. 201.

⁹⁷ Leibniz, Gerhardt, VI, p. 131; Jalabert, p. 137.

when once wound up also carries out its motions of itself".⁹⁸ Hence the dogmatic illusion results in necessitarianism. But necessitarianism runs against our moral conscience. "A man may dissemble as much as he will in order to paint his unlawful behavior, which he remembers, as an unintentional error, as mere oversight, which can never be entirely avoided, and consequently as something to which he was carried along by the stream of natural necessity, and in this way to make himself out as innocent. But he finds that the advocate who speaks in his behalf cannot silence the accuser in him when he is conscious that at the time when he committed the wrong he was in his senses, i.e., he was in possession of his freedom".⁹⁹ What is revealed through the paradoxical sentiment of repentance is not the actual psychological presence of freedom but its obligatory postulated presence at the base of our past responsibility taken as the *ratio essendi* of the imputation of our acts to a moral agent.

The dogmatic illusion then is really a moral fault, or, at best, an intellectual habit got for the purpose of casting the veil of necessity over our moral faults. To deliver us from this illusion, it is not necessary for intuitionism to appeal directly to the experience of freedom, but it must simply indicate the phenomenon of that freedom, self-blame,¹⁰⁰ which alone is the object of experience in the matter. But Kant's transcendental theory of freedom has a universal bearing for intuitionism precisely because, according to it, all our intellectual errors are faults in that reflection leads us to feel them as such. For Epicurus' impatience of opinion and Descartes' prejudice and precipitation of judgment confront us with facts that accuse us. Even if there is nothing positive about error, formally it still bears witness of our imperfection and poses a problem for us that postulates our freedom by implication. Thus the firm resolve to take only experience into account appeals to a faculty going beyond that experience, and it has been rightly said that it was Epicurus who introduced the transcendental concept of freedom into philosophy. And this very concept is the keystone of intuitionism. Psychology and anthropology give rise to it, though it does not fall within their jurisdiction. It is characteristic of that paradoxical exigency imposed by our finite reason. It is Fichte who best described it in saying: "To be free is nothing, to become free, everything".¹⁰¹

This anthropological exigency may be seen as intuitionism's ultimate requisite. That is how Epicurus took it in placing the gods in

⁹⁸ *Critique of Practical Reason, op.cit.*, p. 203.

⁹⁹ *Ibid.*, p. 204.

¹⁰⁰ *Ibid.*, p. 204.

¹⁰¹ Quoted by Gueroult, 1930, I, p. 269.

the inter-worlds and in depriving them of all care and knowledge as regards our actions. He thus insured himself against the difficulties of any possible theodicy by extirpating them at their root. In limiting necessity and modality to the experiences that make us sense them, intuitionism in general tends to reject questions of theodicy because the necessity linking essences as well as existents has, in itself, only an artificial ground. In other words, as long as intuitionism limits itself to a description of knowledge, it systematically rejects the idea of finality that would give necessity a basis in divine right or noumenal law. Descartes' physics is therefore altogether mechanistic.¹⁰² It is true that Kant's physics, as Newtonian, does admit forces; but these forces have lost the metaphysical and finalistic character given them by Leibniz:¹⁰³ they become the organs of universal mechanism.¹⁰⁴

Yet far from being incompatible with a reflection on the finality of error and evil, intuitionism even invites such a reflection. For it explains error as a conflict of faculties: opinion's out-distancing sensation, the reciprocal encroachments of the understanding and the faculty of judgment, the confusion of pure reason and the understanding applied to empirical intuition. But even if we admit with Epicurus that everything there is results from the interplay of the atoms, once that interplay has issued in the formation of the soul, the soul must surely owe the conservation of its arrangement for a certain time to its internal viability, that is, to the absence of incompatibility that that conservation reveals between the relative stability of that arrangement and the global situation of the other atoms and bodies. It has been pointed out that Lucretius borrowed the notion of natural selection from Empedocles to explain the persistence of the living.¹⁰⁵ But then what natural selection accounts for the conflict between opinion and sensation? For it can be presumed that if the conflict were without survival value nature would not have maintained it. And here, through the fact of the existence of error, the problem of theodicy arises again.

But this problem can only be solved, if at all, by a comparison of the drawbacks of error, formidable ones since error is an obstacle to wisdom and happiness, with the advantages procured as a general rule

¹⁰² "Preoccupied with founding the autonomy of a purely mathematical physics, [Descartes] puts the accent on what God's incomprehensibility comports in the guise of absolute freedom taken as a power or decision subject to no rule of any sort, a sovereign power of *yes* and of *no*, of doing and not doing, of doing thus or otherwise. He thus radically separates science and theology, the material world and teleology." (Gueroult, 1953, II, p. 215).

¹⁰³ Leibniz, Gerhardt, VI, p. 321; Jalabert, pp. 335-6

¹⁰⁴ Vuillemin, 1955, *passim*.

¹⁰⁵ Furley, 1970.

by the cause of error, i.e., the excessive broadness of opinion over sensation. It will be said for example that that excess allows the living to foresee dangers and therefore aids in conserving life. But it is seen that theodicy engages in speculation going beyond sensations. It is this then that represents the extreme limit of intuitionism, going further even than the adventure through which credence was granted to atomism.

Descartes reaches the same limit in connection with his own system when he tries to clear God of the responsibility for error. "Preoccupied with integrating the human experience (formal error, feeling) into the total system of philosophy, and affective psychology and ethics into the system of the sciences, he reintroduced into the created world, through the principle of the best, finality, theodicy, and consequently theology, although it was natural theology... Divine freedom's infinitude can no longer be conceived of then as the *absoluteness of a decisive power*, of a power of arbitrary pronouncement of *yes* and *no*, for God can not subscribe to nothingness: He cannot deny being. There is an ascendancy therefore, in the species, of the idea of the perfect or of the infinite essence over pure freedom. Whether this primacy comes from the divine will itself, arising by virtue of its nature as unlimited omnipotence, or is imposed from without under the constraint of a dominant understanding, as for Malebranche or Leibniz, makes certainly more than just a nuance of difference. But on either hypothesis the fact of that primacy remains indubitable"¹⁰⁶

That balance of freedom and perfection in God dominates the subtle and complicated interplay of psychology and metaphysics throughout the Cartesian theodicy.¹⁰⁷ On the one hand, the incomprehensibility of the divine decrees would have as limit the imperfection of the work and therefore the formal reality of error, which psychology requires. On the other hand, Divine Perfection demands that that error be an absence of being and that the formal imperfection it introduces in nature cancel out as an ingredient of a perfect whole subject to the principle of the best.¹⁰⁸ But if in the final analysis divine freedom submits, so to speak, to its positive perfection like the incomprehensibility of creation submits to the assent we owe to the visible finality of that creation in virtue of the divine veracity, this double subordination is rather induced from signs than produced by the pseudo-necessity of the principle of the best. It is an object of rational faith rather than one of reason.

¹⁰⁶Gueroult, 1953, II, p. 215-6.

¹⁰⁷Gueroult, 1953, I, p. 300 *sq.*

¹⁰⁸*Ibid.*, pp. 306-18.

In this respect it is reminiscent of Epicurus' position on finality. Epicurus' greatest disciple, Darwin, also subordinates the mechanism (still called *chance* and having to do with hereditary mutations) to finality (also called *necessity* and having to do with the selection or conservation of mutant strains). Without nature's sanctions in consecrating these changes as biologically adapted and useful, they would disappear without a trace and there would be no evolution. But how do we ascertain the utility and finality of natural selection? There are two criteria given: fecundity and ecological extension. But these criteria are the external signs by which we recognize what life has judged useful. It is from them we induce that a mutation will have been useful. On the other hand, we understand strictly nothing of the nature of that finality. We know only the mechanisms. But from the presence of signs, proliferation or disappearance of a species, we conclude that there must be some finality. The degree of an individual organism's complexity, often taken as a measure of evolution and as capable of furnishing an internal criterion of finality—as Spencer mistakenly believed—plays no direct role in Darwin's appraisal of the situation. It is possible that very complex and efficient organisms should have been incapable of forming fecund and durable species: selection will have eliminated them. Thus, in spite of the ontological subordination of mechanism to finality, Darwinism maintains the epistemological subordination of finality to mechanism. All that we know of the former is taught us by signs taken from the latter. We are not privy to the council of nature.

The same thing goes for Descartes' system as well. Divine freedom is ontologically subordinate to God's perfection, but epistemologically there remains that unfathomableness that results in my having to believe that what obeys a purpose because it is God's creation, not for some intrinsic perfection that I could find in it. That is why error—a point that scandalized Gassendi¹⁰⁹—may be said to have a sort of perfection, because it belongs to this created world. Once it has been demonstrated that God exists and is the Creator it follows that the creation *must* be a perfect work. But no direct examination of that work would lead us, as through a Thomistic analysis of effects, to posit its finality and therefore its dependence on divine will. Recognition of finality in no way detracts from its incomprehensibility, for they are one and the same. And here we are poles apart from Leibnizian dogmatism. A propos of this incomprehensibility Leibniz jibes at Nicole that if faith and reason are both gifts of God, then the combat of faith

¹⁰⁹Gassendi, pp. 418–20.

against reason is a combat of God against God.¹¹⁰ For Descartes there is a faith within reason, and that is what posits the principle of the best that we cannot penetrate, whereas according to Leibniz it would have to be said that it is this principle, which is the highest reason, that enables us to ground faith on reason.

This difference shows up clearly in connection with the example of the duel that Descartes gave Princess Elisabeth to explain how God's omnipotence and human freedom are reconcilable.¹¹¹ "If a king who has outlawed dueling and who quite well knows that two gentlemen of his realm who live in different cities are having a dispute and are so excited against one another that nothing could stop them from fighting if they were to meet; if, I say, this king sends one of them on business on a certain day towards the town where the other is, and he also sends the other on business the same day towards the place where the first is, he very well knows that they can't help meeting, and fighting, and thereby breaking his law, but he doesn't thus force them to do so, and his knowledge and even the willfulness he had to determine them in that manner do not prevent its being as voluntarily and as freely that they fight as they would have done on meeting had he known nothing of their difference and had it been on some other occasion that they crossed paths. And they can be just as justly punished because they have transgressed his prohibition. But what a king can do in the matter regarding some free actions of his subjects, God, who has infinite foreknowledge and power, does infallibly regarding all the actions of man. Before sending us into the world he knew exactly what all the inclinations of our will would be; it is he himself who put them in us; he is also the one who disposed all the other things outside us so that certain objects would be present to us at certain times, knowing that on those occasions our free will would determine us to some one thing or another; and he willed it so, but he did not will to compel it for all that. And as we can distinguish two different degrees of will in the king, one by which he willed that these gentlemen fight, since he brought it about that they meet, and the other by which he did not will it, since he prohibited dueling; so do the theologians distinguish in God an absolute and independent will by which he wills that all things should happen as they do, and another which is relative and has to do with the merit or demerite of men, by which he wills that his laws should be obeyed." Some time earlier Descartes had

¹¹⁰Leibniz, Gerhardt, VI, p. 73; Jalabert, p. 77 and applied to Descartes, Leibniz, Gerhardt, VI, p. 89; Jalabert, p. 94.

¹¹¹Letter of January, 1646, *Correspondence*, A.T., IV, pp. 353-4; see also *Principes*, part I (41).

written to the same princess that "philosophy alone suffices for knowing that the least thought could not enter the mind of a man, that God should not will and should not have willed there from all eternity".¹¹² Leibniz¹¹³ thought that even Calvin had said nothing harder than this last assertion of God's "total" causality over our least thoughts and he excuses Descartes only if divine will is taken in a "permissive" and not in an absolute sense, as Descartes however explicitly maintains.

The conflict comes to a head when Leibniz corrects Descartes' example in order to make it acceptable. He says that it would be necessary "to invent a reason that obliged the prince to arrange or allow that the two enemies should meet".¹¹⁴ That reason is the principle of the best. On the contrary, for Descartes, from the fact that God did something it follows that it was the best. In another letter to Elisabeth he advances considerations on our freedom and on divine providence¹¹⁵ that are contrary by our lights but both equally well founded, and goes on to conclude that that freedom "is not incompatible with a dependency of another nature, according to which all things are subject to God". That other nature is a truth of simple consequence following upon the proof of the existence of God, hence altogether indirect and already resemblant of a rational postulate, of an object of faith.

Just as the usefulness of survival value of a mutation has only an *ex post facto* justification, the finality and insertion of human freedom in the plans of providence has but an indirect and, as it were, retrospective rationale. As Descartes admits, this is because the truths proportionate to our cognitive faculty in so far as it moves spontaneously and of itself in the bosom of truth are of a different nature from those met with by that same faculty when it has to account for error and for the remedies error calls for. Just what is this difference of nature? It is to Kant's glory to have discerned it and made it the touchstone of his philosophical system.

Taking up a remark of Leibniz' on the difficulties added to the question of freedom by the physical concurrence of God and creatures with the will,¹¹⁶ Kant posits the ideality of time as a necessary condition for the solution of the problem. For if divine action were accomplished under the condition of time, necessity would be inescapable. "Therefore, if the ideality of time and space is not assumed, only Spinozism

¹¹²Letter of Oct. 6, 1645, *Correspondence*, A.T., IV, p. 314.

¹¹³Leibniz, Gerhardt, VI, p. 207; Jalabert, p. 218.

¹¹⁴*Ibid.*

¹¹⁵Letter of Nov. 3, 1645, *Correspondence*, A.T., IV, p. 333.

¹¹⁶Leibniz, Gerhardt, VI, p. 122; Jalabert, p. 128; Kant, *Critique of Practical Reason*, p. 206.

remains, which holds space and time to be essential attributes of the First Being itself and the things dependent upon it (ourselves included) not to be substances but merely accidents inhering in it. For if these things exist only as its effects in time, which would then be the condition of their existence itself, the actions of these beings would have to be merely *its* actions, which it performs anywhere and at any time".¹¹⁷ The ideality of time therefore leaves the way open to a solution of the theodicy. It is the noumena, not the phenomena, that God creates. He cannot therefore be the cause of the actions of beings in so far as they are conditioned by time. Creation then doesn't really add a new difficulty to the problem of creatures' freedom, "because creation concerns their intelligible but not their sensible existence, and therefore creation cannot be regarded as the determining ground of appearance".¹¹⁸

The *Critique of Pure Reason* had restricted the usage of the modalities to the capacity of objects being given in an intuition. But it is possible to go further, as in the *Critique of Judgment*,¹¹⁹ and to inquire into the transcendental origin of the modalities, in tracing their relation to our faculty of knowledge. Leibniz distinguished between essences, residing in the divine understanding and representing in themselves the different possibilities along with their measure of degree of perfection, and existents, objects of divine decree and therefore subject to the principle of the best. Kant reduces this ontological opposition to a transcendental one. The understanding's limitation to the conditions of intuition is the condition for our knowledge's being objective. "If our understanding were intuitive it would have no objects but such as are actual. Conceptions, which are merely directed to the possibility of an object, and sensible intuitions, which give us something and yet do not thereby let us cognize it as an object, would both cease to exist".¹²⁰ Distinction of the modalities is therefore founded on the opposition between our two sources of knowledge. If there is a modal logic, if the modalities do not collapse into one, it is not because there is anything objective in the Leibnizian hierarchy of possible worlds, but simply because our faculty of knowing is irremediably discursive and can reach through its synthesis only phenomena given in the ideality of time. "To say, therefore, that things may be possible without being actual, that from mere possibility, therefore, no conclusion whatever as to actuality can be drawn, is to state propositions that hold true for

¹¹⁷Kant, *Ibid.*, p. 207.

¹¹⁸*Ibid.*, p. 208.

¹¹⁹§76.

¹²⁰*Ibid.*, p. 570.

human reason, without such validity proving that this distinction lies in the things themselves".¹²¹

In taking the fact of error to be the result of divergence among the faculties, intuitionism finds the remedy in critically assigning these faculties a domain of validity. But what error shows is that the possibility of transgressing the bounds set is inevitably tied to our human nature. Opinion outstrips sensation, understanding and judgment encroach on one another's domain, reason inexorably disengages itself from the limits of intuition. But the fact is that these proscribed excesses are not of uniquely negative bearing. A classically demonstrated theorem poses a problem for mathematicians just as selective value and finality do for the intuitionist philosopher. Subordinations arise that, seen from the outside, seem to run counter to the method. Finality, for example, imposes itself as a principle of natural science.¹²² But insurmountable difficulties would arise if it were of the same order as mechanical causality whose sway extends over the phenomena. For teleology is inexplicable. Hence, in so far as applied to objects, it does not afford a principle of truly determinant judgment, but only one of merely reflective judgment, even though this last be universal.¹²³

Kant tells us that the same can be said of the connection between freedom, presenting us with the law as an imperative, and actual action. If our will were holy there would be no such distinction. Thus the requisite subordination of mechanism to finality and of particular to universal,¹²⁴ does not express a dogmatic subordination in the objects, of which we could form a determinant conception. It expresses only a regulative principle, necessarily valid for our *human faculty of judgment*. Far from governing creation like the Leibnizian principle of the best, it is a subjective principle depending on the contingent nature of our faculty of knowledge. The illusion common to Diodorus and his dogmatic adversaries, and probably to all of modern modal logic, lies in the confusion that attributes a constitutive use to principles that have no meaning once divorced from cognizing activity and whose legitimate and simply regulative use is to police that cognizance from the inside without presuming to legislate with respect to things in themselves.

¹²¹ *Ibid.*, pp. 570-1.

¹²² *Ibid.*, §68.

¹²³ *Ibid.*, §74.

¹²⁴ *Ibid.*, §76.

Carneades and the Skeptical Nominalism of the Modalities.

In the *De Fato* Cicero sets a scene with Carneades at grips with Chrysippus and Epicurus, dissociating an implication that his two adversaries both admitted, even though they did draw contrary conclusions from it. First the debate itself will be analysed. Then it will be shown that the dissociation Carneades proposes comes down to challenging the fundamental postulate of dogmatism, namely, the Aristotelian definition of truth. He is consequently able to retain all of the Master Argument's premises without burdening himself with Diodorean necessitarianism for all that. Such a conception leads to a non-existential interpretation of the quantifiers as in the theory of ampliation as it is found in Buridan.

8.1 What is the relation between the principle of the excluded middle and the principle of causality (*De Fato*, X-XII)?

It has been said that the discussion of Carneades and Chrysippus "was over two propositions that formed the basis of the latter's conception: (1) there is no motion without cause; but, he said, an act that would be free and contingent without at the same time being determined would be a movement without cause; (2) of two contradictory propositions about the future, one must well be true and the other false, for the rule of contradictories holds in a parallel way of past and present; but a free act supposes that the true can become false, and inversely. Therefore, in one way as in the other, freedom and determination must be reconciled and, since there is an uninterrupted causal chain, the determination must be an eternal pre-determination, involving a plurality of diverse causal series however so that the necessity of the whole leaves

room for individual spontaneity in each of these series and fate does not wrong freedom".¹ Epicurus denied both propositions asserted by Chrysippus. Carneades took a more balanced stance, typical of his philosophy in general, accepting the first of these propositions but limiting his acceptance of the second.

Such an interpretation fails to draw out all that is in the texts. The dispute between Chrysippus and Epicurus becomes a dispute over the materiality of the two principles of causality and the excluded middle,² as if these two principles were admitted as independent. But that is not the case. For here is Chrysippus' reasoning: "If uncaused motion exists, it will not be the case that every proposition (termed by the logicians an *axioma*) is either true or false, for a thing not possessing efficient causes will be neither true nor false; but every proposition is either true or false; therefore uncaused motion does not exist. If this is so, all things that take place take place by precedent causes; if this is so, all take place by fate; it therefore follows that all things that take place take place by fate".³ The polysyllogism with contraposition takes as its starting point the following Epicurean principle:

If there is motion without cause (P_1), there are exceptions to the principle of the excluded middle (P_2),

and immediately furnishes an application of the principle: the proposition saying that 'there will be a certain swerve' is neither true nor false. The principle is true and the consequent P_2 false. In order to criticize his two adversaries then, Carneades begins by specifying the hypothetical principle they both take to be true. If there is opposition between Chrysippus and Epicurus, it is because the latter, accepting the antecedent as true (the existence of motion without cause), is obliged to admit the consequent (invalidity of the excluded middle), whereas Chrysippus, admitting the falsity of the consequent ($\text{not-}P_2$), the excluded middle being always true, is obliged to reject the antecedent, the invalidity of the principle of causality, ($\text{not-}P_1$). As the universal principle of causality ($\text{not-}P_1$) entails the universal precedence of causes (P_3), and that precedence in turn entails fate (P_4), the truth of

¹Robin, 1944, p.127.

²In spite of the fact that the principle is here expressed in the meta-language, it will be spoken of throughout the paragraph as the principle of the excluded middle and not as that of bivalence. For in the debate before us none of the speakers establishes a difference between those two principles and the discussion at no point touches on Aristotle's solution.

³Cicero, *De Fato*, X.20-21, trans. by H. Rackham in the Loeb Classical Library, 1942, pp. 216-217.

the Epicurean principle together with the universality of the excluded middle entails fate.⁴

Carneades contests neither the Epicurean principle nor the falsity of its antecedent. If he refuses to accept fate it is that he rejects one of the particular premises of Chrysippus,⁵ according to which universal causality entails universal precedence of causes. For neither voluntary motion nor even the inert motion of an atom can be properly said to be without cause: it is simply that their cause is not antecedent or exterior.⁶ Chrysippus replies to Carneades' objection that in denying the universality of antecedent causes one is inevitably open to denying the excluded middle, since "future events that have not got causes to produce them cannot be the objects of the assertions".⁷ But the validity of the excluded middle is admitted.

This last contestation shows that Chrysippus takes the contrapositive of the Epicurean principle in a very particular sense: from the truth of a proposition that he rightly holds to be eternal and that he accordingly retrogrades he concludes the eternity of the event's causes. Carneades therefore proposes another interpretation of the contrapositive, according to which the eternity of the truth in no way entails the eternity of the cause. Thus, with Chrysippus and against Epicurus, he admits the universality of the excluded middle and the principle of retrogradation.⁸ With Chrysippus again, he admits the universality of causality. But against him, he rejects the universality of antecedence: "Yet it does not immediately follow from the fact that every statement is either true or false that there are immutable causes, eternally existing, that forbid anything to fall out otherwise than it will fall out. The causes which bring it about that statements of the form 'Cato will come into the Senate' are true statements, are fortuitous, they are not inherent in the nature of things and the order of the universe; and nevertheless 'he will come', when true, is as immutable as 'he has come' (though we need not on that account be haunted by fear of fate or necessity)".⁹

⁴Chrysippus' reasoning may be schematized as follows:

$$\frac{P_1 \supset P_2, \sim P_2, \sim P_1 \supset P_3, P_3 \supset P_4}{\sim P_2 \supset P_4}$$

⁵ $\sim P_1 \supset P_3$.

⁶*De Fato*, XI.24 "...voluntary motion possesses the intrinsic property of being in our power and of obeying us, and its obedience is not uncaused, for its nature is itself the cause of this" (XI.25, *ibid.*, pp. 220-221).

⁷XI.26, *ibid.*, pp. 220-222. Translation altered.

⁸XII.27.

⁹XII.28, *ibid.*, pp. 222-223.

8.2 Aristotle's dogmatic definition of truth called into question (*De Fato*, XIV).

Carneades' parry, which did not touch Chrysippus, did affect Aristotle. For according to Aristotle, it is certain that the truth of a future entails the actual existence of its cause.

The discord between Carneades and Aristotle is all the more remarkable in that their doctrines of cause seem to be very close. When Carneades asserts that "there is a difference between causes accidentally precedent and causes intrinsically containing a natural efficiency",¹⁰ or again "the causes which bring it about that statements of the form 'Cato will come into the Senate' are true statements. are fortuitous. they are not inherent in the nature of things and the order of the universe",¹¹ it is the very language of Aristotle he employs. When he mocks the Stoics for going back and back and back in infinite regress instead of assigning causes,¹² of being ignorant of proximate causes and of confusing the effective cause bringing something about with that without which it could not have been, it is again an Aristotelian argument and distinction he makes his own.

But if in agreement with the Stagirite on the theory of causes, of freedom and of chance, Carneades differs with him on the theory of truth. For he considers the truth of a future event to be conceivable without the immutable existence of its cause. For to maintain with Chrysippus, but also with Aristotle, that the eternal truth of a proposition about the future entails the necessity of fate, is to say nothing. "For it makes a great deal of difference whether a natural cause, existing from all eternity, renders future things true, or things that are going to be in the future can be understood to be true even without any natural eternity. Accordingly, Carneades used to say that not even Apollo could tell any future events except those whose causes were so held together by nature that they must necessarily happen. For what consideration could lead the god himself to say that the Marcellus who was three times consul was going to die at sea? This had indeed been true from all eternity, but it had no efficient causes. Therefore Carneades held the view that Apollo had no knowledge even of these past events which had left behind them no trace of their passage—how much less had he knowledge of future events, for only by knowing the efficient causes of all things was it possible to know the future; therefore it was impossible for Apollo to foretell the fate of Oedipus when

¹⁰IX.19, *ibid.*, pp. 214–215.

¹¹XII.28, *ibid.*, pp. 222–223.

¹²XV. *ibid.*, pp. 228–233.

there were no causes fore-ordained in the nature of things making it necessary for him to murder his father, nor could he foretell anything of the sort".¹³ The truth of a future- or past-tense proposition, dissociated then from the existence of its cause or of its trace, allows neither prediction nor retrodiction. There is a complete separation between the *determination* of events and the *truth* of propositions.¹⁴

Truth is therefore only a formal property of propositions, and not a real one.¹⁵ But if one refuses, as against Chrysippus and Aristotle, to make the formal and the real coincide, one no longer need fear necessity. "Moreover, those who say that things that are going to be are immutable and that a true future event cannot be changed into a false one are not asserting the necessity of fate but explaining the meaning of terms".¹⁶ Between Carneades and Aristotle there is conflict over the relation of truth and actuality, and the first conflict leads to another over the possibility of knowledge. For Aristotle the truth of a statement results from its agreement with the thing in act and therefore with the actuality of its cause. If a statement about the future is true, it is that the determination of that future is actual. For Carneades the truth of a statement, being formal, in no way entails the existence in act of the thing or of its cause. A statement about the future can therefore be true without that future's determination being actual. Both philosophers of course consider that knowledge of the truth implies the pre-existence of causes: Aristotle, because there is no truth without act, Carneades, because the knowledge of truth is impossible in the absence of a pre-existent cause. But because truth entails act for Aristotle, he is able to save the contingency of the future only by depriving propositions about future contingents of the actual possession of a truth value, whereas Carneades says simply that, such propositions being true or false in themselves, that truth and that falsity are inaccessible to our knowledge which is limited to probability.

Imagine a wise man embarking for Puteoli which is four miles away, with a good helmsman and a tranquil sea, and wondering whether he will get there safe and sound. The proposition 'the wise man will get there safely' is true or false from all eternity, just by virtue of the very definition of the notion of proposition. But is the wise man able to know that truth-value? "... surely he has not got the knowledge already grasped in his mind and perceived that he will make the voy-

¹³XIV.32-33, *ibid.*, pp. 228-229; Robin, 1944, p. 125.

¹⁴Robin, *ibid.*, p. 127.

¹⁵Robin, *ibid.*, p. 128.

¹⁶*De Fato*, IX.20, Rackham, 1942, pp. 214-215.

age as he intends? How can he have it"?¹⁷ All that his knowledge can attain is the probability of a safe passage given what he believes he presently perceives, and that probability is enough for determining action; but there is a gulf between it and truth. The outcome of Carneades' skeptical pragmatism is the probable, as it would have been for Aristotelian dogmatism had it classed all contingent events in a single category. But it is because there are propositions about the future that are neither true nor false that the probable exists for Aristotle, while for Carneades the probable results from the gulf there is between the self-containment of truth and the act of our apprehension.

If a theory of degrees of belief were to be developed for Carneades as one of degrees of determination has been for Aristotle, it might be said that the Carneadean probable opens the way to 'subjective' probability while Aristotelian contingency suggests 'objective' probability. We shall refrain from speculating about these developments however, especially since for Carneades, for whom a statement's truth implies no actual determination of the thing, the accent put on the subject in no way reduces the probable to ignorance of what is determinate.

8.3 Carneades and the Master Argument (*De Fato*, IX).

De Fato, IX contains the following argument. 1) Diodorus' necessitarianism renders the difference between past and future as regards necessity purely apparent (IX, 17). 2) Nor is invoking the notion of degrees in the precision of propositions about the future of any avail for giving that appearance some reality: the more general propositions having to do with essence are no more necessary than the more particular ones having to do with accident (IX, 18). 3) If there is no more necessity in past than in future or in essence than in accident, that goes to show that the question is one of grammar, not of ontology. Rather than having to do with the nature of events it has to do with the nature of statements and with the fact that once true, they always have been or will be so. There is nothing for Epicurus to fear therefore from a grammatical necessity of this sort and it is not necessary to invalidate the excluded middle to establish freedom (IX, 19).

Those today who accuse Cicero of having misunderstood Diodorus in failing to recognize that the necessary does not merge with the Diodorean possible, and who seek a grammatical, that is to say formal, way out of the debate, have simply not read this chapter. For in the

¹⁷Cicero, *Academica*, II.XXXI, trans. by H. Rackham in the Loeb Classical Library, 1933, pp. 594-595.

first two parts of the argument, the Orator shows that the Diodorean distinctions, from Diodorus' point of view, not only fail to avoid but even establish necessitarianism. The last part of the argument shows that he knew of one philosophy that maintained that that Diodorean necessity was only apparent or formal and that was the philosophy of Carneades. The mainstay of the argument's first part is the consideration of the change of truth-value. "... it is no more possible for things that will be to alter than it is for things that have happened; but...whereas in the things that have happened this immutability is manifest, in some things that are going to happen, because there immutability is not manifest, it does not appear to be there at all, and consequently, while 'This man will die of this disease' is true in the case of a man suffering from a deadly disease, if this same is said truly in the case of a man in whom so violent an attack of the disease is not manifest, none the less it will happen. It follows that no change from true to false can occur even in the case of the future. For 'Scipio will die' has such validity that although it is said of the future it cannot be converted into a falsehood, for it is said about a human being, who must inevitably die".¹⁸

It is clear that Cicero is here speaking of propositions, that is to say of the equivalence classes of all possible statements about a certain presently future event. Diodorus, on the contrary, limited himself to statements or to propositions as partial equivalence classes *modulo* a given lapse of time, formulated or not, according to the state of our knowledge. Cicero maintains that in going from the latter to the former necessitarianism *in forma* cannot be avoided.

But let us leave grammar and come back to reality. In other words, let us eliminate these purely exterior variations of truth-value, in taking the equivalence class of statements *modulo* a determinate date. According to Diodorus a *proposition* is possible if it is true or will be true. In this case its truth retrogrades; it is necessary as soon as one takes for granted, with Diodorus, the Aristotelian principle of correspondence.

For Cicero then, Diodorus does not escape the necessitarianism for which the Megarians were reproached. He would only avoid it in sticking to a consideration of grammatical forms alone and in refusing to consider the corresponding propositions.

Some have compared Diodorus to Quine. Just as Quine is skeptical of the contemporary modal logics, so was Diodorus skeptical of the Aristotelian modal theory "but offered nevertheless some 'harm-

¹⁸*De Fato*, IX.17; Rackham, 1942, pp. 212-213. [I have slightly revised the Rackham translation of this passage. Translator's note.]

less' senses that might be attached to modal words".¹⁹ It is known that Quine does not admit the 'propositions' that Cicero brandishes against Diodorus. Yet he proposes converting variable statements into eternal ones by incorporating into the statement the date upon which it is made.²⁰ But in the system of Diodorus this substitution will have the same effect as would the introduction of Ciceronian propositions. Eternal 'future' statements will retrograde since they are determinate, unlike the corresponding statements and propositions of Aristotle. Therefore they are necessary.

Now for the second part of the argument. Cicero has already shown that the Diodorean future has the same modal status as the past. Will there be a way out then in distinguishing, among futures, those having to do with essence (Scipio will die because all men are mortal) from those having to do with accident (Scipio will die a violent death)?²¹ Such a way out is precluded in the Diodorean perspective in which there is no distinction between essence and accident. Hence the accident is every bit as necessary as the essence.

Carneades' conclusion follows from this universal modal collapse. If we are no more able to separate future from past than accident from essence, if every proposition is subject to Diodorean necessity, it must be because that necessity is a fact of language and not a trait of nature, and that the immutability of the truth-values of propositions has no ontological bearing.

It is only in appearance that Diodorean necessitarianism follows from dialectic, *i.e.* logic, which is a neutral art. The illusion is due to a dogmatic interpretation of logic mixing the formal with the real. Interpreted without dogmatism the Master Argument has no force.

Carneades though, in the Ciceronian passage, fails to specify in just which sense the Master Argument is without force. Does he mean that, since Diodorus' necessitarianism has only grammatical bearing, we should accept his conception of the possible as what is or what will be? Or are we on the contrary to keep the third premise without fearing contradiction with the other two? There is one sentence of Cicero's that suggests by implication that Carneades favored the second

¹⁹Prior, 1967, p. 16.

²⁰Quine, 1960, §40, pp. 191-195.

²¹It seems impossible to determine precisely whom the second part of Carneades' argument was aimed at. It will be remembered however that for Aristotle there is no possibility of a science of the accidental. The accidental is that which happens sometimes, as its being cold in the dog-days; and it is precisely its specificity, with respect to essence, that preserves against necessitarianism (*Metaphysics*, K, 8, 1064^b30-1065^a 14).

position. "Hence if, while it is consistent for the Stoics, who say that all things happen by fate, to accept oracles of this sort and all other things connected with divination, yet the same position cannot be held by those who say that the things which are going to happen in the future have been true from all eternity, observe that their case is not the same as that of the Stoics".²² The partisans of Carneades admit the eternal truth of propositions about future contingents. It would seem then—and this impression is reinforced by the entire context of the discussion between Carneades and the Stoics—that these partisans go along with the Stoics in admitting the eternal truth of a possible that will not be realized, but without running the risk of necessitarianism for all that. What is dangerous about the third premise on the dogmatic interpretation is that the truth of the negation entails the act of the contrary event. Once that act is suspended, necessity vanishes.

We must now turn to the logical consequence of Carneades' skepticism. The late scholastic philosopher, Buridan, will provide the occasion for doing so.

8.4 From Carneades to the logics of "fictive" names: Buridan's ampliation.

There is a passage in Buridan that specifies what could have been Carneades' position regarding the Master Argument's third premise: "There is a third distinction, namely, that something is said to be corruptible, either because it is already existent and can not exist, or, in another sense, because it can exist and then afterwards no longer exist. But, in this second sense, there are infinitely many corruptible things that never will be corrupted; for there are infinitely many things that can be engendered but will never be engendered, and even though all things that can be engendered are, by the same token, corruptible, they will never have to be corrupted if they have not been previously engendered. And that is why the only question that arises here arises with respect to corruptible things that already exist".²³

The words 'there are' in this passage have no existential import. A particular proposition can be true without there existing any subject that the asserted property is attributed to. If, in fact, a proposition must be able to be true without for all that there existing or having existed or having to exist *in act* the state of things that verifies the proposition, then it must be that actual existence is completely dissociated from a property's being predicated of a subject and that, as

²² *De Fato*, XV, 33; Rackham, 1942, pp. 228–231.

²³ Buridan, 1942, p. 119.

Carneades would have had it against Chrysippus and Aristotle, truth entails no ontological commitment.

In his theory of ampliation, Buridan systematically developed this conception of a logic thus freed of existential involvement or rather of one rendered explicitly sensitive to the conditions of existence and inexistence.

The theory of ampliation (*ampliatio*) seems to meet the logical need of specifying the ontological status of subjects in two sorts of propositions that can be true when their subject has no actual supposition at the time when the proposition is asserted. These two sorts of proposition are 1/ those about the future and the past as well as those about the possible, and 2/ those depending on a verb expressing a "propositional attitude" (believe, imagine).

Buridan remarks, in his *Consequentiae*: "Some maintain that every true proposition is true because, whatever its manner of meaning, the thing or things are as is signified. As for me, I believe that the following proposition is not true by virtue of what is said, namely, that if Colin's horse that walked well is dead this proposition is true 'Colin's horse walked well', and it is not in the thing as the proposition signifies, since the thing is corrupted... but rather that if that proposition is true it is because it was in the thing as the proposition signifies that it was... Likewise this is true 'Something that never will be can be', not because it is as the proposition signifies but because it can be as the proposition signifies it can be; and so it is evident that according to the diverse sorts of proposition it is appropriate to assign the causes of their truth in different ways".²⁴

Ampliation consists formally in extending the supposition of the subject term to things not existing at the time the proposition is asserted, the extension being made by way of an inclusive disjunction with the present time. 'Some man will die', for example, is to be translated as 'For some x , x is a man or x will be a man, and x will die'.²⁵ But how are the quantifiers to be interpreted? Some have thought that ampliation came down to hypostatizing all subjects that have existed, will exist or can exist by putting them into a pool from which the speaker or the Creator pulls them out as need be.²⁶ Interpreted in this way, the theory of ampliation would inevitably lead to positing the existence of inexistent beings. Indeed, the second category of verbs involving ampliation includes verbs in the present tense which have

²⁴ *Id.*, *Consequentiae*, I, 1; quoted by Moody, 1953, pp. 53-54.

²⁵ Moody, 1953, p. 54 and p. 56 where Albert of Saxony, *Logic*, II, ch. 10, is quoted.

²⁶ Prior, 1967, p. 143.

the power of being transitive with respect to past, future or possible things as well as with respect to present things;²⁷ such are the verbs 'to understand', 'to be acquainted with', 'to think'. The reason for this, as Albert of Saxony explains, is that when a thing is understood, "the act of understanding terminates in that thing just as well, when the thing is something which has existed, or will exist, or can exist, as it does when the thing exists at the same time as the act of understanding it".²⁸ The second use of ampliation suggests that its mechanism would lead to taking terms such as 'chimera' and 'golden mountain' as designating objects having a being distinct from existence. It would thus evoke, in the Middle Ages, the "extravagances" Russell imputed to Meinong and attribute to his own early writings.²⁹

Without a doubt, such an interpretation must often have tempted philosophers. Not all of them, however, succumbed. In contrasting the legitimate consequence, "If I am now eating bread, bread now exists", to the illegitimate one, "If I am thinking now of a rose, a rose now exists", Albert of Saxony clearly shows his disregard for the notion of a pool of beings which would have included the imaginary rose.³⁰ And Buridan explicitly rejects such an interpretation. When the supposition of the subject does not presently exist, we haven't the right to go from truth to existence, though we always have the right to go from present existence to truth. We must therefore reject conditionals of the sort 'If it is true that Colin's horse has run, Colin's horse exists', and, 'if it is true that a possible will never be realized, a possible exists that will never be realized'.

Truth is not a ground of existential import. But truth can combine with what today's ordinary logic calls "existential quantification". Buridan himself declares that "there are an infinite number of corruptible things that will never be corrupted". It's the "there are", i.e., the existential quantification, that is the issue here and gives rise to the difficulties mentioned. The point then is to determine just what sense to give to this notion of logical existence which is altogether other than actual existence.

It obviously does not have its standard or objective sense,³¹ but a purely nominal one. If it is the ways of signifying that dominate ampli-

²⁷Moody, 1953, p. 56.

²⁸Moody, 1953, p. 57.

²⁹Russell, 1956, p. 45; Kneale and Kneale, 1962, p. 262 as regards William of Shyreswood.

³⁰Moody, 1953, p. 57.

³¹Moody is right in rendering the existential quantifier by "for some *x*" rather than by "there is an *x*".

ation it is obviously impossible to give it an objective and homogeneous scope, which seems rather reserved to the way of signifying of the past. In Diodorus' third premise then, the existence of a possible that will never be realized and that must be assured to ground the assertion that it is true that a possible will never be realized is an existence merely in the sense in which a mathematician would say that something exists if it is non-contradictory. To say here that there is a possible is simply to say that it is possible.³²

For Aristotle, the truth of a temporal proposition entails the assignable character of its act at the appropriate time. Such a consequence is homogeneous. It applies to all times indifferently. That is why Aristotle is obliged, in order to preserve the asymmetry of time, to consider the truth-value of propositions about the future in contingent matter to be indeterminate. By contrast, ampliation guarantees the asymmetry from the outset in utilizing a quantification whose actual scope must be reexamined for each mode of signification. Truth here becomes inoffensive in the sense that no conclusions can be drawn from it mechanically. On the contrary, certain "received" laws of modal and temporal logic must be called into question.³³ One law of modal logic, for example, is the following:³⁴

If it is possible for all x that x have a certain property, then every x possibly has that property.

Buridan objects that it could be that everything was God, in the case in which He had created nothing, without it's being the case that everything could be God. In like manner, he objects to the law of temporal logic:

If it has been the case that everything had a certain property, then everything has had that property,

where the antecedent 'It was true (before the creation) that everything was God' can be true, and the consequent 'it is true that everything has been God' false.³⁵

Theories of ampliation therefore deviate from standard quantificational logics. The modern logic that seems the most suited to ex-

³²Moody, 1953, pp. 57-58; thus for Albert of Saxony, the ampliation required for the necessity of scientific propositions indicates that the quantification is over what exists and what can be.

³³Prior, 1967, p. 138.

³⁴Hughes and Cresswell, 1972, p. 144. The demonstration of the law in question ($\vdash M(x)\varphi x \supset (x)M\varphi x$) does not involve the Barcan formula.

³⁵*Sophismata*, ch. 4, sophism 13; quoted by Prior, 1967, p. 138: $\vdash P(x)\varphi x \supset (x)P\varphi x$.

pressing their requisites is Lesniewski's "ontology" or logic of names. Besides singular names naming extant individuals and common nouns, it admits fictive names. It does not maintain the standard interpretation of the quantifiers, but distinguishes between 'There is an x such that fx ' and 'For some x , fx '. As a consequence, it is obliged to introduce new logical functors and arrives at theorems such as 'For some x , x does not exist' and at laws of identity which are very deviant with respect to those of standard logic.³⁶ It is also possible, in the so-called "free" logics,³⁷ to maintain the usual interpretation of the quantifiers and identity, in modifying the laws of quantification in such a way as to purge logic of all existential presupposition attaching to the use of names. The complications accepted in diverging from the standard notion of quantification, or in maintaining it while rejecting the ontological interpretation of proper names and of the word 'all', have the advantage of purifying the logical instrument of all compromise with existence and of clearly expressing the separation of logic and ontology.

But consider the meaning of the logical quantifier 'there is' devoid of any ontological involvement. Saying that there is a possible that will not be realized must be taken as being about a possible that assuredly cannot be realized in the real world, though it can be "realized" in a possible world which is an ideal alternative to this real world. But what can a possible realization as opposed to a real realization amount to if not to that phantom of existence, to that logical 'there is' extraneous to ontology and capable of coexisting with the actual existence of the opposite reality? Does the non-dogmatic interpretation of truth, in other words, automatically constitute a challenge to the principle of conditional necessity? To clarify the positions of Buridan and Carneades, we must take up the question of this challenge again.

8.5 Carneades does not abandon the principle of conditional necessity; he simply deprives it of the ontological involvement conferred upon it by the dogmatic interpretation of truth.

The apparent ambiguity of the skeptical position regarding conditional necessity arises from the fact that, if truth and reality in themselves are inaccessible, it seems difficult to maintain that reality entails necessity while it obtains, though it can be maintained, on the other hand,

³⁶Lejewski, 1957-1958, *passim*.

³⁷In the sense developed by Lambert, for instance in "On Logic and Existence", 1965, pp. 135-141. On the relation between "free logics" and Lesniewski's ontology see Lambert and Scharle, 1967.

that phenomenal or apparent reality, which alone is accessible for the skeptic, does entail an apparent or phenomenal necessity in its turn, so that the principle of conditional necessity is maintained without its entailing dogmatism. Briefly, there are two possible interpretations of skepticism: the first, as a rupture between reality or truth and necessity; the second, as a rupture between reality, truth or necessity for us and reality, truth or necessity in themselves. The first case leads to a rejection of the principle, whereas the second can accommodate it. We will show that it is the second interpretation that must be chosen. In depriving the existential quantifier of its objective signification, the skeptic is not telling us that being is not necessary but that being, and the supposed necessity it entails, is inaccessible.

In denying the existence of cataleptic presentations, what Arcesilas, whom we know to have been influenced by Diodorus,³⁸ is contesting is that sense presentations should be able to *necessarily* command our assent. On the one hand, it is impossible to distinguish absolutely between the true and the false; on the other, our assent remains always free. When Carneades, starting from this critique, develops his theory of degrees of probability, he too is combating the idea of an absolute necessity revealing itself to us in the true.

This allows us to make better sense of the paradoxical transition from Platonism to skepticism in the New Academy. For Plato himself had also contested the view that presentation could attain to certainty. Of the sensible we have only opinion, and opinion can never become science.³⁹ It would seem that the New Academy, more and more attentive to sensation and less and less preoccupied with the ideas, was able to find inspiration in a mutilated but still authentic Plato.

The apparent ambiguity of skepticism results from the following dilemma. On the one hand, Carneades says that the truth of a proposition about the future does not entail its necessity. This is one of the lessons of the Ciceronian *de Fato*. To be consistent, as he does claim to be in his adherence to the principles of dialectics, he must then reject the principle of conditional or hypothetical necessity, for if it is already true at t that a certain thing will happen at t' ($t' > t$), it will be true at t' that that thing will happen at t' . But suppose the validity of the

³⁸Sedley, *op. cit.*, pp. 82-83.

³⁹Thus according to Cicero (*Academica*, II, XXIII (74)), Socrates and Plato are the ancestors of skepticism.

The same Platonic theme is found in Aristotle too: of the sensible there is no science, but only opinion. But as a consequence of the immanence of Aristotelian form and of his theory of first substances, excepting supra-lunar beings, the significance of this Platonic theme is limited (Cherniss, 1962, p. 239, pp. 340-343 sq.).

principle of conditional or hypothetical necessity: if it is already true at t that a certain thing will happen at t' , it will be necessary at t' that, while t' , that thing will happen at t' . But from the truth at t of the occurrence of that thing at t' , we should not have been able to conclude to its necessity, even if only conditionally.

On the other hand, Carneades holds that we cannot distinguish the true from the false. But this thesis, which is deliberately skeptical, enables us to avoid necessitarianism while at the same time retaining the principle of conditional necessity. For what difference would this principle in fact make, since, not being able to attain truth, we are not able to attain the necessity supposed to follow from it either? Here we can conclude from its being "true" at t that a certain thing will happen at t' ($t' > t$) that it will be necessary at t' , while t' , that that thing happen. But since that truth is only "for us" and we cannot conclude from it anything about the nature of things in themselves, the necessity here has only to do with our representation of the occurrence of the thing and not with that occurrence itself.

It is possible to decide between these two interpretations and we have every right to acquit skepticism of the accusation of ambiguity leveled against it.

In the first place there is no passage in the skeptics attesting to abandoning the principle. If it was abandoned it only would be so by implication. But Carneades never ceases reminding us ironically that, as for its contents, the world of the skeptic is not different from that of the Stoic. "For this wise man of whom I am speaking", says Cicero, "will behold the sky and earth and sea with the same eyes as the wise man of your school, and will perceive with the same senses the rest of the objects that fall under each of them. Yonder sea that now with the west wind rising looks purple, will look the same to our wise man, though at the same time he will not 'assent' to the sensation, because even to ourselves it looked blue just now and tomorrow it will look grey..."⁴⁰

In fact, abandonment of the principle is not required by skepticism. If in saying that propositions about the future have an immutable truth-value we are not insisting on necessity but simply defining the sense of the terms, will it not be the same in the case of saying of what is that it cannot not be, while it is? The conditional necessity of what is is a trait of language, not affecting reality itself.

⁴⁰ *Academica*, II, XXXIII (105); Bréhier, 1962, pp. 235-236.

There is a passage in the *Academica*⁴¹ that can help us show how Carneades was able to formally accommodate conditional necessity without being forced into dogmatism.

As the passage on the sea just quoted shows, all our sense knowledge—and it can only be this that is involved where the principle of conditional necessity is at stake—is given to us by way of appearances or sense-presentations. A sense-presentation is a witness, veridical or not,⁴² that is to say, naturally and immediately associated with a mark that makes it the presentation of an object represented. If the object represented is effectively associated with the presentation, it will be said, as with the Stoics, that that presentation can be perceived; Zeno defined it as “a presentation which, coming from a real object, bears the mark, the impression, the image of the object”.⁴³ If the real object is missing it will be said that the presentation cannot be perceived.

The argument can now be reconstructed as follows.

- (1) There is such a thing as a false presentation. Since it is a matter of sensation here, Epicurus contests this assertion, though it is admitted by Stoics and Academics alike. Epicurus maintained that if a single presentation were false the whole of science would collapse, and concluded from this that all presentations are true. The Academics, who were in agreement with Epicurus on his conditional and with the Stoics on the existence of the illusions of sense, will prove the impossibility of science.
- (2) A false presentation cannot be perceived. This second principle is admitted by Epicureans. Stoics and Academics

Let us now reason on the hypothesis:

H. There is a true presentation, A.

Since it is true it can be perceived and a real object corresponds to it.

It is here that the principle of conditional or hypothetical necessity comes into play.

N. H. While true presentation A persists, it is necessary that there exists the real object corresponding to it.

- (3) There is no true presentation coming from the senses to which there can not be compared a presentation differing in no way from it but which cannot be perceived.⁴⁴ The Stoics denied this third proposition.

⁴¹ *Ibid.*, XXVI (83); Bréhier, 1962, p. 225.

⁴² *Ibid.*, XXV (81); Bréhier, 1962, p. 224.

⁴³ *Ibid.*, XXIV (77); Bréhier, 1962, p. 222.

⁴⁴ For the sake of the argument I reverse the order here of these last two propositions reported by Cicero.

To illustrate its meaning we can consider the perception of identical twins. "If therefore a person looking at Publius Servilius Geminus used to think it was Quintus, he was encountering a presentation of a sort that could not be perceived, because there was no mark to distinguish a true presentation from a false one".⁴⁵ If A is a supposed presentation of *a*, there is a presentation B as close as one would to A and, ultimately, indiscernible from A, such that B represents a *b* different from *a*. But B presents itself as a mark or an impression or an image of *a* and therefore cannot be perceived. The dispute between Stoic and Academic centers on this point. As Cicero said "I imagine that Zeno was sharp enough to see that if a presentation proceeding from a real thing was of such a nature that one proceeding from a non-existent thing could be of the same form, there was no presentation that could be perceived".⁴⁶ "Proceeding from a non-existent thing" here is obviously to be taken in the sense of proceeding from another thing than from that which is represented as real. The skeptical position gets its force from the fact that the Stoics admitted the existence of false presentations.⁴⁷

In virtue of H and NH, it is possible to compare to A which is necessary while it is true, a presentation B, indiscernible from A, and which cannot be perceived.

(4) Among those presentations which do not differ from one another, it is impossible that some should be perceived and others not. This is nothing but an application of the principle of indiscernibles to presentations. If A is indiscernible from B, B should have all the properties A has. Stoics, Epicureans and Academics agree on this.

Therefore, on the supposition of (1), of H and NH, since B cannot be perceived, A cannot either. If there exists, therefore, a true presentation and consequently a conditionally necessary one, it cannot be perceived. What will be possible then is to make conjectures about the probable, not to give one's assent to the true. There is therefore no presentation such that a perception of the real follows from it⁴⁸ and there is therefore no presentation either such that a perception of what is conditionally necessary follows from it. What threatened freedom was dogmatism, not conditional necessity. The latter may be retained once the former is renounced.

⁴⁵ *Academica*, II, XXVI (84); Bréhier, 1962, p. 225.

⁴⁶ *Ibid.*, XXIV (77); Bréhier, 1962, p. 222.

⁴⁷ *Ibid.*, XXV (79); Bréhier, 1962, p. 223.

⁴⁸ *Ibid.*, XXXI (99); Bréhier, 1962, p. 232. As Hamelin remarks (1978, p. 30), in agreement with Aristotle on the definition of truth, "Carneades denatures the notions of truth and falsity".

Platonism and Conditional Necessity.

If it is Aristotle's refutation of the *Timaeus* that occasioned Diodorus' formulation of the Master Argument, it is probable that Plato should have denied, by implication at least, one of the principles advanced by Aristotle and tacitly used by Diodorus.

A candidate fitting this bill is the principle of conditional necessity, whose justification is to be found in the analysis of local motion. It is in reflecting on the soul and its pure movements that Plato and the Platonists were led to challenge conditional necessity. In another context, but for similar reasons, Duns Scotus systematized these doubts. We shall also analyze his conception.

9.1 Platonism and the principle of conditional necessity.

In the *Timaeus* Plato himself denied the principle of conditional necessity by implication. The world, being produced and composed by the demiurge out of the primitive disorder, must be susceptible as is everything composite, to being destroyed, even if it is indissoluble except by the power that united it.¹ It is the demiurge's will, constantly opposing this destructive force in virtue of his love of order, that insures the perennality of the world. The demiurge addresses the created gods as follows: "Gods, children of gods, who are my works, and of whom I am the artificer and father, my creations are indissoluble, if so I will. All that is bound may be undone, but only an evil being would wish to undo that which is harmonious and happy. Wherefore, since ye are but creatures, ye are not altogether immortal and indissoluble, but ye shall certainly not be dissolved, nor be liable to the fate of death, having

¹ *Timaeus*, 32C.

in my will a greater and mightier bond than those with which ye were bound at the time of your birth".²

Plato, as will Aristotle, posits a sort of permanence of the modal status of beings. A composite being is by nature perishable and will therefore always remain so. From the fact that it was engendered it always will be able to perish. Yet in the case at hand that force of destruction will never be realized because the will of the demiurge will constantly keep it in abeyance. So far from excluding the capacity of not existing at the same moment, as conformity to the principle of conditional necessity would require, existence is compatible with that capacity, and in virtue of divine will it will remain so for the rest of time. The demiurge's address to the gods implies therefore, on the part of Plato, a revocation of the principle of conditional necessity³ that was nearly unanimously espoused by the Ancients. This stems, for Plato, from the gap there is between the ideas and the sensible world. In so far as it is sensible, necessity characterizes the strictures of mechanism and materiality and therefore expresses only the nature of the image.⁴ It would be foolish to conclude from the strictures on the image the existence of corresponding strictures regarding the ideas.

The Platonic refusal of conditional necessity is therefore independent of the particular way in which the creation of the world is inter-

² *Timaeus*, 40E-41C; Jowett trans. in Hamilton, 1963, p. 1170.

³ In the *Platonic questions*, VIII, 1007 (Plutarch's *Moralia*, XIII, Part I, Cherniss, 1976, I, p. 89), Plutarch writes in the same vein "... as they came into being together [the universe and time], together they will also be dissolved again if any dissolution overtake them, for what is subject to generation cannot (be) apart from time just as what is intelligible cannot apart from eternity either if the latter is always to remain fixed and the former never to be dissolved in its process of becoming". Since they were born together the universe and time will perish together. That is to say, they conserve the capacity of decaying together, even if (by divine will) they are maintained perpetually in being. The reasoning here differs from the demiurge's address above only in that here is posited the compatibility of a constant association of being with a no less constant association of the corresponding contrary capacity. As Cherniss has pointed out to me, the principle of the conservation of modal status is implied in a passage of the *Laws* (818 A7-E2; and Cherniss, 1962, pp. 608-9) where Plato distinguishes two sorts of necessity: a divine necessity to which the gods are subject and which has to do with the objects of arithmetic, geometry and astronomy, and a human necessity having to do with what exists sensibly. A consequence of this distinction is the invalidation of conditional necessity, for the demiurge is not subject to "human" necessity.

⁴ "The nature of this world is blended of intelligence and necessity. What is good in it comes from god, what is evil from the primordial nature as Plato says for referring to matter as a simple substance as yet unadorned by god." (Plotinus, *First Ennead*, VIII, 7, 4-7). Cf. also Plato, *Timaeus*, 48A.

preted, which from what Atticus⁵ says was already a matter of dispute among the ancients. If, with Atticus himself, the literal interpretation is adopted,⁶ the productive power of the demiurge will be placed above the obstacles materiality set up against it. If, with the Platonic *Statesman*, creation is taken in a symbolic sense with the accent put on an indefinite succession of such "creations",⁷ the deity will be seen as indefinitely thwarting the capacity of disorder and destruction that haunts the images by their very nature. Such a capacity, as will be pointed out by Aristotle in the *De Caelo* (281^b15–25), will have to be posited at the same time as the contrary actuality, for the ascendancy of form applies to an infinite time.

That such really was the doctrine of the Platonists finds verification in a passage of the *City of God* in which St. Augustine, criticizing them, shows that starting from their principles one cannot refuse the

⁵A second-century A.D. Platonist engaged in the general polemic of the day with the Aristotelians. Cf. Taylor, 1969, p. 443.

⁶On the opposition between Plato and Aristotle, fr. 4 (1977, pp. 50–4 (801a–804b)).

⁷"Then, when a long time had passed, it began to rest from this tumult and confusion; convulsion gave place to calm, and the world went on its way, settling down into its normal course with superintendence and sovereignty over itself and all it contained, mindful to the best of its power of the lessons of its maker and father. Now it discharged this task at first with much exactness, but more carelessly as time went on, the reason of this being the corporeal strain inwrought in the primal fabric of its structure, which, before it was brought to the order we see today, was steeped in all manner of lawlessness. From its fashioner, indeed, the world has received only good; all the violence and wrong it has within it or breeds in its creatures, it has as a relic of that its ancient state. So while the captain was still there to help in rearing the creatures within it, it brought forth little that was amiss and much that was good; when parted from him, in the years just after the severance, it still orders all things excellently well, but in process of time, as forgetfulness comes over it, the old discord prevails ever more and more till, in the fullness of the days, the world runs wild, producing little that is good with a great admixture of the contrary, and so comes in peril of perishing with all its contents. So there comes the moment when God, who first made an ordered world of it, beholds it in these straits; concerned that it shall not be broken by the buffetings of the storms of disorder, and so founder in the boundless ocean of unlikeness, he takes his place again at the tiller, turns back all that had run to disease and dissolution in the age just past while the world was left to its own devices, restores its order once more, and makes it safe against death or senescence." (*The Statesman*, 273 A–C, trans. Taylor). Mugler, 1960, p. 170 sq., draws a radical line of distinction between the demiurge's redressment of the world in *The Statesman* symbolizing only "the structural information of the world and the regulative power it has over the diffuse forces of necessity" and the diachronic intervention of the World-soul of the *Timaeus* and panpsychism of the *Laws*. But then he is obliged to construe the expressions in *The Statesman* making mention of the world's memory of the demiurge's instructions, for instance, as a simple anthropomorphism (p. 192), whereas it may have been for Plato the sign of the presence of a soul.

resurrection and eternity of the beatified body. "Will it be said that this dissolution does not take place [for the created gods] because God, whose will surmounts all obstacles as Plato says, does not will it so? Who then is to prevent God from not willing it for earthly bodies too, since He can ensure that what has a beginning should exist without end, that what is made up of parts should remain indissoluble, that what is derived from the elements should not return to them? Why would He not ensure that earthly bodies be imperishable"?⁸ As Macrobius will say, we must distinguish between two different immortalities: one proper to the soul and belonging to it by nature, the other proper to the world and thus to bodies which are capable of death but are defended from it by the intervention of another.⁹

The Platonists thus challenged the principle of conditional necessity universally admitted by their adversaries. Are we to conclude from that that their doctrine of creation, taken figuratively or not, opened the way for the Christian doctrine, and that they alone among the Ancients established a clear-cut distinction between "real" and "logical" possibility that would be systematized by Duns Scotus and would make modal logic as we know it today possible?¹⁰ That would mean ignoring that "real possibility" expresses a plausible view of things. It is not formal logic but philosophy that explains the different sense that it took on.

The thesis underlying the principle of conditional necessity for the adversaries of creation is that of the substantiality of the sensible. How would a spontaneous and sempiternal movement of heavenly bodies and atoms be conceivable if the "real possibility" of their rest was supposed? When Aristotle postulates the eternity of the world—identical to the universe for him—and when Epicurus postulates the eternity of the universe—compatible with the vicissitudes of the different worlds according to him—, such an eternity exclusive of all real possibility of corruption, is in no way distinguishable from perennality in time, which is itself indefinite, being nothing more than the measure or accident of an indefinitely continued movement.¹¹ Suppose then that we take away every sort of real necessity from corruptible things and from the particular and partial movement underlying it. In this case, the

⁸ *De Civitate Dei*, XIII, 17, St. Augustine, VII, 1695, pp. 336-7.

⁹ *Aut enim ideo est immortale quid, quia per se non est capax mortis, aut quia procuracione alterius a morte defenditur* (Macrobius, 1952, II. 13).

¹⁰ Such is the upshot of Faust's otherwise erudite and penetrating monograph (1931-1932), but which leaves out Plato and reduces the Platonists to obligatory intermediaries between the Greek confusion and the Christian clarity and distinction.

¹¹ Plotinus, *Third Ennead*, VII, 7-10.

necessity attributed to indefinitely lasting beings (as a result of the substantiality of the sensible) would have to be completely and unconditionally withdrawn from generable and corruptible beings. But then the causality of the heavenly bodies and the atoms would lose its efficacy and would no longer save the phenomena in view of which their sempiternality had been posited. Although the Stoics admit of no permanence with the exception of the divine logos, they nevertheless reach the same result. For this logos lives indestructibly in a time, conceived of as an interval of motion. To eliminate conditional necessity, that is to say, the necessary existence of any finite phase in the history of the world, would be to destroy divine continuity and to usher the void into this world.

It is in virtue of the connections he establishes between eternity and time that the Platonist will refuse these consequences. Since there is no such thing as sensible substance, we cannot assimilate eternity to perennality in time,¹² and the atemporality of the intelligible bears but a distant analogy to time which, even when universally quantified,¹³ affords only a mobile image of itself. Everything in space and time is generable and corruptible,—and what it is could in no way be founded on an immaterial sensible, on invisible atoms or even on a divine intramundane breath. Since what is sensible is the image of an extramundane reality extraneous to time, it must be possible for it, so as not to transgress its status as image, to be other than it is while it is.

Let us begin with the substantial and eternal intelligible. The world Soul must then produce time and the universe together.¹⁴ Hence the Soul will bear to time the same relation which the intelligible Being bears to eternity.¹⁵ As Plotinus says, “the universal Soul, in producing the sensible world, moved, not by the intelligible movement, but by one that is only an image of it, and in striving to render this movement similar to the first, first rendered itself temporal in engendering time instead of eternity, and then submitted its work to time in embracing

¹² “When we say that Being *always is*, that there is not a time in which it is and another in which it is not, it is only for the sake of expressing ourselves more clearly. In saying *always* we don’t take the word in its absolute sense; but if we use it to show that Being is incorruptible it may mislead the mind, taking it out of unity to make it scan the diverse.” (*Ibid.* VII, 6).

¹³ “But Being, whose nature it is to have no need of the future nor to be related to another time, whether measurable or indefinite and existing indefinitely, Being, which is already self-sufficient, is that very Being our intelligence seeks and not a simple equivocation. Its existence does not come from some quantity or other, but it exists prior to all quantity and admits of no sort of quantity in itself.” (*Ibid.* VII, 6).

¹⁴ *Timaeus*, 38.

¹⁵ Plotinus, *Third Ennead*, VII, 11.

the whole of existence and all the revolutions of the world within it".¹⁶ Suppose then that the Soul ceases to act and to exercise its power and withdraws within itself and in eternity. Succession, time, all images would disappear at once.¹⁷ Such is the secondary being of images, of time, and therefore of sensible movement and of all that partakes of quantity. There is a possible world in which the Soul would return into its life of pure contemplation and where that secondary being would disappear. It must therefore not be necessary. Even the sensible things that will last forever, dependent as they are on the activity of the Soul, are without necessity in spite of their perennality.¹⁸ There is no absolute sensible necessity. *A fortiori* there is no conditional sensible necessity.

9.2 Consequences of the connection between conditional necessity and the substantiality of the sensible for modality, causality and freedom.

To admit conditional necessity is to suppose the substantiality of the sensible. The only motion one is then obliged to admit is sensible motion. Aristotle says: "there is no such thing as motion *over and above* the things".¹⁹ Both Stoics and Epicureans followed him on this point. The analysis in this paragraph will be limited to Aristotle's own doctrine and to that of the Peripatetics in general, since they are the ones who had to systematically articulate the consequences of a decidedly anti-Platonic principle and the consequences to be expected from a return to Platonism as well.

If all motion is sensible and subject to the principle of conditional necessity then there is a real distinction, i.e. an incompatibility, between real potentiality and achieved reality. For in virtue of conditional necessity, a substance at time t has the capacity of being at a later time t' in a different situation or state from the one it is presently in at t . This capacity of being belongs to substance therefore only in the form of privation. "For each thing of this kind is capable of being at one time actual, at another not. Take for instance the buildable as buildable. The actuality of the buildable as buildable is the process of building. For the actuality of the buildable must be either this or the

¹⁶ *Ibid.*, VII, 11.

¹⁷ *Ibid.*, VII, 12.

¹⁸ *Ibid.*, VII, 12. "The universal sphere would not exist either as it does not exist prior to time, because it is in time that it exists and moves."

¹⁹ *Physics*, III, 1, 200^b32.

house. But when there is a house, the buildable is no longer buildable. On the other hand, it *is* the buildable which is *being* built. The process then of being built must be the kind of actuality required. But building is a kind of motion..."²⁰ Since the formulation expressing the real potential contains both the present privation and the present capacity of future realization, it would be contradictory to posit that realization presently.

This disposition of the modalities making motion a real 'imperfect'²¹ actuality, subjects the universe to an extrinsic causality. Take the point of view of the movable. It is presently deprived of a property or a situation that it is capable of having later. Since what it is in actuality now is only a privation with respect to what it is in potency, this cannot be the cause that will actualize what it potentially is, and the onset that will be the source of that actualization has its origin in something else. The moved which is deprived of what it is in potency and the movent that actually possesses what that potentiality is a potency for²² are not identical then, unless it be accidentally. In other words, the movable, in so far as it is moved, is not to be confused with what moves it, otherwise there would be complete actuality and not motion. But if that is the case, then everything that is moved is moved

²⁰ *Ibid.*, III, 1, 201^b7-13.

²¹ *Ibid.*, III, 2, 201^b29. Motion is an actuality, though an imperfect one. Saint Thomas comments (Marietti, 1965, 2305, p. 546): "The reason is that it is the actuality of something incomplete, that is to say, of what is possible or what is in potency. For if it were a complete actuality it would eliminate any potentiality, which is the capacity in matter for a given determination. That is why complete actuality is the actuality of what is in act and not of what is potentially. But motion is the actuality of something potential since it does not suppress the potentiality. In so far as there is motion, there is a potentiality in what is moving for that to which it tends by way of the motion. But it is only the potentiality that was the potentiality of being moved that motion suppresses. And even this is not totally suppressed since the moved is still capable of being moved, given that all that is moved will be moved in virtue of the division of continuous motion".

²² *Physics*, VIII, 5, 257^b9-10. "The movent on the other hand is already in activity: e.g. it is that which is hot that produces heat: in fact, that which produces the form is always something that possesses it."

by something else. There is no self-mover.²³ The known desirable is an unmoved mover; desire is a moved mover.²⁴

The nature attributed to the soul derives from these principles. The soul is not a self-mover. Nor can it be moved except accidentally, for otherwise, since all motion is divisible, its simplicity would disappear along with its essential immobility.²⁵ For the human soul, sensation and knowledge are to be accounted for by its being moved to what it is only potentially by an agent that is actually the object of that potentiality. For sensation, which is potentially all the sensible forms, this actualizing agent is provided by the external sensible objects. In the case of thought, it is necessary to posit an internal efficient cause to actualize all the intelligible forms of which the agent is capable. Such is the agent intellect—whatever difficulties for Aristotelianism may be entailed by the necessary actuality of the forms through which that intellect produces knowledge.²⁶ The freedom of such a soul, if admitted,

²³ *Physics*, VIII, 5, 257^a 31–b 13.

"Aristotle's reason for refusing to be content, as Plato was, with the notion of a self-mover, is that in so far as it moves, it must already have a certain character, while in so far as it is moved, it must have that character only potentially, and actually not have it. *E.g.* that which warms itself must be warm in order to impart warmth, and cold in order to receive it. The law of contradiction, therefore, forces us to analyze the self-warming into a part which is warm and a part which is cold, *i.e.* self-imposed change turns out to be change imposed by one thing on another." (Ross, *Aristotle's Metaphysics*, I, CXXXIII, n.5).

The formula for the moved at t is:

$$M_t p_{t'} \sim p_t \cdot t < t'$$

At the same moment t the movent has the property expressed by ' p_t '. What leads to the contradiction pointed out by Ross is conditional necessity, since if the formula for the moved at t could be

$$M_t p_t \sim p_t,$$

there would be no need to appeal to a movent other than itself for it to produce the action. Aristotle's argument in *Physics* VIII, 5 is a dilemma (Cherniss, 1962, pp. 390–1, n.210). The continuity and therefore the divisibility of motion renders a self-mover impossible. Either the self-mover moves itself as a whole, but then it would both undergo and cause the same motion at the same time, and motion would no longer be an incomplete actuality. Or self-motion would result from a partial motion which would be reciprocal (with A moving B and B moving A, in effect reproducing the previous confusion of action and passion) or ordered. If self-motion is partial and ordered, a part A will move a part B without being moved by it. But if A moves itself in moving B the same difficulty will arise again: A would undergo the motion it causes. Therefore A must move B while remaining immobile itself. The mainstay of the argument is the essential separation of movent and moved, in other words, conditional necessity. For the Thomistic doctrine on this point see Effler, 1965, pp. 180–91.

²⁴ Saint Thomas, *Summa Theologiae*, Part I, quest. 80, art. 2; Gilson, 1952, p. 583.

²⁵ *De Anima*, 408^a 34–b 30; Cherniss, 1962, p. 402 sq.

²⁶ I'm following Cherniss quite literally here, *ibid.*, p. 169.

cannot be conceived of as a capacity for non-motivated decision. Free action then will consist simply in the play of intellectual representations informing the will, the explicative motive of our actions being the representation of the good.

9.3 The consequences of abandoning the principle of conditional necessity and the substantiality of the sensible world for the Platonic and Platonistic theories of modality, causality and freedom. The same abandonment entails similar consequences for Duns Scotus.

For Plato, sensible motion, which is attributed to the unorganized agitation of the receptacle, far from representing the actuality, even if only imperfect, of substances, expresses the simple deficiency that prevents the sensible image from reflecting the idea exactly. It lacks the minimum consistency required of an image for reflecting an idea. By contrast, there are other motions of a spiritual nature—thoughts, desires and feelings—which are characteristic of the human or universal soul and prior with respect to physical motions.²⁷ Soul is at once contemplation of the ideas and principle of motion. Its motive force, which is immediately manifest in the case of spiritual motions, mediately so in that of sensible ones, depends for its regeneration on the energy got from contemplating the ideas. It can also happen that the soul, which normally animates a body, spends at least some time in the purely intelligible world. At such a time it exercises its contemplative function without exercising its motive function; and this seems to establish a hierarchy between the two functions. Even within the act of intellection a distinction must be made between thought in motion and thought at rest once contact with the intelligible is attained. It is this spiritual motion and rest, recognized as positive phenomena, that must reflect the two ideas of motion and rest.²⁸

The Platonists will modify this doctrine. Plotinus, for example, insisting on the superiority of the contemplative life of the soul over its motive activity, will put the accent on the insubstantial character of

²⁷ For instance *Laws*, 898 C; Cornford, 1960 (1935), p. 246; Cherniss, 1962, p. 402.

²⁸ *The Sophist*, 254 D-F; *Parmenides*, 129 D-E; Cherniss, 1962, p. 439.

the sensible.²⁹ The invalidity of conditional necessity here will be all the more evident.³⁰

²⁹The organizational activity of the soul presupposes an immutable contemplation of order (Bréhier, 1961, p. 34). "The part of the soul which is the first is on high; always near the summit in an eternal plenitude and illumination, it remains there and participates, the first, in the intelligible; the second part of the soul, which participates in the first, proceeds eternally, a second life issuing from the first, activity projecting in all directions and nowhere absent. In proceeding, the soul leaves its superior part in the realm of the intelligible, that the inferior part leaves; for if its course caused it to abandon that superior part it would no longer be everywhere but only in the place its course reached." (*Third Ennead*, VIII, 5). In comparing the life of the One to a cosmic tree, Plotinus distinguishes the principle of life which remains in the roots, immobile, from the life which circulates throughout the whole tree (*Ibid.*, 10). The same metaphor can apply to the two "parts" of the soul. It is because it lives in intelligible eternity, out of all time and space, that the soul can act ubiquitously and animate an extended body. It is only the successive manifestations of this body in sensible motion that are subject to the principle of conditional necessity.

This is because, following the Platonic teaching, motion, which is a supreme intelligible genus along with rest (*Sixth Ennead*, II, 7), exists *qua* intelligible prior to time and to space (*Ibid.*, II, 16). For an idea could not admit of the conditions of possibility and *a fortiori* of definition that characterize only sensible being. "The Motion which acts upon Sensible objects enters from without, and so shakes, drives, rouses and thrusts its participants that they may neither rest nor preserve their identity—and all to the end that they may be caught into that restlessness, that flustering excitability which is but an image of Life.

"We must avoid identifying Motion with the objects moved: by walking we do not mean the feet but the activity springing from a potentiality in the feet. Since the potentiality is invisible, we see of necessity only the active feet—that is to say, not feet simply, as would be the case if they were at rest, but something besides feet, something invisible but indirectly seen as an accompaniment by the fact that we observe the fact to be in ever-changing positions and no longer at rest." (*Sixth Ennead*, III, 23, trans. Stephen MacKenna).

The relation of the sensible to the intelligible is therefore comparable to the relation between the two functions of the soul. The soul moves but even in moving remains in contemplation of the idea. The self-mover, which is none other than the soul then, 'comes' to the sensible but does not 'remain' in it. Motion goes toward the mobile but does not inhere in it as to be severed from its mover, the invisible soul, that diffuses it like a breath upon the agent it animates.

³⁰The categories of the sensible world are not to be conceived of as being on the same simple pattern as those of the intelligible world. In the intelligible world motion and rest are two distinct and even contrary supreme genera; soul and idea, whose meeting is defined by contemplation, both belong to the realm of being. But is there any ground for positing the same opposition or even distinction in the sensible world? Are sensible immobility (ἡρεμία) and motion opposed in the same way that intelligible repose (στάσις) and motion are? Not at all.

"Stability in that [the intelligible] realm does not arise from the fact that what is of a nature to move is not moving; what is There is stable because Stability has taken hold of it; in so far as it has Motion, it will never cease to move: thus, it is stationary under the influence of Stability, and moves under the influence of Motion. In the lower realm, too, a thing moves in virtue of Motion, but its Rest

Duns Scotus' system lies outside the orthodox Platonic tradition, and is even radically hostile to it. He never relents criticizing Avicenna, the neo-Platonist for whom salvation is to be expected from philosophy as a matter of course. It is not that Avicenna is wrong in appealing to the dignity of human knowledge, to our soul's affinity with the intelligible, in short, to its need for God. The error lies in attributing to the possible intellect a natural light which it does not have in *act pro statu isto* but for which it is owing to Christian revelation and theology.³¹ Aristotle described man as he is, Avicenna as he ought to be. The first was unaware of grace; the second confused it with nature. Both were unaware of freedom, the one confining it to the miscarriages of nature, the other to the supposed necessity of creation. But in dissipating the Platonic illusion that attributes a supernature to nature, Scotus denounces Aristotle's mistake of reducing nature to its present state and exalting that state to the level of essence. For the Platonists, the invalidity of conditional necessity required the compatibility of the actual with the simultaneous contrary potential which transcended nature. With Scotus this requirement finds a ground which resides in the excess of theology over metaphysics.

Saint Augustine extended to the union of human body and soul that perpetuity that the Platonic demiurge had established for the union of the world soul and the world. What is dissoluble by nature was therefore to coexist with an elective indissolubility. Scotus takes over the same thesis in other terms. To deny the validity of conditional necessity is to assert that a state of fact brought about by God can forever defy the natural possibility of the contrary state. But if we are bound by theology to believe in the reality of that state of fact, we

is caused by a deficiency; it has been deprived of its due motion." (Plotinus, *Sixth Ennead*, III, 27, trans. S. MacKenna, pp. 517-8).

Sensible immobility is therefore a limit of sensible motion: it is not its contrary. But in so far as the motion is an ordered and regular one, in so far as it is of the sort that only a soul can impart to a body, we find Plotinus in the same situation as that described by Plato in the *Timaeus* for the motion of the Heavens. This motion can come to an end in virtue of the possible dissolution of the world, but the Demiurge does not allow that dissolution. For Plotinus, the case is the same for the organized movements of a living being: they can cease, but in so far as animated by the soul, the soul will not allow them to cease. The inert world of Platonic necessity has been compared to entropy (Mugler, *op. cit.*): the demiurge must constantly reintroduce information to maintain the world in dynamic equilibrium. The Plotinian soul exercises the same office. The universe is constantly prey to a possible dissolution or immobilization; but the energy supplement afforded it by the soul, nurtured on ideas, prevents that's happening. There is a possible then that is never realized, because of the will of the demiurge and the soul; and as it never can be realized it must coexist in the instant with the contrary actuality.

³¹Gilson, 1952, pp. 17-24, pp. 38-43.

are bound by logic to postulate that such a state is possible by nature. There is therefore in the human composite, just as in the cosmic one, a natural passive potential for incorruptibility. No natural active potentiality however comes to activate that passive one. From the philosophical point of view then that passive potentiality will remain a dead letter. The theologian, whose views are not limited simply to nature, postulates then that to every passive potentiality—understanding by that what a thing is capable of at best, given its essence as union with the soul for the body—there corresponds a natural active or *free* potentiality.³² In short, what thwarts conditional necessity are perfections not inscribed in the nature of beings; and the passive possibilities instituted by these perfections will only become real through the actuality of a free cause.

Instead of invoking spiritual motions as does Plato in saying what it is that retains Socrates in his prison, the point of departure for Scotus will be the passive potentialities revealed in contingent unions. The superior part of something contingent will include the passive potentiality of not being subject to the constraints of the union. Thus, even if the human soul was created in union with the body, it must be conceived of as capable of being created for itself in a first instant of nature, with its role as part of the union relegated to a second instant.³³ As for the inferior part, the fact of the union assures it of its capacity of being united to the superior part. Since a free active potency is required for actualizing these passive capacities, the immortality of the soul as well as the resurrection of the body will depend on such. The theologian makes the metaphysician stipulate the passive capacities of beings. Matter itself as receptive of form is not reducible to a simple indeterminate capacity. Its natural inclination to another form confers positive being on it.³⁴ From this disposition results the multiplication of formal distinctions. The body, for instance, must be capable of being

³² "According to the Theologians, to say that there is an active natural potentiality corresponding to every passive natural one is to say something false, for the nature of the higher beings is capable of more perfection than that within the power of active natural potentiality. Nor does this passive potentiality exist in vain, for it can be actualized by a free agent as well as by a natural one; and if it is a matter of acting outside of itself the free agent is even more efficacious and has more power than the natural one, since the free agent is infinite, which the natural agent is not. That proposition must therefore be taken as follows: to every natural passive potentiality there corresponds either a natural or a free potentiality that reduces it to act; and to this I agree." (*Rep. Par.*, L IV, d.43, q.3, n.18; Gilson, 1952, p. 645; Vivès, XXIV, p. 519).

³³ Gilson, 1952, pp. 480–1; see 4.2, p. 80 *sq.*

³⁴ *Op. Ox.*, L II, d.12, q.1, n.11, Vivès XII, p. 558; *Rep. Par.*, L II, d.12, q.1, n.10, Vivès, XXIII, p. 6.

united to the intellective soul which is its 'essential form'. But there is such a gulf between them that an 'organic bodily form' seems necessary as the ultimate disposition of matter to receive the intellect.³⁵ The multiplication of forms will result from the requisite specification of every potentiality become positive. But the hierarchy ordering that matter and those forms increases, rather than diminishes the distance between them. As strong as the bond between them is, there is nothing that could prevent God from creating matter without form. For Plato the bonds established by the demiurge's creation took precedence over the constraints of the sensible world. Divine power, for Scotus, takes precedence over the bonds formed by creation. How could conditional necessity subsist in the face of that power?

In connecting sensible motion with a mobile, Aristotle is drawing the consequence of his modal definition of motion as imperfect act, identical with the process of realizing form in matter. But let us, with Plato, consider the motion of *things* as secondary. A thing can be moved by other things that move, but the ultimate cause of its motion cannot exist in any other thing.³⁶ We must posit its model in the idea of motion and its realization in the life of the soul. But if motion cannot be reduced to any other given than motion itself, there is no longer a reason for its modal analysis in terms of imperfect actuality. On the one hand, the idea of motion is pure actuality in virtue of its immateriality. On the other hand, the soul, which takes its model from that idea and produces the spiritual motion and secondarily the physical one, is free from all potentiality for it is not a subject susceptible of contrary determinations: it is the self-mover in act. Only physical motion then falls under the Aristotelian analysis. It alone requires the distinction between the mover and the moved.³⁷

Scotus has similar thoughts on the matter. He says that it seems impossible for an angel to move if the motor and mobile are always different and if what is in act cannot be in potency. But the impossibility is only an apparent one stemming from the application to spiritual things of what is only valid for material things in general.³⁸ We must

³⁵ *Rep. Par.*, L IV, d.11, q.3, n.22, Vivès, XXIV, pp. 125-6.

³⁶ Cherniss, 1962, p. 453.

³⁷ *Ibid.*, p. 441, p. 453.

³⁸ *Op. Ox.*, L II, d.2, q.10, n.1, Vivès, XI, p. 523; *ibid.*, d.25, q.1, n.12, Vivès, XIII, pp. 207-8. "When it is said that mover and moved must necessarily be distinct by their subject, what is said is true only of corporeal things: I think that even here moreover it is not necessarily true. But I say that it is simply false for spiritual things. Otherwise God could not create a single Angel, bare in its nature, and which, left to its nature, could comprehend its essence, thereby being a same mover and moved indistinctly as to the subject."

distinguish between the modal incompatibility and the substantial incompatibility of potency and act in the instant.³⁹ As a mode, potency has to do with a being that doesn't actually exist, but that can exist. In this case there is incompatibility between being potentially and being actually at the same instant.⁴⁰ By contrast, as substance or principle, the potency is either a material or receptive cause or an efficient cause. But once the causal point of view is taken the incompatibility of potency and act no longer holds in general. Potentiality and actuality can coincide in the same instant precisely because they are not superposed exactly in all respects. It is only in univocal agents that there is an incompatibility between potency and act. In equivocal agents on the other hand this does not obtain.⁴¹ For as regards these, the subject in

³⁹Effler, 1965, p. 182.

⁴⁰*Op. Ox.*, L. II, d.25, q.1, n.12, Vivès XIII, p. 208. "When you say that this is the agent in act, that the patient in potency, etc., I say that if *act* and *potency* are taken as two first differences of being, they divide also everything that is. Consequently, they divide any given numerically identical being. Thus it is a contradiction for one and the same thing to be simultaneously in act and in potency relatively to a same thing, because the potency, in so far as it is a distinct difference of being from act, is necessarily inclusive of the opposition or negation of that with respect to which it is posited a potency, as the white in potency is not the white in act, as long as it is in potency. And so it is impossible that something thus existing in potency should actualize itself."

⁴¹*Ibid.*, n.13, p. 208. "But *act* and *potency* may be taken in an entirely different way, namely as dividing the potency or the active principle into the univocal and the equivocal. For as regards univocal agents it is true that one is the patient in potency, another the agent in act, and it is therefore impossible that a same thing should be both formally such-and-so in act and simultaneously such-and-so in potency. As regards equivocal action however, this need never be the case, for here the agent must be more noble and virtually so."

Consider the example of 'intensive quality', i.e., of the *intension* and *remission* of forms. Albert the Great and St. Thomas had held that in the case of intension or remission the original form or degree was destroyed and a new form, more or less intense, was created in its place. Their position was based specifically on the principle of conditional necessity. The moments of motion are incompatible: "each *ubi* is destroyed when another *ubi* is occupied" (Maier, 1951, pp. 60-1). It is this analogy with local motion that Duns Scotus combats when he asks whether the pre-existing charity perishes in its entirety when a different degree of charity is attained, so that there is no numerically identical reality that remains in the different degrees of charity (*op. cit.*). The subject receiving a form in different degrees is equivocally one: it is more or less perfect. What it has then to one degree it can, at the same moment, have to a higher degree. Consequently, "... if you said that nothing can be identically and at the same time potentially other and have in actuality that other thing either eminently or virtually, I say that that is false, for that comes down to saying that such a thing is not capable of its perfection. For we see that nature has given to that to which it gave the potentiality of being augmented the active potency of effecting the augmentation and of sustaining it that it might be conserved" (Scotus, *ibid.*, p. 207). The intellectual soul, therefore, has the potentiality of different perfections which are not distinct as to the subject.

potency and subject in act are only equivocally the same, as is required by the imputability of action.

The objection against the existence of a self-mover is therefore removed.⁴² "Soul, however, as self-motion in which there is no distinction of substrate and activity can have no potency of motion in this Aristotelian sense which involves the possibility of contrary determinations but as a positive process must have a pattern in reality of which it is the manifestation".⁴³ The life of the soul is not the attribute of a substance. We must not say that soul moves, but rather that it is self-mover or self-motion.

Once the principle of *omne agens est praesens passo* is given up, the way is clear for the freedom of the human will.⁴⁴ The proposition assigning an external cause to every volition had been condemned⁴⁵ in 1277. The faith posits that "nothing other than the will is total cause of the will in the will".⁴⁶ The will continues therefore to exist

"Therefore it is absurd that a very noble form, such as the intellective soul, should not have the active as well as the receptive potentialities of its accidental perfection. And because in such forms there cannot be given an active and passive potency distinct from the subject, as they are not organic potencies, they are therefore not distinct by the subject and will therefore be united here without distinction as to the subject, without however being deprived of formal distinction." (*Ibid.*, n.13, Vivès XIII, p. 208).

⁴²Duns Scotus, *ibid.*, n.14, pp. 208-9 "Likewise, it has been shown in book four that a separated soul has the power of moving itself to another place, for to move thus is suited to a very imperfect being, and what are distinct perfections in an inferior nature must be unified in a more perfect nature to which the inferior one is ordered."

⁴³Cherniss, 1962, pp. 441-442.

⁴⁴*Op. Ox.*, L II, d.25, q.1, n.2, Vivès XIII, pp. 197-198.

⁴⁵Gilson, 1952, pp. 377-379.

⁴⁶*Op. Ox.*, L.II, d.25, q.1, n.22, Vivès XIII, p. 221. The condemnation was of "the proposition that the soul wills nothing unless it is moved by something else. Whence it follows that it is false that the soul moves itself. This is wrong if we understand moved by something else, namely the desirable or the object, to mean that the desirable or the object is the entire reason for the movement of the will." That the soul is necessarily moved by something else was the position of Scotus' adversary, Godfrey of Fontaines. From this argument he distinguishes that of "another Doctor" who is St. Thomas: "As concerns the proof that it is the same thing to say that mover and moved are indistinct as to subject as to say that there is a self-mover, it seems that another Doctor would concede that the will is moved by an object in so far as apprehended by the intellect, and yet that the object as apprehended by the intellect or the intellect showing said object are not distinct from the will as to subject; and yet he would not concede that mover and moved here are one and the same thing." Scotus refutes both opinions. "As for me, I say not only that they are indistinct as to subject, but that one and the same thing simply can be mover and moved." (*Op. Ox.*, L.II, d.25, q.1, n.12, Vivès XIII, p. 208; see above Chapter 4.2, note 30, p. 81).

as a power at the same moment in which it produces its acts. For it is the equivocal, not the univocal, cause of the acts of volition, being more noble and eminent than they. Abandonment of the principle of conditional necessity and of its consequence, the principle of external causality, revolutionizes the theory of freedom in substantializing the will as the equivocal and autonomous principle of its acts. Conditional necessity, even there where it does apply, *i.e.* in the world of phenomenal effects, does not then have an absolute validity, but only a validity *secundum quid*.⁴⁷ It is a sort of necessity taken on contingently by what is contingent by nature.

The Master Argument is ineffectual here. Duns Scotus and Plato do give different explanations of the phenomenal world, and no matter how the descent of the ideas into things is represented it is not comparable to the contingent creation of the Christian God. But the Idea does have this much in common with the Creator: what necessity there is, whether logical or real, remains within it; and whatever is produced outside of it by any mode whatsoever, *i.e.* be it necessarily or contingently, is itself devoid of necessity. Conditional necessity belongs to the laws of nature; but the laws of nature are decidedly contingent.

Aristotle's hylemorphism supposed the validity of the principle of conditional necessity: (1) the ideas are never separated from the sensible, (2) every motion is a sensible motion, (3) the mover is never identical with the moved, (4) self-motion is a contradictory concept, (5) the hierarchy of the causes within motion points at a prime, immobile mover, (6) finally every motion is the act of a potency as such (potential teleology).

Abandoning the principle of conditional necessity (1) makes sense of a world of separate ideas or of separate or separable spiritual sub-

⁴⁷Gilson, 1952, p. 586. The consequences of that assertion have been examined. To admit extrinsic causality only is to leave the way open to infinite regression. "In accepting that the first action in the will should be caused by an object, whatever be the object that must be posited according to you, that action is purely natural. Therefore it is not within the power of the will. And indeed as Augustine says, it is not in our power not to be touched by what we see. If therefore after that act I can move my intellect to consider this or that, I ask by what act. Not by that which I have just spoken of, since it is not within the power of the will. Therefore it must be by another. I ask where that other is from, if it is through the will itself or through an object or phantasm? If it is through the will, then I have what I advanced since that will is within its power and effectively is through it, and for the same reason the same is true of the first volition. Or again it is through something else, namely an object or a phantasm. And if this is so it will then be a natural act and consequently, it will be no more within the power of the will to thus command the intellect to consider this or that than it was to produce the first act" (*Op. Ox.*, L. II, d. 25, q. 1, n. 7, Vivès XIII, p. 201).

stances,(2) pure spiritual motions then become the paradigms, for which (3) the mover can be identical with the moved, (4) self-motion defines freedom, (5) the hierarchy of the ideas or of the spiritual substances supposes a system of reflection or emanation or creation from the One or the Perfect Being to the least images or creatures, (6) finally every motion is the act of such a power (actual teleology).

Epilogue

To write the history of a problem is to reconstruct, insofar as possible, the arguments whose conjunction it is that raises the difficulty and to examine the solutions that have been proposed in challenging either one of these arguments or an implicit and generally accepted premise.

The historian, as such, does not judge history. But to refuse giving an epilogue would be to do violence to thought. Was the problem raised a genuine one and were the solutions proposed in fact acceptable ones?

An epilogue is not a conclusion. It goes beyond the limits of what is strictly historical. Once this step has been taken we should not be in the least surprised to find ourselves at grips with the philosophers' disputes.

The historical analysis has allowed us to conclude that the Master Argument rests on the incompatibility of three premises (four if we distinguish conditional necessity from the contraction of the diachronically possible) which all have to do more with physics than with logic, since they are all analyses of movement in the most general sense, that is to say, of change. The clear sign of this is the double temporal index affecting them all.

This being the case, one might expect on the part of the philosopher a properly physical inquiry upon which to base his judgment. It is in this way that we shall proceed.

There are two things however that complicate matters here. The first is in connection with the very nature of the Master Argument. For the movements and capacities that the argument addresses itself to in the first place are "rational" movements and capacities that bring deliberation into play. The study of such objects as these belongs to ethics before belonging to physics. Let us note nevertheless that human decisions inevitably fit themselves into the course of nature, secondly that nothing prevents nature, insofar as it is recognized to be

contingent, from posing the same problem for analysis that freedom does, and finally, that the naturally contingent, without pretending to impose its own particular features on freedom, may suggest analogical ways of rendering this latter possible.

The second source of complication, extraneous to the Master Argument, is one of circumstance. The majority of philosophers today, either because of their lack of familiarity with the sciences or because they deny that there is any separation between scientific and ordinary languages, postulate that it is sufficient to correctly analyze the common language with which they express themselves and which has incorporated wisdom and truth in order to dissipate the confusions, the paradoxes, the pseudo problems and imagined solutions transmitted by the history of philosophy.

Aristotle, already, led into the real examination of a question by a preliminary linguistic or dialectical examination. Let us follow his lead.

10.1 The impasse of natural language.

Faced with the three premises of the Master Argument, common sense, along with the philosophers who follow it, rejects the solution of Cleanthes and that of Diodorus out of hand, since natural language seems to block any application of the possible to the past and to require in the domain of what is naturally contingent, just as well as in that of human decision, that there be some possibles left unrealized. Little inclined to enter into the logical quibbles of Chrysippus or to flee towards the Platonic ideas, not to mention the other solutions of the paradox, given that all these solutions do violence to common sense and to language, they are invariably brought back to the second premise. Brief examination makes them reject it. Don't we ordinarily say, after all, that what is possible today for us to do the day after tomorrow will turn out tomorrow to be no longer possible?¹

On the other hand, the axiom of conditional necessity must be rejected because it precludes consideration of the only thing upon which we act, of the somewhat extended or specious present, in favor of abstractions such as those of spatio-temporal points of situation, those of the instantaneous velocities and impulsions of kinematics or dynamics, all illusions, that obstruct the experience of freedom.

The first objection seems to be so strong that one wonders why Chrysippus, who goes to such lengths to avoid the premise of contraction did not think of it. But then just what was the sense of the word

¹See above, 5.5, pp. 124-125.

possible in the context of physics about which the Ancients reason? The analysis of the Chrysippean modalities has shown what it was. The absence of internal contradiction is not generally sufficient for recognizing a possible; there must be conditions of reality as well, conditions about which the philosophers are not in agreement but which require, in any case, that the realization of the possible should not be prevented by obvious external hindrances. As Aristotle puts it, "a thing is capable of doing something if there will be nothing impossible in its having the actuality of that of which it is said to have the capacity".² As the condition has to do with a being that, not walking, has the capacity of walking, or that, walking, has the capacity of not walking, the most general impossibility we can imagine is that of a possible that would come to be contracted and realized at the very instant at which the contradictory state of affairs would be the case.

Let us then follow the suggestions of ordinary language. Let us forget the requirements of reality that the Ancients attached to the notion of the possible. Let us consider now a possible at t , but such that from an instant $t' < t$ on it will be clear that it will not be reduced to a contracted possible: we will not have $M_{t'}p_{t'}$. Since the contraction of this possible between now and t' has not even been envisaged, this amounts to a denial of premise (B). We then posit, subject to the validity of (A):

$$(A) \text{ and } (\sim B) (\exists t) \sim L_N \sim p_t \cdot (t_1)(N \leq t_1 \leq t \supset \sim M_{t_1}p_{t_1}).$$

What is the meaning of this assertion that formally contradicts Aristotle? It will be seen in examining the two limiting cases that arise when N and t are identified and when $t_1 = t$ is precisely fixed. In the first case we reach a contradiction: p_N is impossible and its negation is not necessary. In the second case we assert, for example, that it is not impossible, if it is 6 o'clock, that I should not take a certain train at 8:02, although at no intermediate instant am I faced with the concrete possibility of taking it. In short, the event considered is a verbal possible, a word that makes no commitment. For Aristotle, for Diodorus, for Chrysippus, even though there is no logical contradiction in the concept this word expresses, even if we consider the concept possible, it "is" not a possible as long as the impossibility of the contraction at the stated instant is not excluded and as long as the diverse hindrances that are liable to intervene have not been removed.

This is what we sometimes express in explicitly adding the clause of non-hindrance to the statement of diachronic possibility. Aristotle,

² *Metaphysics*, Θ , 3, 1047^a24-25.

who knows what saying something means, has been seen to mention this given.³

A being has a potency insofar as this is a potency to act, not an absolute power, but subject to certain conditions, among which will be included the absence of external obstacles. For a possible to be authenticated such, the hindrances to its realization must first be ruled out. It is because it receives them instead of ruling them out that the first objection invalidates (B) so readily.

Henri Bergson and William James, in the name of metaphysics and psychology, and often in thinking that they were criticizing logic and everyday language, defended an idea similar to that expressed by the objection made against conditional necessity. It may be doubted whether the testimony of language can be cited in an unequivocal fashion on the matter. Aristotle, in any case, to whom we owe the distinction between the limits and the parts of time and who on other occasions rested arguments on the distinction, did not look to it for a solution to the aporia on freedom but reserved the use of it to extricating us from the labyrinth of the continuum. The fact is that the axiom of conditional necessity does not seem on its own to commit us to an interpretation in terms of instants. All that is required by contraction is the identity of the two indices. While and for as long as the possible lasts, the actuality lasts. The validity of the axiom in no way depends on the particular conception one adopts as regards the relation between the continuum and its elements.

10.2 A probabilistic reconstruction of the Master Argument: Diodorus' solution.

Let us leave grammar behind and go on to physics.

The aporia of Diodorus imposes no condition relative to the distinction between natural events and human decisions. When determinism and universal causality are assumed, as they are in classical physics, the most natural and simplest solution seems to be that proposed by Diodorus. From this point of view, a possible destined not to be realized is not a possible, but only a word.

But since the aporia imposes no conditions either as regards determinism or causality, it must be applicable to all events, including those we take to be contingent and to which we are able at most to attribute a probability. It can be seen from statistical mechanics, and from the reflections of a Laplace before that, that classical physics does not reject such a notion. We have shown that it was called for

³ *Metaphysics*, Θ, 5, 1048^a16–21, quoted on page 125.

in ancient physics by how dangerously close Aristotle's position came to Diodorus'.⁴ That is why we had to try to interpret the Aristotelian notion of contingency in terms of probability.

One of the reasons that kept Aristotle from reaching the notion of probability lies in the special teratological status he accords to the notion of "rare" event.⁵ To profitably investigate the relations between the possible and the probable, while at the same time giving all their weight to the objections raised against the Master Argument's second premise, it will be of interest to reflect on the Poisson probabilities and the Poisson law, which have to do specifically with such events as the emission of a radioactive particle, the arrival of a customer at a ticket window, the occurrence of a typing error on a printed page. The Poisson model applies when the three following conditions are satisfied:

1/ The numbers of events considered in a given region are independent (The region in the example to be taken up here is a time interval with the year as unit).

2/ The probability of an event's occurring in a region of size h is approximately proportional to h for small h , independently of the location of the region.

3/ The probability of more than one event's occurring in a region of size h is negligible in comparison with the probability of one event's occurring, for small h .⁶

Taking τ as the average value of time in which a system remains in its state or its life span (τ being the inverse of the expected number of events per unit time), it is demonstrated that then the number of events in the interval t follows the Poisson distribution of parameter: $\frac{t}{\tau}$

$$\pi(n \text{ events in the interval } t) = e^{-\frac{t}{\tau}} \frac{(\frac{t}{\tau})^n}{n!}$$

$$n = 0, 1, 2, \dots$$

It will be observed that once n becomes notably greater than $\frac{t}{\tau}$, $\pi(n)$ tends rapidly to 0. We have:

$$\pi(n+1) = \pi(n) \cdot \frac{\frac{t}{\tau}}{n+1}$$

⁴See above, 6.6, p. 149 *sq.*

⁵See above, 6.9, pp. 159–160.

⁶P.W. Lindgren, G.W. McElrath, *Introduction to Probability and Statistics*³, MacMillan, London, 1969, p. 68, where the authors give mathematical precision to the sense of the terms "approximately proportional" and "negligible". E. Borel, *Éléments de la Théorie des Probabilités*, Paris, Albin-Michel, 1950, p. 186–187.

When $\frac{t}{\tau} = 1$, and in forcing the last decimal place:⁷

$\pi(0)$	$= 1/e$	$= 0.368$
$\pi(1)$	$= \pi(0)$	$= 0.368$
$\pi(2)$	$= 1/2 \pi(1)$	$= 0.184$
$\pi(3)$	$= 1/3 \pi(2)$	$= 0.061$
$\pi(4)$	$= 1/4 \pi(3)$	$= 0.015$
$\pi(5)$	$= 1/5 \pi(4)$	$= 0.003$
$\pi(6)$	$= 1/6 \pi(5)$	$= 0.0005$
$\pi(7)$	$= 1/7 \pi(6)$	$= 0.00007$
$\pi(8)$	$= 1/8 \pi(7)$	$= 0.000008$
Total:		0.999578

Proponents of the Master Argument's third premise, those who dissociate the possible from its realization, will think they find in these figures an argument in their favor. They will say that whatever the number n of occurrences of the event in the unit of time considered, the other numbers of occurrences $k \neq n$ will be excluded. The probability of k occurrences however is possible in the unit of time. But there being k occurrences is excluded, which falsifies premise (B) in its conjunction with (A) and (NH).

It will suffice, however, to consult the list of purely theoretical numbers corresponding to the probabilities of the occurrences of the event in question to rule out this sophism and reestablish the conditions called for when we want to translate a statement about the possible with double temporal indexation into terms of probabilities. There are indeed two differences that are obvious. 1/ We are not at all surprised at the occurrence first of the event which was far from having the greatest probability. 2/ In repeating trials, we would be surprised to find that the statistics exactly verified the theoretical probabilities: the existence of deviation, and increasing deviation in absolute value as the number of trials increases, belongs analytically to the concept of probability.

The thing is, the possible is ordinarily an individual event, the naval battle tomorrow, without the set of naval battles entering explicitly into its concept. By contrast, the probable has to do with repeatable trials and its concept is inseparable from that of the set of like events. It loses all its significance once divorced from what constitutes its specifically collective relation to possible experience.

What is more, the theory fixes this relation ideally to an infinite number of trials. Such is the role of the law of large numbers that

⁷Borel, *op.cit.*, p. 189.

determines the relation between probability and observed frequency. In its "weak" form, this law states, with f standing for the observed frequency of the expected event in \mathcal{N} trials, that the probability that the relative frequency $\frac{f}{\mathcal{N}}$ differs from the probability of the event by more than a small fixed number ε can be made as small as one likes by increasing the number of trials:

$$\lim_{\mathcal{N} \rightarrow \infty} p\left(\left|\frac{f}{\mathcal{N}} - \pi\right| > \varepsilon\right) = 0 \quad \text{for all } \varepsilon > 0.$$

Keeping in mind the collective feature of probability and its essential relation of corroboration to a frequency, it is to the law of large numbers that we must compare Diodorus' law of the possible. That law, which is equivalent to the conjunction of (A, B and NH), says that if it is possible at N that p at t , then there is a t_1 such that $N \leq t_1 \leq t$ and p at t_1 . But just what are we asserting when we use the Poisson distribution to attribute the probability $\pi(2) = 0.184$ to the occurrence of two events in the unit of time? In the list of probabilities given above the total did not reach 1. To obtain a total of 1 as the theory requires, we shall follow the Table of values of the Poisson probabilities in forcing the last decimal place of $\pi(7)$, the final probability of the list. (We shall return later to the error thus introduced, which does not affect the theoretical argument). We obtain then the following list:

$\pi(0)$	= 0.3679
$\pi(1)$	= 0.3679
$\pi(2)$	= 0.1839
$\pi(3)$	= 0.0613
$\pi(4)$	= 0.0153
$\pi(5)$	= 0.0031
$\pi(6)$	= 0.0005
$\pi(7)$	= 0.0001
Total	: 1.0000

With the total equal to one we can now imagine an urn in which there have been mixed 10.000 tickets, proportionately distributed over the eight designated classes and appropriately marked. If we make 10.000 drawings without replacement, necessarily 1.839 tickets of the third class will be drawn. If, on the other hand, the urn contains a million tickets marked according to the required proportion and suitably mixed, 10.000 drawings will give a frequency that differs from 1.839 and a relative frequency slightly different from 1.839/10.000. To assign that probability to the event of class 3, even in leaving room for the absolute deviation that can grow as \mathcal{N} grows, is to assure that if the relative frequency deviates from the probability by a number $\varepsilon > 0$,

increasing \mathcal{N} will suffice for reducing that deviation. The annual occurrence of two events has the probability 0.1839 if, and only if, in a long series of years and for like populations, it is probable that the limit of the difference between relative frequency and probability tends toward 0. There is, therefore, no probability without a determination of the conditions of realization of the probable event; and these conditions of non-hindrance affect the probabilities of rare events as much as those of frequent ones.

What then, in probabilistic terms, would correspond to the statement (C) which asserts the existence of a never realized possible? It would be a statement asserting that, no matter how long the sequence of trials chosen relatively to a positive probability nor how great the number of trials become, the probability that the relative frequency of the expected event should differ from the probability of the event by more than a small number, ε , cannot be made as small as one likes. And this statement contradicts the theorem of Jacques Bernoulli.

Thus when proper care is taken to translate the Master Argument's premises into probabilistic terms, the law of large numbers leads analytically to an acceptance of Diodorus' solution and incrimination of premise (C). To apply probabilities to the natural sciences is to bind these latter to frequencies. The sense in which the words "real" and "virtual" are to be taken here still remains to be specified. For in the probabilistic reconstruction of the Master Argument, the two contradictory statements expressing it are both asymptotic. Their truth depends on a passage to the limit for an infinite number of trials. A probability is not defined by a frequency after the manner in which Diodorus defined a possible by one or more realizations; and the passage of the relative frequency to the probability is itself only probable. The realizability to which the possible is reduced is to be understood as a virtual sequence of virtual events on a plane of abstract representation. As happens every time a scientific concept is defined, we have changed the sense of the term from that which it had in ordinary language. The individual possible takes its place in a "reality" going beyond experience, since it is made up of a virtual sequence of virtual events. But the infinite future postulated by Diodorus also has a transcendence comparable to that of this sequence.

10.3 The special status of premise (C): Chrysippus' solution and the 'Unique Law of Chance'.

Is the question settled then?

It would be if we confined ourselves to the theoretical domain of the probability calculus, translating the Master Argument's premises into propositions of that calculus, as we have done. It would be possible, however, while still maintaining that the law of large numbers translates the conjunction of premises (A), (B) and (NH), to refuse the translation proposed for premise (C). It might be argued that that premise is not on the same level as the others. It bears not on a theoretical relation of probability to experience, but on a practical one. What is true in theory need not be true in practice, as otherwise, our experience being finite by definition, we could not apply the probability calculus.

Aristotelians could be tempted by the following argument. The law of large numbers reduces probability to frequency only if there exists an irreducible theoretical probability. Because of its irreducibility the relevant sentence is comparable to the premise (C) and is not "already" provided with a truth value.

The temptation must be resisted. First the probable validity of the law of large numbers means that no experience suffices for its legitimation while the realization of the possible falsifies (C). Secondly, while Aristotle denies that (C) as well as (\sim C) have actual truth value, the law of large numbers—even if it is a theoretical law—applies to experience only by choosing (C) against (\sim C). In themselves probabilities justify rather Diodorus' than Aristotle's contingency.

Let us therefore turn to statistics. Consider those given for births of quadruplets in France⁸ from 1945 to 1962.

⁸Quoted by J.L. Boursin-P. Caussat, *Autopsie du hasard*, Bordas, Paris, 1979. The mean of the distribution, equal here to the variance (square of the standard deviation), is equal to $\frac{7}{4}$.

Year	Number of quadruplet births	Year	Number of quadruplet births
1946	4	1955	1
1947	1	1956	0
1948	1	1957	0
1949	2	1958	0
1950	1	1959	0
1951	2	1960	2
1952	0	1961	2
1953	0	1962	0
1954	1		

The Poisson Law solves the problem of repeated trials when, with a large number of trials, the probability is quite weak for each of them. We shall naturally adopt the hypothesis then that the statistic at hand obeys the Poisson Law. Moreover, there are 17 births of quadruplets in 17 years. Since the average is equal to 1, we shall specify the hypothesis in setting $t = \tau$ in the Poisson distribution. What remains to be done now is to compare the frequencies to the theoretical probabilities just presented.

Let us now consider one of the statistical tests devised for answering the question as to whether or not to accept a Poisson distribution of parameter 1 in the case of our statistic.⁹ For each class of events the frequencies $f_i (i = 1, \dots, 17)$ are noted. These are compared in turn to the corresponding mathematical expectations (products of the corresponding probabilities p_{0i} furnished by the null hypothesis H_0 —that is to say, the hypothesis of the Poisson distribution—by the number of years, 17). From this the difference between the observed and expected frequencies is deduced:

$$|f_i - 17p_{0i}|$$

⁹Lindgren, McElrath, *op. cit.*, p. 240.

Number of quadruplet births in 17 years	0	1	2	3	4	5	6	7
Observed frequencies	7	5	4	0	1	0	0	0
Expected values	6.25	6.25	3.13	1.04	0.26	0.07	0	0
Difference	0.75	1.25	0.87	1.04	0.74	0.07	0	0

It will be noted that except for the numbers 6 and 7 the expected values all differ from the observed frequencies with the deviation in excess of 1, the value of the standard deviation, for the numbers 1 and 3. The test will be based on the value of a mean square deviation:

$$\chi^2 = \sum_{i=0}^{i=7} \frac{(f_i - 17p_{0i})^2}{17p_{0i}}$$

which gives:

$$\frac{(0.75)^2}{6.25} + \frac{(1.25)^2}{6.25} + \frac{(0.87)^2}{3.13} + \frac{(1.04)^2}{1.04} + \frac{(0.74)^2}{0.26} + \frac{(0.07)^2}{0.07} = 3.92$$

Under asymptotic conditions for the number of trials,¹⁰ the statistic would obey a distribution χ^2 with 6 degrees of freedom. If χ^2 is too large there will be reason to cast doubt on the hypothesis H_0 . The question that arises is for which set of values $\chi^2 > M$ (M being constant) the hypothesis H_0 is to be rejected, with the constant M devised to assign the probability α of rejecting H_0 when H_0 is true.¹¹ In other words, to fix M is to fix the probability α that one will accept of being mistaken in rejecting a true hypothesis:

$$\begin{aligned} \alpha &= \pi(\text{reject } H_0 | H_0) \\ &= \pi(\chi^2 > M | H_0) = 1 - F_{\chi^2} M. \end{aligned}$$

¹⁰"The exact distribution of this statistic

$$\chi^2 = \sum_{i=1}^{i=n} \frac{(f_i - \mathcal{N}p_{0i})^2}{\mathcal{N}p_{0i}}$$

is not simple and it depends on the p_0 's in the null distribution (H_0); but it was found that this dependence disappears as the sample size becomes infinite, and that the distribution for $n = \infty$ can be computed. It is a 'chi-square distribution with $n - 1$ degrees of freedom' ". (Lindgren, McElrath, *op. cit.*, p. 240).

¹¹Lindgren, McElrath, *op. cit.*, p. 240.

Let us decide, for example, to run the risk of being mistaken with a probability of $\alpha = 0.10$. In the table for χ^2 with $7 - 1 = 6$ degrees of freedom, we shall then choose M such that $F_{\chi^2} M = 0.90$, which gives $M = 10.6$. Since $\chi^2 = 3.92 < 10.6$, the test calls for acceptance of the hypothesis of a Poisson distribution.

Even apart from asymptotic considerations, the decision to accept or reject a statistical hypothesis is never exempt from risk. All that can be demanded of it is that it be informed by the measure of the risk involved. Then too, this measure should be exact. But what is it in fact that we see? First we calculated the Poisson Law of parameter 1, in forcing the last decimal place, for 9 degrees of freedom: the approximative total of the probabilities fell short of 1. Next we consulted a table in order to specify the sense of the word *probability* in such a distribution. That table was limited to 7 degrees of freedom. Since the values not listed by the table were smaller than $1/2(0.001)$, it "rounded off" the value of $\pi(7)$ in conformity with the table's degree of precision.¹² Whatever the degree of precision required of a table, all values going beyond that degree of precision will be disregarded. Without such disregard, and thus without there being imposed some limit to the degree of precision, the very formulation of a statistical test would become impossible and the probability calculus would have no application to experience.

This necessary approximation has attracted little attention from the men of science, with the exception of Emile Borel. It is, so to speak, the counterpart of the law of large numbers. As an idealization of a theoretical order, the law of large numbers embeds a finite sequence of trials in an infinite one. As a practical idealization, the "law" of approximation enjoins us to disregard a part of probability or a sufficiently small probability. The law of large numbers translates Diodorus' definition of the possible into probabilistic terms. It will be seen that the law bearing on approximation gives consistency to Chrysippus' definition of the probable.

Borel writes: "The practical applications of the results [bearing on repeated trials] are dominated by an empirical law that ... must be regarded as absolutely certain and that might be called the *unique law of chance*. That law, which is altogether simple, can be expressed as follows: *Events with an extremely weak probability are never realized*, but this statement should still be made explicit by indicating just what

¹²Boursin, Caussat, *op. cit.*, p. 324. Instead of $\pi(7) = 0.00007$ we take $\pi(7) = 0.0001$, in introducing an error of $0.00003 < 1/2(0.001)$.

is meant by *extremely weak probability*?¹³ The degree of precision involved is naturally relative to a given scale. What is negligible on a human scale is one thing, what is negligible on a terrestrial scale another, and what is negligible on a cosmic or supercosmic scale is yet other. The first list of probabilities given stopped at $\pi(8) = 0.000008$. The tables stop at $\pi(7)$. They disregard $\pi(8)$ and introduce an error of the order of 10^{-6} . This is the order of precision that Borel says is negligible on the human scale.

We shall not go into the justification of these scales. In doing so we would find ourselves confronted with the Megarian paradox of the heap and with undecidable concepts.¹⁴ We shall limit ourselves to pointing out that the theoretically true statement, affirming that it suffices to reach a sufficiently large number n of trials for the weakest probability to be realized,¹⁵ has every chance of being devoid of any practical signification. For "if we turn our attention to the simplest and most elementary experiments, which are by far the most frequent (impacts of infinitesimal particles, the ultimate elements of matter and energy) and to time intervals far surpassing the life of our solar system, the number of times that an experiment can be repeated is no more than a power of 10 with an exponent of less than a thousand".¹⁶ The realization of an event corresponding to a very small probability would require, in order for the number of repeatable trials to be sufficient, a cosmic homogeneity rendering possible the formation of the same combinations; and this is nothing but a puerile and anthropocentric representation of an infinite universe.¹⁷

In short, an event with a small enough probability will never occur because the physical conditions rendering possible the repeatability of the trial will have disappeared. Such destruction of the physical conditions of the probability falsifies the conjunction of the axioms (A), (B) and (NH); it reinstates premise (C). It is in conformity with the doctrine of Chrysippus and with the destruction of deictic statements that doctrine invoked¹⁸ and for which the unique law of chance provides an unexpected warranty.

¹³Borel, *op. cit.*, p. 100; Boursin, Caussat, *op. cit.*, p. 288.

¹⁴See their formulation by Diodorus, a probable critic of the Stoics, above, 3.5, pp. 64-66; on the unique law of chance and the paradox of the heap, see Boursin, Caussat, *op. cit.*, pp. 290-291.

¹⁵This is the "mathematician's argument" referred to by Borel, *op. cit.*, p. 106. The mathematician in question seems to be Poincaré; see, for example, *La Valeur de la Science*, Paris, Flammarion, 1970, pp. 173-174.

¹⁶Borel, *op. cit.*, p. 107.

¹⁷Borel, *op. cit.*, p. 107.

¹⁸See above, section 5.2, p. 107 *sq.*

Chrysippus' recourse to a non-standard modal system is at once explained and justified. According to the law of large numbers, the realization of the probable, as is required by the conjunction of premises (A), (B) and (NH), is validated, but weakly so. This much is required by the theory of probability: it is impossible for an event with a positive probability, no matter how slight, not to occur some day at least once; but this does not imply that it is necessary for it to occur some day at least once, if the circumstances rendering possible the repeated trials from which the event can result no longer obtain. On the other hand, according to the unique law of chance, one can, and even must, disregard the realization of an event whose probability is sufficiently small. Thus premise (C) is validated, but weakly so too. And this is a requisite for the application of probability theory to experience. It is not necessary for an event with a sufficiently small, although positive, probability to occur some day at least once, which does not imply that it is positively possible for it not to occur any day at least once, for this latter would usher in a conflict between the unique law of chance and the law of large numbers.¹⁹

Whatever their respective merits, the two solutions of Diodorus and Chrysippus are not of equal value. They haven't the same value in that the second can be reduced to the first. It will be possible to see this in taking a closer look at the law of chance. The elimination of $\pi(8)$ from the Poisson table for parameter 1 means in effect that the table admits a precision of less than 10^{-6} . We have a right then to replace the approximation statement ' $\pi(8) = 8 \cdot 10^{-6}$ may be disregarded' by the complete and theoretically exact statement 'the mathematical expectation that there will be at least one occurrence of 8 favorable events in the course of 10^6 units if time is equal to 8'.²⁰ The effect of this latter statement, if brought to bear on pure probability theory, would be to eliminate from Diodorus' urn containing all possibles those for which external circumstances of hindrance would postpone the realization for too long a time. Chrysippus, who has recourse to incomplete statements and who distinguishes the impossibility that not from necessity, calls the events thus excluded *possibles that will never be realized*. Once the statements are completed the distinction is dissolved and the excluded events are called *impossibles of a scale*. The argument was thought to give a practical validation of contingency. In fact it has only succeeded in narrowing its domain.

¹⁹See above, p. 121. The conjunction ($Lp \cdot M \sim p$) is contradictory because $M \sim p \supset \sim Lp$. By contrast, the conjunction ($\sim M \sim p \sim Lp$) is consistent because we do not have the right to conclude Lp from $\sim M \sim p$ nor $M \sim p$ from $\sim Lp$.

²⁰ $1 \times 8 \cdot 10^{-6} \times 10^6 = 8$.

10.4 Contingency and ignorance: The statistical mix.

The fact that it is possible to reconstruct an equivalent of the Master Argument in probabilistic terms goes to show that there is some notion of contingency that is not incompatible with the Diodorean premises and that leaves the principles of logic intact. It is along these lines, moreover, that Diodorus resolved the aporia. He denied axiom (C) and gave a *sui generis*²¹ definition of contingency. What remains to be done within that conception transposed into terms of probability is to determine the part due to nature and that due to human ignorance.

Let us replace the image of two mutually exclusive possibles, one of which is but a shadow of reality, by a dichotomous contrivance that will force nature to recognize a degree of possibility for them both. To make the image concrete think of a stream of projectiles shot from a canon too inaccurate to control their angular dispersion and subjected to passing through a screen pierced by two holes in the manner of Young's experiment and then reaching a detector that absorbs and counts them. But any other apparatus would do just as well on condition that there be passage from an initial state i to a final state f by means of a well defined alternative: either passage by way of A or passage by way of B, with distinct and exclusive A and B.

When it is waves—not particles—that are submitted to the test of Young's apparatus, the experiment gives rise to the remarkable and paradoxical phenomenon of interference: crests and troughs cancel each other out in superposition, as Posidonius had understood. But wave propagation doesn't shock our physical intuitions. It is continuous and non local. The wave passes through both holes and that is why there is interference. There is no analogue here of a choice between two mutually exclusive possibles. The resultant intensity I_{AB} measured at the detector is not reducible to the sum of the intensities I_A and I_B that would have been obtained respectively in obturating first hole B, then hole A. It contains an interference term that depends on the difference of phase between the two waves diffracted at A and B.

Let us return now to particles.

The detector registers the arrival of a particle at point x on the detection screen. This event can be described by the following diagram (read from right to left):

$$\langle f_x | i \rangle.$$

There is a transition from the initial state (emission of the projec-

²¹See above, p. 42. The infinite time there assigned for the realization of the possible evokes the abstract and virtual character of statistical series.

tile by the canon) to the final state (arrival on the detector at point x). Furthermore, this transition can be analyzed. It is the sum of two possible transitions: one in which the particle passes through the intermediary state $|u_a\rangle$ (passage through hole A), the other in which it passes through the intermediary state $|u_b\rangle$ (passage through hole B):

$$\langle f_x|i\rangle = \langle f_x|u_a\rangle\langle u_a|i\rangle + \langle f_x|u_b\rangle\langle u_b|i\rangle.$$

Attribution of corresponding probabilities involves, in all cases, the elaboration of a density function making it possible to speak of the probability of the detection of a particle at point x and of conventions that allow of assigning numbers to the elements of the diagram of transitions that will be apt to suitably define the probabilities sought. Of these conventions we shall retain only what is of importance for the discussion of the Master Argument. The expression for the transition $\langle f_x|i\rangle$ is generally a complex number, and its probability is equal to the square of that expression, $|\langle f_x|i\rangle|^2$, which is always a real number.

Experiment shows that every time we observe, or can observe, by which branch of the alternative the particle has passed, the total probability is the sum of the probabilities of the two branches:

$$(1) \quad \pi(\langle f_x|i\rangle) = \pi(\langle f_x|u_a\rangle\langle u_a|i\rangle) + \pi(\langle f_x|u_b\rangle\langle u_b|i\rangle).$$

This equality²² corresponds to the classical hypothesis according to which every transition is observable and corresponds to a well defined path. We shall see that, with the validity of the Master Argument, it entails Diodorus' solution. Quantum particles, on the other hand, obey a different law. The total probability is the square of the sum of the transitions—we are speaking now of probability amplitudes—so that here there appears a term of interference that did not figure in expression (1):

$$(2) \quad \pi(\langle f_x|i\rangle) = \pi(\langle f_x|u_a\rangle\langle u_a|i\rangle + \langle f_x|u_b\rangle\langle u_b|i\rangle).$$

This equality²³ disallows observation and the observability of the intermediary state. It is impossible to determine by which branch of the alternative the transition is effected. Consequently, there is not a well defined path corresponding to a transition. We shall see that in this

²²That is to say:

$$|\langle f_x|i\rangle|^2 = (|\langle f_x|u_a\rangle|^2 \cdot |\langle u_a|i\rangle|^2) + (|\langle f_x|u_b\rangle|^2 \cdot |\langle u_b|i\rangle|^2).$$

²³That is to say:

$$|\langle f_x|i\rangle|^2 = (|\langle f_x + u_a\rangle|^2 \cdot |\langle u_a + i\rangle|^2) + (|\langle f_x + u_b\rangle|^2 \cdot |\langle u_b + i\rangle|^2) + 2\langle f_x|u_a\rangle\langle u_a|i\rangle\langle f_x|u_b\rangle\langle u_b|i\rangle.$$

It is this last term, called the interference term, that makes the difference.

case the Master Argument loses its cogency and physics can do without Diodorus' solution.

Let us first consider the case of classical particles. The frequency f_x (the number of particles reaching the detector at x) is measured, and in shuttering hole B the frequency f_{xA} (the number of particles reaching x after passing through hole A) is measured, as is the frequency f_{xB} with hole A shuttered. If \mathcal{N} is the number of particles having passed through one of the two holes, then $f_x|\mathcal{N}$, $f_{xA}|\mathcal{N}$ and $f_{xB}|\mathcal{N}$ measure the corresponding relative frequencies that will have to be compared to the respective theoretical probabilities $\pi_x, \pi_{xA}, \pi_{xB}$. There are three weak laws of large numbers that follow from this, the first resulting from the sum of the other two. When the particles are projectiles in a Young apparatus, it is a simple matter to reconstruct the Master Argument. The projectiles collected on the detector screen at x fall into two disjoint classes, though without our being able to distinguish those having passed through A from those having passed through B. We are nevertheless certain, now that the event has taken place, that each individual projectile considered has passed either through A or through B. In the first case the probability that it passed through B is zero, and *vice versa* in the second case. Thus axiom (A) is verified. We are justified then, as regards the consequent of axiom (B), in retaining only that disjunct that has to do with the future. But at the moment of firing, the probability π_x attributed to a certain particle now collected at x may be broken down into the two non null components π_{xA} and π_{xB} . The weak law of large numbers applied to π_{xA} and π_{xB} guarantees that the probabilistic version of the conjunction of axioms (A.B.NH) holds for both of these probabilities, notwithstanding the fact that they are contraries: each of the two "degrees" of the possible contracts and is realized with the frequency sought as the number of trials is increased at will.²⁴

²⁴Let us shutter B and count f_{xA} at x . The law says that

$$\lim_{\mathcal{N} \rightarrow \infty} p(|\frac{f_{xA}}{\mathcal{N}} - \pi_{xA}| > \varepsilon) = 0.$$

The contrary event (passage through B) is excluded due to the shuttering of B. This is what verifies the second term of the conjunction in axiom (C). But π_{xB} is then equal to 0 and the first term of the conjunction ($(\exists t)(\sim L_N p_t \cdot N \leq t)$) is therefore falsified. It could likewise be shown, in shuttering A, that the law of large numbers for π_{xA} verifies the axioms (A.B.NH) while falsifying axiom (C).

One might think that with the two holes A and B left open and the two probabilities π_{xA} and π_{xB} each greater than zero the Master Argument would cease to apply. It would be argued that since the law of large numbers verifies the conjunction of axioms (A.B.NH) for π_{xA} in probabilistic terms, the contrary event is excluded though without its probability being null, and this is a state of things that pleads in favor of axiom (C). But this would be to confuse two experiments that are in fact independent. For just what does it mean to say of a particle collected at x that at the moment of its emission it had probability π_{xA} of passing through A and probability π_{xB} of passing through B, A and B both being open? Take one such particle. At the very latest at the moment of passage through the holes, the possible will have had to synchronically contract, in conformity with axiom (B), and will have had simultaneously to be realized, in conformity with axiom (NH). The possibility of the contrary choice is *nothing* but a word once the contraction is made. Are we obliged then to recognize that before the contraction, in the time interval separating the emission of the particle from its passage through the hole, there was a contrary possibility and that since that contrary possibility was destined never to be realized, the probabilistic model is seen to verify axiom (C)? The symmetry of the Young apparatus requires that $\pi_{xA} = \pi_{xB}$ for x equidistant from A and B. Isn't this an argument of weight for attributing equi-possibility to both the excluded and the realized events?

But what is the meaning of the concession made? Have we really the right to put off the choice—that is to say, the synchronic contraction and realization—until passage through the holes? This would be to misunderstand the meaning of equality (1). For indeed, with all due precaution taken as regards times of emission and the counting of particles, we can postulate that collecting the particles simultaneously at x that have come through one or the other hole with both holes open comes down to the same thing as collecting successively at x those particles having come through A with B shuttered, then those having come through B with A shuttered. As this second experimental case decides for Diodorus' solution, the first will then do so as well. In other words we can imagine that the particles are already partitioned at the source into two exclusive classes much as would be the balls of two different colors in an urn. The emission of a particle is like a drawing in a statistical mix. If \mathcal{N} is the total number of particles, $\mathcal{N}\pi_{xA}$ of them are marked "destination x via A" and $\mathcal{N}\pi_{xB}$ of them "destination x via B". Reduced to its simplest expression the Master Argument, together with Diodorus' solution, asserts only that it is impossible for one ball

in the urn to be two different colors or for one particle to be marked for two destinations and two distinct paths.

Classical probability theory, expressed by (1), assigns each particle to one of the two exclusive classes f_{xA} , f_{xB} at the outset. This assignation is another way of incriminating axiom (C). The Diodorean solution it implies is characteristic of classical physics. 1/ It is extensional and thereby compatible with the language of physics. 2/ It is simple, as can be seen by comparing it with the other solutions examined. 3/ It excludes no elementary laws, though without requiring a knowledge or application of them. It is thus compatible with the principle of determination and with the principle of causality which were defended by all ancient philosophers with the exception of Epicurus. 4/ It leaves room in science for ignorance. Laplace, one of the founders of the probability calculus, put it in charge of that ignorance. A superior intelligence, aware of initial conditions and of the limiting conditions of a phenomenon would be able to foresee its future exactly. Probabilities realize "the most felicitous supplement to the ignorance or weakness of the human mind".²⁵

10.5 Contingency and nature: The state of superposition.

The fact that probability in the classical sense, that is to say relative to our ignorance only, does not challenge the validity of the Master Argument but even calls for the solution of Diodorus is something that becomes clear as soon as we try to imagine a remedy for that ignorance. If we observe the passages through hole A or if we go back to follow the paths taken at the moment of their emission, assignment of the particles either to class f_{xA} or to class f_{xB} for any x is immediately given: probabilities become useless.

If it were possible then to go back to the state of each particle at the moment of its emission we could in principle (or a superior intelligence in the place our own could) answer the two questions we ask: 'where is the particle going?' 'through which hole does it pass?'. These two questions may be represented in the form of two alternatives with the first admitting an infinite and even continuous number of choices, the second being dichotomous. When we imagine a superior observer capable of observing all the conditions of a phenomenon, the hypothesis that all physical magnitudes are in principle simultaneously observable

²⁵Laplace, *Essai philosophique sur les probabilités*. "Were it not for our ignorance", said, Poincaré, "there would be no probability, there would only be room for certainty". (*La science et l'hypothèse*, Paris, Flammarion, 1968, p. 196).

comes down to an up-front distribution of the different particles into their class f_{xA} or f_{xB} (with x indexed over a continuous set of values). Since the hypothesis remains impractical for a finite observer such as us though, incapable as we are of answering the two questions asked exactly and of assigning each particle its point of destination and place of passage, we are reduced to assigning probabilities to these two magnitudes. Instead of answering 'this particle has destination x by way of hole A' we must content ourselves with answering 'this particle has the probability π_{xA} '. The difference between the two answers reflects no difference with respect to the state of nature. It simply reflects a difference relative to our knowledge of that state. And this latter difference would disappear if our observation of the conditions of the phenomenon were complete.

Within mechanics there is a class of phenomena, the extent of which has recently been measured by chaos theory, that casts doubt on the practical possibility of a complete observation. There are those phenomena that are sensitive to the initial conditions.²⁶ A very small difference in these conditions and imperceptible to the observer provokes with the divergence of the resultant trajectories our inability to predict them. The indiscernibility of observables makes one think of the paradox of the twins and of the skeptical solution, or rather reduction, of the Master Argument.²⁷

These doubts, nevertheless, have no bearing on law (1). From our state of ignorance they draw unexpected consequences in classical mechanics about the predictability of phenomena. They in no way modify either the idea of a physical state or the idea of the observable. The solution they can bring to the Master Argument remains external to the aporia itself. Rather than invalidating it, they render its significance unclear. This uncertainty, in turn, justifies a maxim of indifference. What difference does the invalidity of axiom (C) make if in situations of instability I generally remain incapable of distinguishing a possible that will be realized from one that will not?

But suppose now the physical world to be bound by the laws of quantum mechanics and no longer obeying (1) but obeying (2). The

²⁶P. Bergé, Y. Pomeau, Ch. Vidal, *L'ordre dans le chaos*, Paris, Hermann, 1984 (*passim*, and p. 287).

²⁷See above, p. 219; P. Suppes, *Probabilistic Metaphysics*, Oxford, Blackwell, 1984, p. 129 (where the author remarks that "the matter is sometimes discussed as if the whole problem resides in the initial conditions, but this is not the case": in Laplace's doctrine moreover the explicit concept of a superior intelligence was called into question, in dealing with the three-body problem, before the implicit concept of a superior observer was.

existence of the intercrossed term in (2) doesn't come from a modification of the probability calculus. It results from the fact that it is no longer probabilities that are added but probability amplitudes. It attests that during its flight the particle is in state of superposition.²⁸ It invalidates Diodorus' argument in changing the sense of its premises.

On the classical interpretation axiom (A) signified both the necessity of the past and the impossibility of realizing the possible in the past. Obviously definition (2) of the probability of the transition $\langle f|i \rangle$ in no way changes the relation of the probability to the time parameter: the impossibility of realizing the possible in the past is respected and there is nothing to hope for, for the invalidation of (A), in the idea of eternal return. Nevertheless, since the intercrossed term of interference in (2) would disappear if we were to observe through which hole the particle passed, it is impossible to say for a particle detected at x whether it passed through hole A or hole B. Letting t' stand for the time of the particle's passage through the holes, whereas formerly we could say that it either passed through A at t' with the probability 1 and then its probability of passing through B at t' was nil, or *vice versa*, that retrodiction now ceases to be true. (A) is thus falsified in the sense in which it proclaims the necessity of the past and in which the poet said that God could not make it so that what had been done had not been done.

Axiom (B) subordinates the diachronic possible to its contraction. If we compel nature to choose then between two dichotomous possibles, $M_N p_t$ and $M_N \sim p_t$, the positive and negative diachronic possibles will have to contract into $M_{t_1} p_{t_1}$ and $M_{t_2} \sim p_{t_2}$ with $N \leq t_1 \leq t$ and $N \leq t_2 \leq t$. But in virtue of (NH) contradiction will be avoided only if $t_1 \neq t_2$. Consequently, two contrary synchronic possibles cannot be superposed in time. In quantum mechanics, by contrast, where conditional necessity no longer has its former sense, the state of synchronic superposition of two contrary amplitudes of probability is entirely legitimate so long as we do not measure through which hole the projectile has passed, which would destroy superposition. At the moment of passage, t' , we can say that there is a non-null amplitude of probability that the particle is passing through hole A and a non-null amplitude of probability as well that it is passing through hole B. All that is required is that there be no observation that comes to transform the

²⁸We obtain the expression of this state in projecting the initial state $|i\rangle$ on the pair of intermediate states $|u_a\rangle$ and $|u_b\rangle$:

$$|i\rangle = |u_a\rangle\langle u_a|i\rangle + |u_b\rangle\langle u_b|i\rangle = C_a|u_a\rangle + C_b|u_b\rangle.$$

state of superposition into a state proper. Thus axiom (B) turns out to be truistically confirmed and even reinforced since the synchronic contraction extends to all moments preceding that of the reduction.

Axiom (C) is verified by a possible that is not realized. But since the term of interference precludes our observing through which hole the particle has passed, it is impossible to say which of the two contrary probability amplitudes has been realized, so that there ceases to be an exclusion between axiom (C) and the conjunction of axioms (B and NH) and, furthermore, axiom (C) is strictly speaking undecidable.

What of axiom (NH)? It is verified when there is reduction of the wave-particle, that is to say, when the process of measurement selects one particular value of the observable. If such a reduction takes place at the moment the particle passes through one of the holes, the interference effect is destroyed for detection. If it takes place only at the moment of detection, the interference effect is produced. In other words, nature chooses the statistical mix or superposition accordingly as we observe or do not observe where the particle passes. To decree along with classical mechanics that the state of a physical system is composed of a set of physical magnitudes and that these magnitudes are all simultaneously observable is to choose law (1). To choose law (2) is to define the general state of a physical system by superposition: such a state is not observable. An observable is associated with the action of an operator on it and it is that action that produces a proper state.

What dismantles the Master Argument's premises is the new distinction in the history of modal notions between a probability and a probability amplitude. Classical physics was content with the opposition 'This particle passes through A' versus 'This particle has the probability π of passing through A'. This opposition has nothing to do with ontology: it incorporates what is due to our ignorance into the determination of natural phenomena. Instead of attributing a property or magnitude to a physical system, we attribute it a disposition or propensity to have that property or magnitude. Probability measures that disposition or propensity that belongs to the system in act. A probability amplitude is something altogether different. We can compare it to an embryonic probability as the inventors of the infinitesimal calculus compared the "moment" of motion to an embryonic motion that an integration would bring to a state of "whole" motion. But the comparison limps. For the probability amplitude, which is generally a complex quantity, does not figure among the elements of reality. To obtain a probability we must multiply two conjugated probability amplitudes. This means that, when we attribute that amplitude to

a system, it is attributed neither as an actual property or magnitude nor as an actual disposition or propensity to having such property or magnitude, but as a purely virtual disposition or propensity to having it. The second-order potentiality, as it were, thus put into play is no longer the measure of an ignorance that might have some chance of being only provisional. It is physical. It describes nature.

BIBLIOGRAPHY

- Albert of Saxony, *Perutilis Logica Magistri Alberti de Saxoniam*, ed. Petrus Aurelius, Sanatus, Venice, 1522.
- Alexander of Aphrodisias, *In Aristotelis Analyticorum Priorum librum I Commentarium*, CAG II, P. 1, ed. Wallies, Berlin, 1883.
– *Praeter Commentaria Scripta Minora, Quaestiones, De Fato, De Mixtione*, CAG Suppl. 2, P. 2, ed. Bruns, Berlin, 1982. (Numbers in parentheses refer to “Traité du Destin et du Libre Pouvoir”, in Nourrisson, *Essai sur Alexandre d’Aphrodise suivi du Traité...*, Didier, Paris, 1870.)
- Ammonius, *In Aristotelis de Interpretatione Commentarius*, CAG IV, P. 5, ed. Busse, Berlin, 1897. (Numbers in parentheses refer to the Latin translation of William of Moerbeke: G. Verbeke, Ammonius, *Commentaire sur le Peri Hermeneias d’Aristote*, Louvain-Paris, 1961, *Corpus latinum Commentariorum in Aristotelem Graecum*, t. I.)
– *In Aristotelis Analyticorum Priorum librum I Commentarium*, CAG IV, P. 6, ed. Wallies, Berlin, 1899.
- St. Anselm, *Ein neues unvollendetes Werk des hl. Anselm von Canterbury*, ed. F.S. Schmitt. (Beiträge zur Geschichte der Philosophie und Theologie des Mittelalters, 33/3, Münster, Aschendorff, 1936, 24.16.)
– *Opera omnia*, ed. F.S. Schmitt, Edinburgh, Nelson, 1961, photomechanical reproduction by F. Fromann (G. Holzboog), Stuttgart, 1968.
- Aristotle, *Aristotle’s Metaphysics*, Text and Commentary by W.D. Ross, Oxford, Clarendon Press, 1924, 2 vol. .
– *The Works of Aristotle*, transl. into English by Edghill, ed. Ross, vol. I, Oxford University Press, 1928.
– *Aristotle’s Prior and Posterior Analytics*, W.D. Ross, Oxford,

- Clarendon Press, 1949
- *Aristotelis Opera*, ex. rec. I. Bekkeri, ed. O. Gigon, Berlin, de Gruyter, 1960, 2 vol.
 - *De la génération des animaux*, texte et traduction P. Louis, Paris, Les Belles Lettres, 1961.
 - *Categoriae et Liber de Interpretatione*, ed. L. Minio-Paluello, Oxford, 1949, 3^o ed. 1981.
 - *Du Ciel*, texte établi et traduit par Paul Moraux, Paris, Les Belles Lettres, 1965.
 - *Métaphysique*, traduction avec commentaires, nouvelle édition, par J. Tricot, 2 vol., Paris, Vrin, 1953.
 - *Physique*, texte établi et traduit par H. Carteron, 2 vol., Paris, Les Belles Lettres, 1931.
- Arnim, J. von, *Stoicorum veterum fragmenta (S.V.F.)*, Lipsiae, I (1905), II (1903), III (1903), IV (1924).
- Atticus, *Fragments*, texte établi et traduit par E. des Places, Paris, Les Belles Lettres, 1977.
- St. Augustine, *St. Augustini opera*, Paris, Muguet, 1695, 8 vol.
- Bailey, C., *Epicurus*, Oxford, Clarendon Press, 1926.
- Baudry, L. *Lexique philosophique de Guillaume d'Ockham*, études des notions fondamentales, Paris, Lethielleux, 1958.
- Becker, A., "Bestreitet Aristoteles die Gültigkeit des "tertium non datur" für Zukunftsansagen", *Actes du Congrès international de philosophie scientifique*, VI, Paris, 1936, pp. 69–74.
- Becker, O., "Zur Rekonstruktion des "Kyrieion Logos" des Diodoros Kronos", *Erkenntnis und Verantwortung* (Festschrift für Theodor Litt, Düsseldorf, 1961).
- Bergé, P., Pomeau, Y., Vidal, Ch., *L'ordre dans le chaos*, Hermann, Paris, 1984.
- Bergson, H., *La pensée et le mouvant*, essais et conférences, Paris, Alcan, 1934.
- Blanché, R., "Sur l'interprétation du Kyrieion Logos", *Revue philosophique de la France et de l'Étranger*, n^o 155, 1965, pp. 133–149.
- Boethius, *Commentaria in Librum Aristotelis Peri Hermeneias*, Secunda editio, Leipzig, ed. Meiser, 1877–1880.
- Bollack, J., Bollack, M., Wismann, H., *La lettre d'Epicure*, Paris, Editions de Minuit, 1971.
- Borel, E., *Éléments de la Théorie des Probabilités*, Paris, Albin-Michel, 1950.
- Boudot, M., *Logique inductive et probabilité*, Paris, A. Colin, 1972.
- "Temps, nécessité et prédétermination", *Les Etudes philosophi-*

- ques, n° 4, oct-déc., 1973, pp. 435-473.
- Boursin, J.-L., Caussat, P., *Autopsie du hasard*, Paris, Bordas, 1979.
- Boyancé, P., "Epicure et M. Sartre", *Revue philosophique*, 143, 1953, pp. 426-431.
- Bréhier, E., *La théorie des Incorporels dans l'Ancien Stoicisme*, 2° ed. Paris, 1928.
- *Chrysippe et l'Ancien Stoicisme*, Paris, P.U.F., 1910, 2° ed. 1951.
 - *La philosophie de Plotin*, Paris, Vrin, 1928, 2° ed. 1961.
 - *Les Stoiciens*, textes traduits par Bréhier, E., édités sous la direction de Schuhl, P.-M., Paris, Gallimard, 1962.
- Brochard, V., *Etudes de Philosophie ancienne et de Philosophie moderne*, Paris, Vrin, introduction de Delbos, V., 1954.
- Bunge, M., "Possibility and Probability", in Harper and Hoocher (ed.), *Foundations of Probability Theory, Statistical Inference and Statistical Theories of Science*, Dordrecht, Reidel, vol. III, 1976.
- Buridan, J., *Johannes Buridani Sophismata, Consequentiae Buridani*, Denidel, A., de la Barre, N., Baligault, F., Paris, 1495/6.
- *Quaestiones super libros quattuor de Caelo et Mundo*, ed. Moody, E. A., Cambridge, Massachussetts, Mediaeval Academy of America, 1942.
- Burks, A.W., "The logic of causal propositions", *Mind*, n° 5, 60, 1951, pp. 363-382.
- Celluprica, V., *Il capitolo 9 del "De Interpretatione" di Aristotele*, Rassegna di studi 1930-1973, Società editrice Il Mulino, Bologna, 1977.
- Chalcidius, *Timaeus a Calcidio translatus commentarioque instructus*, ed. Waszink, J.-H., (Plato Latinus, t. IV), Leyde, 1962.
- Cherniss, H., *Aristotle's Criticism of Plato and the Academy*, New York, Russel and Russel, 1962.
- *Plutarch's moralia*, t. XIII, Part. I, II, Cambridge-London, Loeb, 1976.
- Chevalier, J., *La notion du nécessaire chez Aristote et chez ses prédécesseurs particulièrement chez Platon*, Paris, Alcan, 1915.
- Cicero, *Academica*, ed. J.S. Reid, London, 1885.
- *De Fato*, ed. Plasberg-Ax, 1938.
 - *De natura deorum*, ed. A.S. Pease, Cambridge, 1955.
 - *De divinatione*, ed. A.S. Pease, Darmstadt, 1963.
- Cornford, F.M., *Plato's Theory of Knowledge*, London, Routledge, 1935, 2° ed. 1960.
- Cournot, A., *Essai sur les fondements de nos connaissances et sur les caractères de la critique philosophique*, ed. J.-Cl. Pariente, *Oeuvres*, Paris, Vrin, t. II, 1975.

- Crombie, J.M., *Examination of Plato's doctrine*, vol. II, Plato on Knowledge and Reality, London, 1963.
- Cumont, F., *Astrology and Religion among the Greeks and Romans*, New York, London, Putnam, 1912.
- *La théologie solaire du paganisme romain*, Mémoires présentés à l'Académie des Inscriptions et Belles-Lettres, Paris, Imprimerie Nationale, t. XII, 2° partie, 1909.
- *Les religions orientales dans le paganisme romain*, Paris, Geuthner, 4° ed. 1929.
- Descartes, R., *Oeuvres et Correspondance*, publiées par Adam et Tannery, Paris, Le Cerf, 1897-1913 (referred to as A.T.), 13 vol.
- Diels, H., Kranz, W., *Die Fragmente der Vorsokratiker*, 3 vol., Berlin, Weidmann, 1959.
- Diogenes Laertius, *Vitae Philosophorum*, ed. G. Cobert, Paris, Firmin-Didot, 1878.
- Döring, K., *Die Megariker*, Kommentierte Sammlung der Testimonien, Amsterdam, Gruner, B.R., 1972.
- Duhem, P., *Le système du monde*, Paris, Hermann, t. I, 1913 (1° ed.), 1951 (2° ed.); t. II, 1914.
- Duns Scotus, *Ioannis Duns Scoti Opera Omnia*, editio nova... a P.P. Franciscanis... recognita, Parisiis apud L. Vivès, 1891-1895.
- Effler, R., *Duns Scotus and the Physical Approach to God*, Studies in Philosophy and the History of Philosophy, vol. 3, John Duns Scotus, ed. Ryan and Bonansea, Washington, the Catholic University of America Press, 1965, pp. 170-190.
- Epictetus, *Epicteti dissertationes*, ed. Schenkl, collect. Teubner, Leipzig, 1° ed., 1898, Minor, 2° ed., 1916, Maior.
- Epicurus (see Bailey)
- Eusebius, *Werke*, Achter Band, *Die Praeparatio Evangelica*, Herausg. v. K. Mras, I-II, Berlin, 1954-1956.
- Faust, A., *Der Möglichkeitsgedanke, Systemgeschichtliche Untersuchungen*, 2 vol., Heidelberg, C. Winter, 1931, 1932.
- Føllesdal, D., "Quantification into causal contexts", *Boston Studies in the Philosophy of Science*, vol. II, New York, Humanities Press Inc., 1965, pp. 263-271.
- "A modal theoretic approach to causal logic", *Det Kgl. Norske Videnskabers Selskabs Skrifter*, 1966, Nr. 2, Trondheim, 1966, pp. 1-13.
- Van Fraassen, B.C., "Presuppositions, Superevaluations and Free Logic", in *The logical Way of Doing Things*, ed. K. Lambert, New Haven, Yale University Press, 1969, pp. 67-91.

- Fraenkel, H., *Wege und Formen Frühgriechischen Denkens*, München, C.H. Beck, 2° ed. 1960.
- Frede, D., *Aristoteles und die "Seeschlacht"*, Göttingen, Van den Hoeck und Ruprecht, 1968.
- Frede, M., *Die stoische Logik*, Göttingen, Van den Hoeck und Ruprecht, 1974.
- Fritz, Kurt von, C.R. of Schuhl, P.-M., "Le Dominateur et les possibles", *Gnomon*, 34, 1962, pp. 138-152.
- "Frede, Aristoteles und die "Seeschlacht", *Pseudepigrapha*, Genève-Vandoeuvres, Fondation Hardt, 1972, pp. 241-250.
- Furley, D., *Two Studies in the Greek Atomists*, Princeton, Princeton University Press, 1967.
- "Variations on themes from Empedocles in Lucretius "Poem"", University of London, Institute for Classical Studies, Bulletin n° 17, 1970, pp. 55-64.
- Gassendi, P., *Disquisitio Metaphysica*, texte traduit et annoté par Rochot, P., Paris, Vrin, 1962.
- Aulus Gellius, *Les Nuits Attiques*, ed. et trad. Marache, R., Paris, Les Belles-Lettres, 1978, 2 tomes.
- Genest, J.-F., "La liberté de Dieu à l'égard du passé selon Pierre Damien et Thomas Bradwardine", *Annuaire de la 5° section de l'Ecole Pratique des Hautes Etudes*, Paris, tome 85, 1976-1977, pp. 391-393.
- Geyer, F., Ueberweg, F., *Grundriss der Geschichte der Philosophie*, II, Die patristische und scholastische Philosophie, Darmstadt, Wissenschaftliche Buchgesellschaft, 1958.
- Gilson, E., *Jean Duns Scot, Introduction à ses propositions fondamentales*, Paris, Vrin, 1952.
- Gilson, E., Böhner, P., *Christliche Philosophie*, 2te Aufl., Paderborn, Schöningh, 1954.
- Goldschmidt, V., *Platonisme et pensée contemporaine*, Paris, Aubier, 1970.
- *Le système stoïcien et l'idée du temps*, Paris, Vrin, 3° ed., 1977.
- *La doctrine d'Epicure et le droit*, Paris, Vrin, 1977.
- "La théorie épicurienne du droit", *Archives de philosophie du droit*, tome 26, *L'utile et le juste*, pp. 73-91.
- Granger, G.-G., *La théorie aristotélicienne de la science*, Paris, Aubier, 1976.
- Gueroult, M., *L'évolution et la structure de la Doctrine de la science chez Fichte*, Paris, Les Belles Lettres, 1930, vol. I & II.
- *Descartes selon l'ordre des raisons*, vol. I: L'Ame et Dieu; vol. II: L'Ame et le Corps, Paris, Aubier, 1953.

- *Malebranche*, vol. I: La vision en Dieu, Paris, Aubier, 1955; vol. II: Les cinq abîmes de la Providence, Paris, Aubier, 1959: a) L'ordre et l'occasionalisme; vol. III: les cinq abîmes de la Providence, Paris, Aubier, 1959: b) La nature et la grâce.
- *Spinoza*, vol. I: Dieu, Paris, Aubier, 1968; vol. II: L'Âme, Paris, Aubier, 1974.
- *Etudes sur Descartes, Spinoza, Malebranche et Leibniz*, Hildesheim, New York, Georg Olms, 1970.
- Hamelin, O., "La logique des Stoïciens", *Année Philosophique*, Paris, 101, p. 26.
- *Essais sur les éléments principaux de la représentation*, Alcan, Paris, 2^e ed. by Darbon 1925.
- *Sur le De Fato*, publié et annoté par M. Conche, Paris, Editions de Mégare, 1978.
- Hintikka, J., *Time and Necessity*, Oxford, Clarendon Press, 1973.
- Hughes, C.E., Cresswell, M.J., *An Introduction to Modal Logic*, London, Methuen, 1972, 2^e ed.
- Hume, D., *A Treatise of Human Nature*, London, ed. Green and Grose, new edition, 1890.
- *Traité de la nature humaine, oeuvres philosophiques choisies*, traduction M. David, Paris, Alcan, 1930.
- *Dialogues sur la religion naturelle*, traduction M. David, Paris, Pauvert, 1964.
- Kant, I., *Kant's Gesammelte Schriften*, hrsg. von der Akademie der Wissenschaften zu Göttingen, Band I-XXXV, Berlin, Reimer, 1910-1917. Berlin und Leipzig, W. de Gruyter, 1910-1955/1966.
- *Critique de la raison pure*, édition et traduction Tremesaygues-Pacaud, Paris, P.U.F., 1944 (1967).
- *Critique de la raison pratique*, traduction Picavet, Paris, P.U.F., 1943 (1976).
- *Critique du jugement*, traduction Gibelin, Paris, Vrin, 1946.
- Kneale, M. & W., *The Development of Logic*, Oxford, Clarendon Press, 1962.
- Lambert, K., "On Logic and Existence", *Notre Dame Journal of Formal Logic*, vol. 6 (1965), pp. 135-141, n^o 2, avril.
- Lambert K., Scharle, T., "A translation theorem for two systems of free logic", *Logique et analyse*, 10^e année, 39-40 (déc. 1967), pp. 328-341.
- Laplace, P.S., *Essai philosophique sur les probabilités* (1813), Paris, Gauthier Villars, 1921, 2 vol.
- Lejewski, V.C., "Zu Leśniewskis Ontologie", *Ratio*, 1957-1958, pp. 50-77.

- Leibniz, G.W., *Die Philosophischen Schriften...*, ed. Gerhardt, C.I., Berlin, Weidmannsche Buchhandlung, 1875, 1890, 7 vol.
- *Nouveaux essais sur l'entendement humain*, Introduction, Note et Appendice par Lachelier, H., Paris, Hachette, 1886.
 - *Essais de théodicée suivis de La monadologie*, Préface et Notes de Jalabert, J., Paris, Aubier, 1962.
 - *Oeuvres*, éditées par Prenant, L., avec introduction, tables et commentaires, Paris, Aubier, 1972.
- Lindgren, A.W., McElrath, G.W., *Introduction to Probability and Statistics*, London, MacMillan, 1969.
- Longpré, E., *La philosophie de B.J. Duns Scot*, Paris, 1924.
- Lucretius, *De natura rerum* (commentaire exégétique et critique), 3 vol., Ernout, A., Robin, A., Paris, Les Belles Lettres, 1925-26-28.
- *De la nature*, tomes I et II, livres I à VI, traduction Ernout, A., Robin, A., Paris, Les Belles Lettres, 1947.
- Lukasiewicz, J., *Aristotle's Syllogistic from the standpoint of modern formal logic*, (1^o edition, 1951), Oxford, Clarendon Press, (2^o edition, 1957) (french translation by Mrs. Zaslowsky, Paris, Armand Colin, 1972).
- "On determinism", *Polish Logic*, 1920-1939, edited by Storrs McCall, Oxford, Clarendon Press, 1969.
- Macrobius, *Commentaire sur le songe de Scipion*, translation Stahl, W.H., New York, London, Columbia University Press, 1952.
- Maier, A., *Zwei Grundprobleme der scholastischen Naturphilosophie, Das Problem der intensiven Grösse, Die Impetustheorie*, 2. Aufl., Roma, Edizioni di storia e letteratura, 1951.
- Marcus Aurelius, *Pensées*, texte établi et traduit par Trannoy, A.I., préface de Puech, A., Paris, Les Belles Lettres, 1925, 2^o edition, 1964.
- Mates, B., *Stoic Logic*, Berkeley, Los Angeles, University of California Press, 1961.
- Mignucci, M., "Sur la logique modale des Stoiciens", *Les Stoiciens et leur logique*, ouvrage collectif, Paris, Vrin, 1978, pp. 317-346.
- "Ως ἐπὶ τὸ πολὺ et Nécessaire dans la Conception Aristotélicienne de la Science", *Aristotle on Science: the Posterior Analytics*, *Studia Aristotelica*, 9, ed. Antenore, Padova, 1981, pp. 173-203.
- Moody, E.A., *Truth and Consequences in Medieval Logic*, Amsterdam, North-Holland Publishing Co., 1953.
- Moreau, J., *Le sens du platonisme*, Paris, Les Belles Lettres, 1967.
- Mugler, Ch., "Sur quelques particularités de l'atomisme ancien", *Revue de Philologie*, N^o 79, 1953, pp. 141-174.
- *La physique de Platon*, Paris, Klincksieck, 1960.

- *Platon et la recherche mathématique de son époque*, van Berkhoven, Naarden, 1969 (Reprint of the P.H. Heitz edition published at Strasbourg in 1948).
- Nemesius, *De natura hominis*, graece et latine, Matthaei, Halae Magdeb., Gebauer, 1802.
- Ockham, W., *The Tractatus de "Praedestinatione et de Praescientia Dei et de Futuris Contingentibus"*, ed. Boehner, P., Franciscan Institute Publications, n° 2, St. Bonaventure, New York, 1945.
- *Predestination, God's Foreknowledge and Future Contingents*, translated by Adams and Kretzmann, New York, Appleton Century Crofts, 1969.
- Oesterle, J.T., *On Interpretation (Aristotle), Commentary by St. Thomas and Cajetan*, Milwaukee, Marquette University Press, 1962.
- Origen, *Contra Celsum*, Origenes, *Werke*, Band 1-2, ed. Koetschau, P., Leipzig, 1899-1913, 3 vol.
- Pauly, A., *Pauly's Real-Encyclopädie der classischen Alterthumswissenschaft*, Stuttgart, Metzler, 1893.
- Philoponus, *In Analytica priora commentaria*, ed. Wallies, CAG XIII 2, Berlin, 1905.
- Plato, *Oeuvres complètes*, tomes I-XIII, Paris, Les Belles Lettres, 1920-1964.
- Poincaré, H., *La Science et l'Hypothèse*, Paris Flammarion, 1968.
- *La valeur de la Science*, Paris, Flammarion, 1970.
- Popper, K., *La logique de la découverte scientifique*, traduction Rutten-Devaux, Paris, Payot, 1973.
- Plotinus, *Les Ennéades de Plotin*, par Bouillet, M.-N., Paris, Hachette, 1857, 3 vol.
- *Plotini Opera*, ed. Henry, P., Schwyzer, H.-R., Paris, Desclée de Brouwer, Bruxelles, L'Édition Universelle, 1951, 3 vol.
- *Ennéades*, éd. et trad. par Bréhier, E., Paris, Les Belles Lettres, 1964, 7 vol.
- Plutarch, *Plutarqui Moralia*, Leipzig, Teubner, tomes I-VIII, 1953-1974.
- *Plutarch's Moralia*, tome XIII, Parts I & II, english edition and translation by Cherniss, H., Cambridge London, Loeb, 1976, 2 vol.
- Prior, A.N., *Time and Modality*, Oxford, Clarendon Press, 1957.
- *Formal Logic*, Oxford, Clarendon Press, 2nd edition, 1962.
- *Past, Present and Future*, Oxford, Clarendon Press, 1967.
- Quine, W.v., *Word and Object*, Cambridge, Massachussetts, M.I.T. Press and John Wiley and sons, 1960.

- *The Ways of Paradox and other Essays*, New York, Random House, 1966.
- Rescher, N., Urquhart, A., *Temporal Logic*, Wien, New York, Springer, 1971.
- Robinson, R., “Plato’s consciousness of fallacy”, *Mind*, 1942.
- Robin, L., *La théorie platonicienne des idées et des nombres d’après Aristote, étude historique et critique*, Paris, Alcan, 1908.
- *Etudes sur la signification et la place de la physique dans la philosophie de Platon*, Paris, Alcan, 1919.
- *Pyrrhon et le Scepticisme grec*, Paris, P.U.F., 1944.
- *Platon*, Paris, Alcan, 1935, nouvelle édition mise à jour et complétée par Schuhl, P.-M., 1968.
- Rodis-Lewis, G., *Epicure et son école*, Paris, Gallimard, 1975.
- Russell, L.B., *The principle of Mathematics*, London, Allen-Unwin, 1956.
- *Logic and Knowledge*, ed. Marsch, R.C., London, Allen-Unwin, 1956.
- Sartre, J.-P., *Situations III*, Paris, Gallimard, 1949.
- Sedley, D., “Diodorus Cronus and Hellenistic philosophy”, *Proceedings of the Cambridge Philosophical Society*, 203, New Series, 23, 1977, pp. 74–120.
- Sextus Empiricus, *Opera*, English translation by Rev. R.G. Bury, London, Heinemann, W.; Cambridge, Massachusetts, Harvard University Press, 1967–1971, 4 vol.
- Schuhl, P.-M., *Le Dominateur et les possibles*, Paris, P.U.F., 1960.
- Simplicius, *In Aristotelis Physicorum libros quattuor posteriores commentaria*, CAG X, Berlin, Hermannus Diels, 1895.
- *In Aristotelis de Caelo commentaria*, CAG VII, Berlin, Heiberg, 1894.
- *In Aristotelis Categorias commentarium*, CAG VIII, Berlin, Kalbfleisch, 1907.
- Stobaeus, *Anthologii libri duo priores seu Eclogae physicae et ethicae*, Berlin, Wachsmuth, 1884.
- Storrs, Mc Call ed., *Polish Logic 1920–1939*, Oxford, Clarendon Press, 1967.
- Suppes, P., *Probabilistic Metaphysics*, Oxford, Blackwell, 1984.
- Tannery, P., *Mémoires scientifiques*, publ. par Heiberg & Zeuthen, Toulouse, Privat, Paris, Gauthier Villars, 1912–1950, 17 vol.
- Tarski, A., McKinsey, J.O., “Some theorems about Sentential calculi of Lewis and Heyting”, *Journal of Symbolic Logic*, XIII, 1948, pp. 13–14.

- Tarski, A., *Logics, semantics, metamathematics*, papers from 1923–1938, trad. Woodger, J.H., Oxford, Clarendon Press, 1956.
- Tatianus, *Tatiani oratio ad graecos*, ed. Schwartz, Heinrich, 1888.
- Taylor, A.E., *Plato, the Man and his Work*, London, Methuen, 2nd ed. 1969 (1926).
- Themistius, *In libros Aristotelis de Caelo Paraphrasis hebraice et latine*, CAG V, Berlin, Landauer, 1902.
- St. Thomas, *Expositio Libri Peryermenias*, in *Opera Omnia*, ed. Leonina, t. I*1, Roma-Paris, 1989 (see also Oesterle).
- *In Aristotelis libros de Caelo et Mundo, de Generatione et Corruptione, Meteorologicorum Exposito*, ed. Spiazzi, Turin-Rome, Marietti, 1952.
- *In duodecim libros Metaphysicorum Aristotelis Expositio*, ed. Cathala-Spiazzi, Turin-Rome, Marietti, 1964.
- *In octo libros Physicorum Aristotelis Expositio*, ed. Maggiolo, Turin-Rome, Marietti, 1965.
- Vlastos, G., “Minimal Parts in Epicurean Atomism”, *Isis*, vol. 56, part 2, n° 184, 1965, pp. 121–147.
- *Platonic Studies*, Princeton, Princeton University Press, 1973.
- Vuillemin, J., *Physique et métaphysiques kantiennees*, Paris, P.U.F., 1955.
- *Mathématiques et métaphysiques chez Descartes*, Paris, P.U.F., 1960.
- *De la logique à la théologie, Cinq études sur Aristote*, Paris, Flammarion, 1967.
- *Le Dieu d’Anselme et les apparences de la raison*, Paris, Aubier, 1971.
- “Kant aujourd’hui”, *Actes du Congrès d’Ottawa sur Kant*, octobre 1974, Ottawa University Press, 1975, pp. 17–35.
- “L’argument dominateur”, *Revue de métaphysique et de morale*, 1979, pp. 225–273.
- “La théorie kantienne des modalités”, *Akten des 5. Int. Kant Kongresses Mainz 4–8*. April 1981, Teil II, pp. 149–167, Bonn, Bouvier, 1981.
- “Le carré chrysippéen des modalités”, *Dialectica*, vol. 37, fasc. 4, 1983, pp. 235–247.
- “Les formes fondamentales de la prédication: un essai de classification”, *Recherches sur la philosophie et le langage*, Langage et Philosophie des sciences, Cahier n° 4, DREUG, Grenoble, Paris, Vrin, 1984, pp. 9–30.
- *What are philosophical systems?* Cambridge University Press, 1986.

- Wajsberg, M., "Untersuchungen über den Aussagenkalkül von A. Heyting", *Wiadomosci matematyczne*, vol. 46, 1938, pp. 45–101.
- Williams, D.C., "The Sea Fight Tomorrow", *Structure, Method and Meaning*, ed. Henle, P., New York, 1951.
- Windelband, W., Heimsoeth, H., *Lehrbuch der Geschichte der Philosophie*, Herausg. v. Heimsoeth, Tübingen, JCB Mohr, 1948.
- Wright, G.H. von, *An Essay in Modal Logic*, Amsterdam, North Holland Publ. Co, 1951.
- "The Master Argument of Diodorus", *Essays in Honour of Jaakko Hintikka* (ed. Saarinen, Hilpinen, Niiniluoto and Hintikka, Dordrecht, Reidel), 1979, pp. 297–307.
 - "Time, Truth and Necessity", *Intention and intentionality*, Essays in Honour of Anscombe, G.E.M., ed. by Diamond, C., and Teichmen, J., Brighton, The Harvester Press, 1979, pp. 237–250.
 - "Diachronic and Synchronic Modalities", *Theorema*, vol. IX/3–4, 1979, pp. 231–245.
- Zeller, E., "Ueber den κυριεύων des Megarikers Diodorus", *Kleine Schriften*, I, 1910.

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